International Journal of Aquatic Science
ISSN: 2008-8019
Vol 12, Issue 03, 2021

On Non-Associative Algebra And Its Properties

Nosirov Sobir Nosirovich[1], Aroev Dilshod Davronovich[2],
Sobirov Avazbek Abdurashid ugli[3]

[1]Kokand State Pedagogical Institute candidate of physical and mathematical sciences, Senior Lecturer
[2]Kokand State Pedagogical Institute PhD, Senior Lecturer
[3]Kokand State Pedagogical Institute Student, Methods of Teaching Mathematics

E-mail: [1]nosirov47@bk.ru, [2]dilshod_aroyev@mail.ru , [3]avazbeksobirovqdpi@gmail.com

Abstract :In this article, we construct a class of non-associative algebras and study their properties. On this case, research has been analyzed both methodological and theoretical aspects of the of non-associative algebras with the properties of the research database. Hence, author makes conclusions with recommendations for the further development prosperity and investigations on non-associative algebras and study their properties

Key words: mapping, isomorphic mapping, permutation, cycle, cycle length

1. INTRODUCTION.

Let $M=\{1,2,3,...,n\}$ be a set and $F:M\to M$ a reflection of a mutual value. The addition between the elements $(x_1,x_2,...,x_n)=x(t)$ and $(y_1,y_2,...,y_n)=y(t)$ of the n-dimensional R_n arithmetic space forms a linear space with respect to the multiplication operations $(\lambda x)(t)=(\lambda x_1,\lambda x_2,...,\lambda x_n)$ for the numbers
$(x+y)(t)=(x_1+y_1,\ x_2+y_2,...,x_n+y_n)$
and $\lambda\in R$.
If we determine the multiplication of the elements $x(t)$ and $y(t)$ in the space R_n by the coordinates, i.e. $(x\cdot y)(t)=(x_1y_1,\ x_2y_2,...,x_ny_n)$, then the set R_n is an algebra whose color (size) is n over the real number field R, that is, the set R_n satisfies the conditions of linear space (8 conditions) and ring (6 conditions). These conditions are as follows [1-9]:
1) + action is associative,
2) + action is commutative,
3) there is a neutral element to the + action,
4) there is an opposite element for each element in R_n relative to the + operation,
5) For $\lambda\in R,\ x(t)\in R_n,\ y(t)\in R_n$ elements the $\lambda(x+y)(t)=\lambda x(t)+\lambda y(t)$ equation is fulfilled.
6) $\lambda_1\mu\in R$ numbers and $x(t)\in R_n$ element for $(\lambda+\mu)x(t)=\lambda x(t)+\mu y(t)$.
7) $\lambda,\mu\in R$ numbers and $x(t)\in R_n$ element for $(\lambda\cdot\mu)x(t)=\lambda\cdot(\mu)x(H)=\mu(\lambda\cdot x(t))$.

8) $(1\cdot x)(t)=x(t)$.

Theoritical background

The above 8 conditions are linear space conditions, the first 4 of which are repeated in the ring conditions, and 2 more of the following conditions are studied.

1) The practice of multiplication is associative, i.e. $((x\cdot y)\cdot z)(t)=(x\cdot(y\cdot z)(t))=x(t)\cdot y(t)\cdot z(t)$.

2) $x,y,z\in R_n$ for elements $((x+y)z)(t)=(x\cdot z+yz)(t)=$ $=x(t)z(t)+y(t)z(t)$; $(x(y+z))(t)=(x\cdot y+xz)(t)=x(t)y(t)+x(t)z(t)$ equations are reasonable.

Furthermore, the product of the two elements in R_n is commutative, and the $e(t)=(1,1,...,1)$ element is a unit element relative to the multiplication operation [2].

Hence, the set R_n forms a commutative loop with a unit element relative to the specified operations.

If K is a circle and its part set satisfies conditions

a) $x,y\in J\Rightarrow x+y\in J$

b) $x\in J, z\in K\Rightarrow x\cdot z\in J$

for $J\subset K$, then the set J is called the ideal of the K circle. The ideals in the K-circle allow us to determine the structure of this circle. The more ideals in the circle, the more complex the circle.

Main part

Determining all the ideals in the R_n circle is not complicated. For example, the set of all elements of the form $(0,0,a_3,a_4,...,a_n)$ with the first two coordinates equal to 0 would be the ideal of the R_n circle. Other ideals of this circle will also consist of elements whose assigned coordinates are 0s.

If one ideal of a circle is not part of another ideal, it is called a maximum ideal. All maximum ideals of the R_n circle will consist of elements with exactly one coordinate 0. However, any ideal of the R_n circle consists of some maximal ideals intersection.

Thus it is possible to determine the construction of all the ideals of the R_n circle.

The result.

If we leave the + operation in the set R_n unchanged and determines the multiplication operation in it by reflecting $F:M\rightarrow M$ with the following equation, $(x+y)(t)=(x_{F_1}\cdot y_{F_1},x_{F_2}\cdot y_{F_2},...,x_{F_n}\cdot y_{F_n})$ in which case this action does not satisfy the associative condition, i.e., it forms an R_n-non-associative circle. In this case, the ideals of the R_n circle are in a different view. If the reflection $F:M\rightarrow M$ is of a single value rather than a reciprocal value, the ideals of the R_n circle become more complex.

We give examples to make the non-associative multiplication operation introduced using F reflection understandable. Let the set $M=\{1,2,3,4,5\}$ and R_5 consist of elements of the form $x(t)=\{x_1,x_2,x_3,x_4,x_5\}$. Let $F:M\to M$ be determined by the reflection $F=\begin{pmatrix}1&2&3&4&5\\2&1&4&5&3\end{pmatrix}$.

We show that the *-production of $x,y,z\in R_5$ elements does not satisfy the associative condition.

$$(x*y)\times z=(x_{F1}\cdot y_{F1},x_{F2}\cdot y_{F2},x_{F3}\cdot y_{F3},x_{F4}\cdot y_{F4},x_{F5}\cdot y_{F5})*Z=$$
$$=(x_2y_2,x_1y_1,x_4y_4,x_5y_5,x_3y_3)*(z_1,z_2,z_3,z_4,z_5)=$$
$$=(x_1y_1\cdot z_2,x_2y_2\cdot z_1,x_5y_5\cdot z_4,x_3y_3\cdot z_5,x_4y_4\cdot z_3)$$
$$x*(y*z)=x*(y_{F_1}z_{F_1},y_{F_2}z_{F_2},y_{F_3}z_{F_3},y_{F_4}z_{F_4},y_{F_5}z_{F_5})=$$
$$=x*(y_2z_2,y_1z_1,y_4z_4,y_5z_5,y_3z_3)=$$
$$=(x_2y_1z_1,x_1y_2z_2,x_4y_5z_5,x_5y_3z_3,x_3y_4z_4).$$

So, $x*(y*z)\neq(x*y)*z$

Let F denote the non-associative circle R_n defined by the reflection in the form $R_n(F)$.

The fact that the writing of the $R_n(F)$ circle ideals is fully defined when the reflection F is mutually exclusive can be expressed as the product of the reflections F in the reflection case, the $R_n(F)$ circle ideals depend on the number of cycles.

For example, If $M=\{1,2,3,4,...,10\}$, $F:M\to M$ is given by the equation

$$F=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10\\2&1&3&8&6&4&5&7&10&9\end{pmatrix},$$

then F is equal to the product of 4 cycles, i.e. $F=(1,2)(3)(48756)(910)$, where the lengths of the first and last cycles are 2, the length of the second cycle is 1, and the length of the third cycle is 5.

All sets of $x(t)\in R_n(F)$ functions that take a value of 0 in one or more cycles would be the ideal of the $R_n(F)$ loop. For example, if the reflection in the $F=(1,2)(3)(48756)(9\,10)$ view the sets of

$$J_1=\left\{(0,0,a_3,a_4,...,a_{10})\,\middle|\,a_i\in R\ i=\overline{3,10}\right\};\ J_2=\left\{(0,0,0,a_4,a_5,...,a_{10})\,\middle|\,a_i\in R\ i=4,10\right\}$$

and so on would be the ideal of the $R_n(F)$ circle.

They can be written for each cycle or multiplication of cycles. In this example, there are 4 ideals of the $R_n(F)$ circle, 4 of which are maximal ideals. By adding to these ideals the inherent ideals of the form $J_0=\{(0,0,...,0)\}$ and $J_n=R_n(F)$, we find all the ideals of the $R_n(F)$ circle when $n=10$ and $F=(12)\,(3)\,(4\,8\,7\,5\,6)(9\,10)$ are present.

2. CONCLUSION.

Below we determine that all the ideals of the $R_n(F)$ circle are written when F a one-valued reciprocal reflection is.

Theorem 1. All elements set in the $R_n(F)$ circle that are equal to 0 in at least one cycle constitutes the ideal of this circle, and conversely, any ideal of the $R_n(F)$ circle consists of elements that assume a value of 0 in one or more cycles.
Proof. The first part of the theorem is proved to be simple, i.e., for a set of elements J that takes a value of 0 in one or more cycles:
1) $x, y \in J$ is $x + y \in J$
2) $x \in J$, $z \in R_n(F)$ is $x * z \in J$ the relationship is directly proven to be reasonable.
Suppose that in proving the second part of the theorem, $F = (i_1, ..., i_k)...(s_1, s_2, ..., s_p)$ consists of the cycles product. Let $J \subset R_n(F)$ be an arbitrary ideal of the circle. Suppose $J \neq \{(0,...,)\}$ and $J \neq R_n(F)$.
If we can show that in the specific ideal J lie $R_n(F)$ all the elements of the set $l_1 = (1,0,...,0)$, $l_2 = (0,1,0,...,0)$, ..., $l_2 = (0,...,0,1)$ in appearance (i.e. the arithmetic basis) lie, then the equation $J = R_n(F)$ is obtained. To prove the theorem, we assume the inverse, i.e., that $J \neq \{(0,...,0)\}$ $J \neq R_n(F)$ and J ideally have an element that takes a value different from 0 at least one point of each cycle.
Suppose that the element whose first coordinate of the first cycle of reflection F is different from 0 belongs to $a = (a_1, a_2, ..., a_n)$ J, let $a_1 \neq 0$. Without limiting the generality, $a_1 \neq 1$ can be obtained. To simplify the notation, we call F the first cycle of reflection (1,2, ..., k). In this case, the ideal J contains an element of the form $a = (1, a_2, ..., a_5, ..., a_n)$, and the element formed by multiplying it by the element $l_1 = (1,0,...,0)$ belongs to the ideal $a - l_1 = (0,1,0,...,0)$.
After multiplying this element by itself exactly k times, it is equal to 1 at only one point of the first cycle and 0 at other cycle points It follows that the elements in figure
$l_1 = (1,0,...,0,0,...,0)$; $l_2 = (0,1,0,...,0,0,...,0)$
$l_3 = (0,0,1,0,...,0,0,...,0)$; $l_k = (0,...,0,1,0,0,...,0)$
belong to the ideal J.
If the same process performed with the first loop is performed for other cycles of reflection F, we obtain that the ideal J corresponds to all the elements of the $R_n(F)$ circle called the arithmetic basis. This contradicts the hypothesis because the $J \neq R_n(F)$ condition existed.
Hence, all ideals of the $R_n(F)$ circle consist of elements equal to 0 in one or more cycles of reflection F. Within these ideals, however, a set of elements equal to 0 at all points in a single cycle will be the maximum ideals.
Any ideal of $R_n(F)$ circle is in a product (intersection) form of some of its maximum ideals.
For a finite set M and a reciprocal of $F: M \to M$ and $G: M \to M$ reciprocal values, if there is a reciprocal reflection $H: M \to M$ satisfying the equation $F = H^{-1}GH$, then the reflections F and G are said to be similar.

For example, reflections $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 4 & 6 & 7 & 5 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 3 & 4 & 2 & 7 & 5 & 6 \end{pmatrix}$ are similar because $F=(123)(4)(56\,7)$ has 3 cycles in reflection, they are the lengths of the first and third equal to 3 and the lengths of the second equal to 1, as well as, $G=(1)(234)(567)$ reflection , there are 2 cycles with 3 length and 1 with 1 length.

In this case, the corresponding satisfactory H reflection $F=H^{-1}GH$ can be constructed as follows

$$H=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 1 & 2 & 3 & 5 & 7 & 6 \end{pmatrix}$$

Then

$$H^{-1}=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 4 & 1 & 5 & 7 & 6 \end{pmatrix}$$

$F=H^{-1}GH$ where $F=H^{-1}GH$ is the equation.

It should be noted here that the H reflection that satisfies the $F=H^{-1}GH$ equation can be constructed in several ways. To do this, it is necessary to find cycles of the same length in the reflection F and G, construct mutually equal reflections between the elements included in this cycle, and combine these reflections.

Theorem 2. For the $R_n(F)$ and $R_n(G)$ non-associative circles constructed with F and G reflections to be isomorphic, the F and G reflections must be similar and sufficient [2].
Let $M=\{1,2,3,...,n\}, \quad n>1$ be a value reflection of set $F:M\to M$.

The points i $i_s(s=\overline{1,k})$ that satisfy the equations $F(i_1)=F(i_2)=...=F(i_k)$ in the set M are called adjacent points. If F is not mutually exclusive, then of course there will be adjacent points. Similarly, there are $t_1,t_2,...,t_p$ points that satisfy the $F(t_1)=F(t_2)=...=F(t_p)=t_1$ equations, which are called F reflection cycles. The number of F reflection cycles can also be more than one.

They are called F reflection cycles. The number of F reflection cycles can also be more than one.
For example,

$$A=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 2 & 3 & 4 & 2 & 5 & 7 & 8 & 9 & 10 & 10 \end{pmatrix};\ F(1)=F(4)=2;\ F(9)=F(10)=10$$

equations are fulfilled, then points 1 and 4 and 9, 10 will be adjacent points. (2 3 4) and 10 points form loops.

Theorem 3. All elements set that assume 0 value in reflection cycle F in the $R_n(F)$ circle is the maximum ideal of the $R_n(F)$ circle.

Proof. Let F be a reflection cycle $i_1,i_2,...,i_k$ points, that is, let the $F(i_1)=F(i_2)=...=F(i_k)=i_1$ equations be satisfied. Let J denote the set of all elements that assume 0 value at these points. The product of 2 arbitrary Z elements in X and R of 2 elements belonging to J

$$(x*z)(t) = x(F(t))Z(F(t))$$

again belongs to J, because F reflects the cycle elements again to the cycle elements, and $x(F(t))$ assumes a 0 value when the function t belongs to the cycle. Hence, the function $(x*z)(t)$ is equal to 0 when t belongs to the cycle. Thus, the set J becomes the ideal of the circle $R_n(F)$.

We now show that this ideal is the maximum ideal, i.e., that the ideal J does not lie within the ideal other than the R circle. To do this, we assume the opposite, that is, that such an ideal $J_1 \subset R_n(F)$ exists and those conditions $J \subset J_1$ and $J \neq J_1$ are satisfied. Under this condition, the function $f(t)$ J_1, which takes a value greater than 0 at least one point of the $i_1, i_2, ..., i_k$ cycle, belongs to the ideal. Let the function $f(t)$ be equal to 1 at point i_1 of the cycle without limiting the generality.

If we multiply this function by the operation * in the $R_n(F)$ circle to the function that takes 0 from all other points at point i_1 in the circle $R_n(F)$, we show that at point i_{k-1} of the cycle the function 1 and at all other points of M 0 also belong to the ideal J_1. And so on, if we repeat this process $k = 2$ times, it follows that the ideal J_1 corresponds to k elements of the arithmetic base, which is equal to 1 at 1 point of the $i_1, i_2, ..., i_k$ cycle and 0 at other points. It follows from the condition $J \subset J_1$ that the elements J_1 which at the $i_1, i_2, ..., i_k$ cycle points of the set M take a value of 0 and at each other point (they are $n-k$) 1 belong to the ideal.

Thus, we have created that the

$$l_1 = (1,0,...,0), \quad l_2 = (0,1,0,...,0), ..., \quad l_n = (0,...,0,1)$$

vectors, consisting of the arithmetic basis of the $R_n(F)$ circle, correspond to the J_1 ideal. This indicates that $J_1 = R_n(F)$ is, i.e., J is the maximum of the ideal.

Taking any point S_1 of the set M, the sequence of points $F(S_1), F(F(S_1)) = F^2(S_1),...$ is followed by the equation $F^p(S_1) = F^q(S_1)$ after one p steps, here $q \leq p$. If point S_1 belongs to any cycle in M, then a sequence of points belonging to the cycle follows.

If point S_1 does not belong to any cycle, then $S_1, \ F(S_1), F^2(S_1),..., \ F^{p-1}(S_1)$ iterative sequence is formed.

It can be proved that for any point t obtained from M, the set of all elements assuming a value of 0 at all points belonging to the iterative sequence $t, F(t),...,F^p(t)$ is the $R_n(F)$ circle ideal. This ideal is part of the maximum ideal generated by the cycle that belongs to this iterative sequence. We call the ideals created by this method the type I ideal of the $R_n(F)$ circle.

For example, if a reflection of the form

$$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 2 & 5 & 7 & 8 & 9 & 9 \end{pmatrix}$$

is given

$1, F(1)=2, \quad F^2(1)=3, \quad F^3(1)=4, \quad F^4(1)=2$

equations are satisfied, elements 1,2,3,4 are an iterative sequence to which the cycle (2,3,4) belongs. The $M=\{1,2,3,4,5,6,7,8,9\}$ set consists of 5 element loops, as well as 9 element loops. Elements 6,7,8,9, as well as elements 7,8,9 and 8,9 are iterative sequences.

Theorem 4. If reflection $F:M \to M$ is given and points $i_1, i_2, ..., i_s$ are adjacent points relative to reflection F, then the elements of the $R_n(F)$ circle that satisfy the condition $a_1 t_{i_1} + a_2 t_{i_2} + ... + a_5 t_{i_5} = 0$ for any real number satisfying the condition $a_1 + a_2 + ... + a_5 = 0$ (at least one $a_i = 0$) constitute its ideal.

Proof. To simplify the notation, suppose that points 1,2,3 are adjacent points to reflect F. Let the equation $a_1, a_2, a_3 = 0$ be satisfied for a_1, a_2, a_3 numbers, one of which is different from 0. We show that a set of $x(t)=(t_1, t_2, t_3, ..., t_n)$ elements satisfying condition $a_1 t_1 + a_2 t_2 + a_3 t_3 = 0$ makes J ideal. We take another $y(t)=(u_1, u_2, u_3, ..., u_n)$ element belonging to J, add them, and make

$(x+y)(t)=(t_1+u_1, t_2+u_2, t_3+u_3, ..., t_n+u_n)$

for the $a_1(t_1+n_1)+a_2(t_2+u_2)+a_3(t_3+u_3)=0$ element, because the $x, y \in J$ relation is reasonable. Since points 1,2,3 are adjacent points, the $F(1)=F(2)=F(3)$ equations are satisfied. The product of $x \in J$ and $f \in R_n(F)$ elements assumes the same value at points $(x*t)(t)$ 1,2,3, i.e. $(x*1)(1)=(x*f)(2)=(x*t)(3)$;

because

$x(F(1)) \cdot f(F(1)) = x(F(2)) \cdot f(F(2)) = x(F(3)) \cdot f(F(3))$;

equations are satisfied. If we set this value to b, based on equality

$(x*f)(1)=(x*f)(2)=(x*f)(3)=b$

we create an equation

$a_1 b + a_2 b + a_3 b = b(a_1 + a_2 + a_3) = 0$

Therefore, a $(x*f)(t) \in J$ relationship is appropriate, i.e. the J set is ideal.

We call the ideals of the $R_n(F)$ circle determined by theorem 4 the type II ideal.

3. REFERENCES.

[1] S.N.Nosirov, D.D.Aroev. About a certain class of non-associative algebra/ Republican scientific conference on "Actual problems of stochastic analysis", dedicated to the 80th anniversary of Academician Sh. Farmonov. Tashkent, February 20-21, 2021, 430-432-p.

[2] A.G. Kurosh / Course of Higher Algebra. The science, M.: 1975.

[3] S.N. Nasirov / Description of the ideals of a class of non-associative algebras. Scientific works of Tashkent State University, issue 418,1972. –p. 232-236.

[4] S.N. Nasirov "On the ideals of a class of finite-dimensional non-associative algebras" Proceedings of the Tashkent State University, issue 461,1974. –p 86-89.

[5] I.M. Gelfand "Lectures on linear algebra" Science, M. 1966.

[6] B.L. Van der Waerden "Modern Algebra" Gostekhizdat, M. 1947
[7] A.G. Kurosh "Lectures on General Algebra" Science, M. 1962.
[8] N.L. Jacobson "Built of circles" IL. 1961.
[9] A. Cardan, S. Eilenberg "Homological Algebra" Il. 1960.
[10] G. Weil "Algebraic number theory" IL. 1949.