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R – Number Of Some Families Of Graphs

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Abstract - Let G(V(G),E(G)) be a simple connected graph. An injective function $f : V(G) \rightarrow \{1, 2, 3, ...\}$ is said to be an R-labeling if it satisfies the following conditions: $|f(u) - f(v)| \geq 2$, if $d(u, v) = 1$; $|f(u) - f(v)| \geq 1$, if $d(u, v) = 2$, for any two distinct vertices $u, v \in V(G)$. The span of an R−labeling, f, is the largest integer in the range of f and is denoted by f_R. The R− number, R(G) or R of G is the minimum span taken overall R−labelings of G. In this paper, we determine the R – number of some families of graphs.

Keywords — R – number, span, λ-number.

1. INTRODUCTION

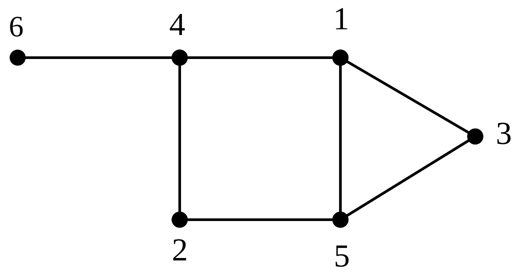
In this paper, we consider only simple, connected, undirected and finite graphs. For basic notations and terminology, we follow [6]. Let $G = (V, E)$ be a simple connected graph. The *distance* $d(u,v)$ between u and v, is the length of a shortest $(u,v)-$ path in G. For any vertex $u \in V$, the *eccentricity*, $e(u)$, of u is the distance of a vertex farthest from u. The *radius* of a graph G is the minimum eccentricity among all the vertices and is denoted by $rad(G)$. The *diameter* of G is the maximum eccentricity among all the vertices and is denoted by $diam(G)$.

For a subset S of V, let $< S >$ denote the induced subgraph of G induced by S. By a *clique* C we mean a maximal subset of V such that $< C >$ is complete. The *clique number* of a graph G, denoted by ω, is the number of vertices in a clique of maximum order in G. The concept of splitting graph was introduced by Sampath Kumar and Walikar[10]. The *splitting graph* of G is obtained by adding a new vertex w for every vertex $v \in V$ and joining w to all vertices of G adjacent to v and is denoted by $S(G)$. The *cosplitting graph* [1], $CS(G)$ is obtained from G, by adding a new vertex w for each vertex v and joining w to all vertices which are not adjacent to v in G. The distance between the vertices in splitting and cosplitting graphs has been discussed in [2].

In 1960's Rosa[9] introduced the concept of graph labeling. A *graph labeling* is an assignment of number to the vertices or edges or both, satisfying some constraint. Rosa named the labeling introduced by him as $\beta-$*valuation* and later on it becomes a very famous interesting graph labeling called *graceful labeling*, which is the origin for any graph labeling problem. Motivated by real life problems, many mathematicians introduced various labeling concepts[7]. Here, we see one of the familiar graph labelings in graph theory.

Let *G(V(G),E(G))* be a graph. A radial radio labeling, *f*, of a connected graph *G* is an assignment of positive integers to the vertices satisfying the following condition: $d(u,v)+|f(u)-f(v)|\geq 1+r$, for any two distinct vertices $u,v\in V(G)$, where $d(u,v)$ and r denote the distance between the vertices u and v and the radius of the graph G, respectively. The span of a radial radio labeling f is the largest integer in the range of f and is denoted by *span f*. The radial radio number of *G*, $rr(G)$, is the minimum span taken over all radial radio labelings of *G*.

For example, a graph *G* and its radial radio labeling are shown in Figure 1.1.



G
Figure 1.1

Here, $rad(G)$=2 and $rr(G)$=6.

The radial radio number of any simple connected graph has been studied in [3], [4], [5] and [11].

Given a simple conneced graph G(V(G), E(G)), an $L(2,1)$ - labeling of G is a function $f: V(G)\to\{0,1,2,3,...\}$ such that $|f(u) - f(v)|\geq 2$, if $d(u,v) = 1$ and $|f(u) - f(v)|\geq 1$, if $d(u,v) = 2$. The $L(2,1)$ – labeling number $\lambda(G)$ is the smallest k such that G has an $L(2,1)$ – labeling with $\max\{f(v): v\in V(G)\} = k$. Inspired by the concept of distance 2 labeing introduced by Griggs [8], we introduce a new concept called *R- labeling* which is defined as follows:

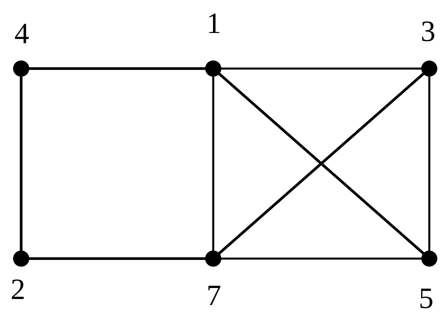
An injective function $f: V(G)\to\{1,2,3,...\}$ is said to be an *R-labeling* if it satisfies the following conditions for any two distinct vertices $u,v\in V(G)$:

$|f(u)-f(v)|\geq 2$, if $d(u,v)=1$

$|f(u)-f(v)|\geq 1$, if $d(u,v)=2$

The *span* of an R−labeling, f, is the largest integer in the range of f and is denoted by f_R. The $R-$ number $R(G)$ of G is the minimum span taken over all $R-$ labelings of G. That is, $R(G)=\min_f f_R$, where the minimum runs over all $R-$ labelings of G.

For example, consider the graph G. One of the R – labelings of G is shown in Figure 1.2.



G

Figure 1.2

The relationship between the radial radio number and the R – number of any given simple graph has been established in [12].

Let G_1 and G_2 be any two graphs. The *join* of two graphs G_1 and G_2 is the graph whose vertex set is $V_1 \cup V_2$ and the edge set is $E_1 \cup E_2 \cup \{uv : u \in V_1, v \in V_2\}$ and is denoted by $G_1 + G_2$. The *union* of two graphs $G_1$and G_2 is the graph whose vertex set is $V_1 \cup V_2$ and the edge set is $E_1 \cup E_2$and is denoted by $G_1 \cup G_2$.

Let $H_{n,n}$ be the graph with vertex set $\{v_1, v_2, ..., v_n; u_1, u_2, ..., u_n\}$ and the edge set $\{v_i u_j : 1 \le i \le n, n-i+1 \le j \le n\}$.

The ladder graph $P_n \times K_2$is denoted by L_n, where $V(L_n) = \{v_i, u_i : 1 \le i \le n\}$ and $E(L_n) = \{v_i v_{i+1}, u_i u_{i+1} : 1 \le i \le n\} \cup \{v_i u_i : 1 \le i \le n\}$

For further details on R – number, one can refer [13] and [14].

I. Some Basic Results

Throughout this section, assume that G is a non trivial simple connected graph on n vertices.

Now, we present some basic traits of $R-$ number.

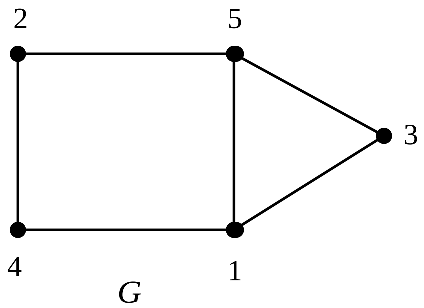
We note that, since the corresponding R – labeling is one to one, no two vertices can get the same label. This forces that, $R(G) \ge n$, for any graph G of order n. Also, this bound is sharp for P_n, $n \ge 4$ and C_n, $n \ge 5$.

We prove these two results later in this section.

Fact 2.1. *If ω is the clique number of G, then $R(G) \ge 2\omega - 1$.*

For, we have, the label difference between adjacent vertices is at least 2. To label the vertices of K_ω, we definitely need the labels 1, 3, ..., $2\omega - 1$. Thus $R(G) \ge 2\omega - 1$.

Remark 2.2. The bound in Fact 2.1 is sharp for the graph shown in Figure 2.1.



G

Figure 2.1

Here, $\omega = 3$ and $R(G) = 5$.

Fact 2.3. *For any graph G* of order *n,*

$n \le R(G) \le 2n - 1$.

For, the lower bound for $R(G)$ is trivial. Also, if G has n vertices, then by assigning the labels 1, 3, ... , $2n - 1$ to the vertices of G, we get an R – labeling of G. This assures that upper bound for $R(G)$ to be $2n - 1$.

The upper bound attained in Fact 2.3 is sharp for the complete graphs K_n, where $n \ge 3$.

In fact K_n is the only graph for which the R – number is $2n – 1$. We prove this in the following theorem.

Theorem 2.4. *Let G be any graph of order n. Then* $R(G) = 2n - 1$ *if and only if* $G \cong K_n$.

Proof

Suppose $G \cong K_n$.

Let $V(G) = \{v_i : 1 \le i \le n\}$. Then define $f : V(G) \to \{1, 2, 3, ..\}$ such that $f(v_i) = 2i - 1$, $1 \le i \le n$. Since $d(v_i, v_j) = 1$, for all $1 \le i \ne j \le n$, we have $|f(v_i) - f(v_j)| \ge 2$. This forces that, f is an $R-$ labeling for G and $f_R = 2n - 1$ Thus $R(G) \le 2n - 1$. But the clique number ω of G is n. Therefore, Fact 2.1 implies that, $R(G) \ge 2n - 1$. Hence $R(G) = 2n - 1$.

Conversely, assume that, $R(G) = 2n - 1$. To show that, $G \cong K_n$. On contrary, assume that, G is not isomorphic to K_n. Then the clique number of G, $\omega \le n - 1$. If $\omega = n - 1$, then for any $R-$ labeling f of G, we have $f_R = \max_{v \in V(G)} f(v) \le 2(\omega - 1) + 1 < 2n - 1$. This forces that, $R(G) < 2n – 1$, which is a contradiction. Hence G must be isomorphic to K_n. ■

Fact 2.5. *If H is a subgraph of G, then* $R(H) \le R(G)$.

For, let f be an R – labeling of G. Then the restricted function $f|_{V(H)}$ is an $R-$ labeling of H. This implies that, $R(H) \le R(G)$.

Now, we turn our attention to find the R – number of paths. One can easily, check that, $R(P_2)=3$, $R(P_3)=4$. For, $n \ge 4$, we determine $R(P_n)$ in the following Theorem.

Theorem 2.6. *For* $n \ge 4$, $R(P_n) = n$.

Proof

Let $V(P_n) = \{v_i : 1 \le i \le n\}$ and let $E(P_n) = \{v_i v_{i+1} : 1 \le i \le n-1\}$. From Fact 2.3, we get $R(P_n) \ge n$, for $n \ge 4$. Now, we will show that, for $n \ge 4$, $R(P_n) \le 4$.

Case 1 Let $n = 2m$, $m \ge 3$, be an even integer

In this case, define $f : V(P_n) \to \{1, 2, 3, ..\}$ such that $f(v_{2i-1}) = i, 1 \le i \le m$; $f(v_{2i}) = m + i, 1 \le i \le m$. Here, for the adjacent pair of vertices (v_{2i-1}, v_{2i}), $1 \le i \le m$, we have $|f(v_{2i-1}) - f(v_{2i})| \ge m > 2$. Also, for the pair (v_i, v_j), $1 \le i \ne j \le n$ with $d(v_i, v_j) = 2$, we have $|f(v_{2i-1}) - f(v_{2i})| \ge 1$. Thus f is an $R-$ labeling of P_{2m}, where $m \ge 3$. This implies that, $f_R = 2m$ and hence $R(P_{2m}) \le 2m$.

Case 2 Let $n = 2m + 1$, $m \ge 2$, be an odd integer

Define $f : V(P_n) \to \{1, 2, 3, ..\}$ such that $f(v_{2i-1}) = i$, $1 \le i \le m + 1$; $f(v_{2i}) = m + i + 1$, $1 \le i \le m$. Proceed as in Case 1, we can prove that f is an $R-$labeling for P_{2m+1}, $m \ge 2$. This gives that, $f_R = 2m + 1 = n$. Hence $R(P_{2m+1}) \le 2m + 1$. This completes the proof. The R – labelings of the paths P_{10} and P_{11} are presented in Figure 2.2. ■

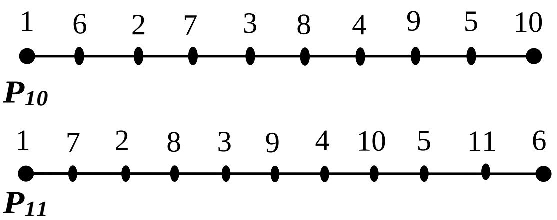


Figure 2.2

Next, we focus on cycles. We can easily verify that, $R(C_3) = 5$ and $R(C_4) = 5$. Now, for $n \ge 5$, we estimate $R(C_n)$ in the following Theorem.

Theorem 2.7. *For* $n \ge 5$, $R(C_n) = n$.

Proof

Let $V(C_n) = \{v_i : 1 \le i \le n\}$ and let $E(C_n) = \{v_1v_n, v_iv_{i+1}\}$.

By Fact 2.3, we have $R(C_n) \ge n$, for $n \ge 5$. It is enough to show that, $R(C_n) \le n$, for $n \ge 5$.

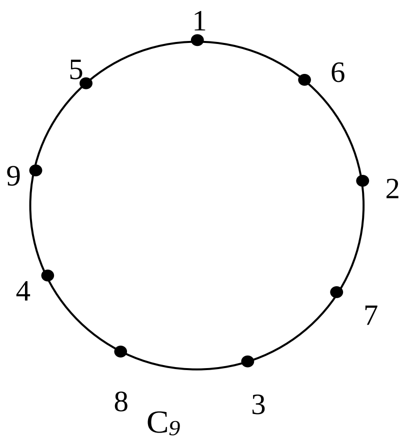
Case 1 Let $n = 2m$, $m \ge 3$ be an even integer.

Here, define $f : V(C_n) \to \{1, 2, 3, ..\}$ such that $f(v_{2i-1}) = i$, $1 \le i \le m$; $f(v_{2i}) = m+i$, $1 \le i \le m$. Since $f(v_i) \ne f(v_j)$, for every i, j, $1 \le i \le j \le n$, f is one to one. We observe that $|f(v_i) - f(v_j)| \ge 2$, if $d(v_i, v_j) = 1$ and $|f(v_i) - f(v_j)| \ge 1$, if $d(v_i, v_j) \ge 2$. This forces that, f is an $R-$ labeling of C_n and $f_R = 2m$ and thus $R(C_n) \le 2m$.

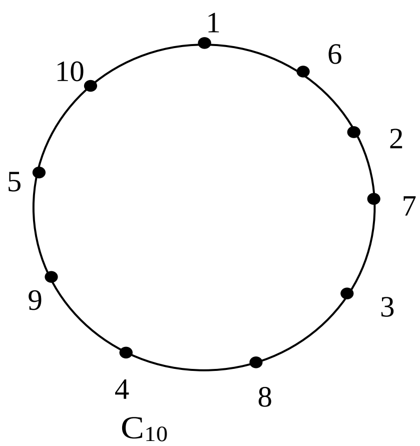
Case 2 Let $n = 2m + 1$, $m \ge 2$, be an odd integer.

If we define $f : V(P_n) \to \{1, 2, 3, ..\}$ such that

$f(v_{2i-1}) = i$, $1 \le i \le m+1$; $f(v_{2i}) = m + i + 1$, $1 \le i \le m$, then as in Case 1, we can prove that, f is an $R-$ labeling of C_n and $f_R = 2m + 1$. Thus $R(C_n) \le 2m + 1$. This completes the proof. The R – labelings of the cycles C_9 and C_{10} are given in Figure 2.3. ■



C_9



C_{10}

Figure 2.3

Theorem 2.8. *For* $n \ge 2$, $R(H_{n,n}) = 2n$.

Proof

Let $V(H_{n,n}) = \{v_i, u_j : 1 \le i, j \le n\}$. By Fact 2.3, we have $R(H_{n,n}) \ge 2n$. It is enough to show that, $R(H_{n,n}) \le 2n$.

Define $f : V(P_n) \to \{1, 2, 3, ..\}$ such that $f(v_i) = i$, $1 \le i \le n$; $f(u_j) = n + j$, $1 \le j \le n$. Here, we observe the following:

$|f(v_i) - f(v_j)| \ge 1$, for every i, j, $1 \le i \ne j \le n$;

$|f(u_i) - f(u_j)| \ge 1$, for every i, j, $1 \le i \ne j \le n$;

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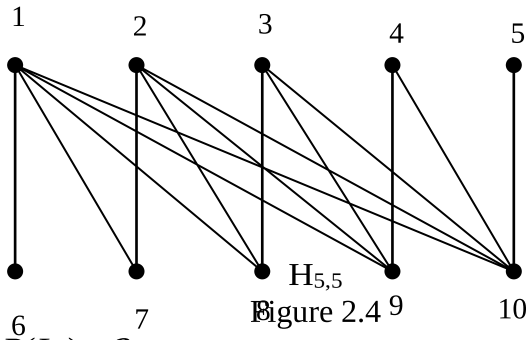
$|f(v_i) - f(u_j)| \geq 2$, for every $i, j, 1 \leq i, j \leq n$.

Thus f is an $R-$ labeling of $H_{n,n}$ and $f_R = 2n$ and hence $R(H_{n,n}) \leq 2n$. This completes the proof. An R – labeling of $H_{5,5}$ is given in Figure 2.4. ■



$H_{5,5}$

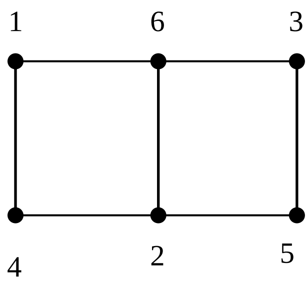
Figure 2.4

Theorem 2.9. *For* $n \geq 3$, $R(L_n) = 2n$.

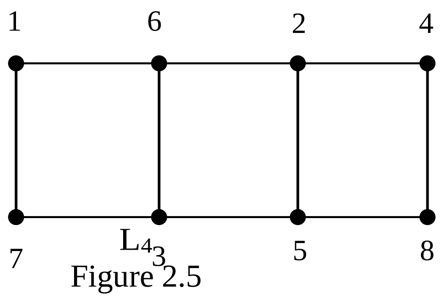
Proof

By Fact 2.3, we have $R(L_n) \geq 2n$. It is enough to show that, $R(L_n) \leq 2n$.

When $n = 3$ or 4, the R – labelings for the graphs L_3 and L_4 are shown in Figure 2.5, from which,



L_3



L_4

Figure 2.5

we conclude that, $R(L_3) \leq 6$ and $R(L_4) \leq 8$.

Now, we assume that $n = 2m$, $m \geq 3$, is an even integer. Then we define $f : V(L_n) \rightarrow \{1, 2, 3, ..\}$ such that $f(v_{2i-1}) = i, 1 \leq i \leq m$; $f(v_{2i}) = m + i, 1 \leq i \leq m$; $f(u_i) = f(v_i) + n$, $1 \leq i \leq n$.

It is obvious that, f is one – one. Here, we can see that,

$|f(v_i) - f(v_j)| \geq 2$, if $d(v_i, v_j) = 1$;

$|f(v_i) - f(v_j)| \geq 1$, if $d(v_i, v_j) = 2$;

$|f(u_i) - f(u_j)| \geq 2$, if $d(u_i, u_j) = 1$;

$|f(u_i) - f(u_j)| \geq 1$, if $d(u_i, u_j) = 2$;

$\left|f(v_i)-f(u_j)\right|\geq 2$, if $d(v_i,u_j)=1$;

$\left|f(v_i)-f(u_j)\right|\geq 1$, if $d(v_i,u_j)=2$, for all i, j, $1\leq i, j\leq n$. This gives that, f is an R – labeling of L_n. Also, *span* f = 2n and hence $R(L_n)\leq 2n$.

Next, we consider the case when $n\geq 5$ is even. Take $n = 2m + 1$, $m\geq 2$. In this case, we define $f : V(L_n)\rightarrow\{1,2,3,..\}$ such that $f(v_{2i-1}) = i, 1\leq i\leq m + 1$; $f(v_{2i}) = m+i + 1, 1\leq i\leq m$; $f(u_i)=f(v_i)+n$, $1\leq i\leq n$. We can easily, check that f is an R – labeling and *span* $f = 2n$. Hence $R(L_n)\leq 2n$. This completes the proof. ■

II. R – NUMBER OF SOME SPECIAL DERIVED GRAPHS

Throughout this section, assume that G is a graph with n_1 vertices and m_1 edges and H is a graph with n_2 vertices and m_2 edges.

Theorem 3.1 Let $G_1, G_2,..., G_k$ be simple connected graphs with R – numbers $R_1, R_2,..., R_k$, respectively and let $|V(G_i)| = n_i$ Then $R(G_1\cup G_2\cup...\cup G_k) = R_1+R_2+...+R_k$, $k\geq 2$.

Proof

Since any R – labeling is one – one, we cannot reuse the label of any vertex in G_i, to a vertex in G_j, for all i, j, $1\leq i\neq j\leq k$. Hence $R(G_1\cup G_2\cup...\cup G_k)\geq R_1+R_2+...+R_k$. Also, since G_1 is disjoint from G_2, the minimum possible labels for the vertices of G_2 should be $R_1+1, R_1+2,..., R_1+R_2$. Proceeding like this, we obtain $R(G_1\cup G_2\cup...\cup G_k)\leq R_1+R_2+...+R_k$. This completes the proof. ■

Theorem 3.2. $R(G + H) = R(G) + R(H)+1$.

Proof

Let $V(G) = \{v_i : 1\leq i\leq n_1\}$ and let $V(H) = \{u_i : 1\leq i\leq n_2\}$.

Then $V(G+H) = V(G)\cup V(H)$ and $E(G+H) = E(G)\cup E(H)\cup\{v_iu_j : 1\leq i\leq n_1$ and $1\leq j\leq n_2\}$. Let f and g be $R(G)$ and $R(H)$ labelings of G and H, respectively. Then define $h : V(G+H)\rightarrow\{1,2,3,...\}$ such that $h(v_i) = f(v_i)$, $1\leq i\leq n_1$ and $h(u_j) = g(u_j)+R(H)+1$, $1\leq j\leq n_2$. Since $d_{G+H}(v_i, u_j) = 1$, $1\leq i\leq n_1$ and $1\leq j\leq n_2$ and $|h(v_i)-h(u_j)| = |f(v_i)-(g(u_j)+R(H)+1)|\geq R(H)+1 > 2$, h is an $R-$ labeling of $G+H$. Also, $h_R = R(G)+R(H)+1$. This implies that, $R(G+H)\leq R(G)+R(H)+1$. Suppose there exists an $R-$ labeling c of $G+H$ such that $R(G+H) = R(G)+R(H)$. Then we can find a pair (v_i, u_j) such that $|c(v_i)-c(u_j)| = 1$, which is a contradiction that c is an $R-$ labeling. This forces that, $R(G+H)\geq R(G)+R(H)+1$. Hence $R(G+H) = R(G)+R(H)+1$. ■

From this result, we deduce the following:

Corollary 3.3

For $m\geq n\geq 1$, $R(K_{m,n}) = m + n + 1$.

Theorem 3.4. *For any graph G on* $n\geq 3$ *vertices*, $R(S(G)) = R(G) + n$.

Proof

Let f be an $R-$ labeling of G and let $V(G) = \{v_i : 1\leq i\leq n\}$ such that $f(v) = R(G)$. Then $V(S(G)) = V(G)\cup\{v_i' : 1\leq i\leq n\}$ and $E(S(G)) = E(G)\cup\{v_i'u : u\in N_G(v_i)\}$. Now, define $g : V(S(G))\rightarrow\{1,2,3,...\}$ such that $g(v_i) = f(v_i)$, $1\leq i\leq n$; $g(v_i') = f(v_i)+1$, $1\leq i\leq n$. Here, we have $d_{S(G)}(v_i, v_i') = 2$. Then $|g(v_i)-g(v_i')| = |f(v_i)-(f(v_i)+i)|\geq 1$ and $|g(v_i)-g(v_i')|\geq|i-j|\geq 1$. This forces that, g is an $R-$ labeling of S(G) and $\max_{v\in V(S(G))} g(v) = R(G)+n$ and hence $R(S(G))\leq R(G)+n$. Since $d_{S(G)}(v_i, v_i')\geq 2$, distinct n positive integers $R(G)+1$, $R(G)+2$,..., $R(G)+n$ are enough to label the vertices of $V(S(G)) - V(G)$. Hence $R(S(G))\geq R(G)+n$.

This completes the proof. ■

Theorem 3.5. *For any graph G on* $n \geq 3$ *vertices,* $R(CS(G)) = R(G) + n$.

Proof

Let f be an R – labeling of G and let $V(G) = \{v_i : 1 \leq i \leq n\}$ such that $f(v_1) = R(G)$ and $v_1 v_2 \in E(G)$. Then $V(CS(G)) = V(G) \cup \{v_i' : 1 \leq i \leq n\}$ and $E(CS(G)) = E(G) \cup \{v_i' u : u \notin N_G(v_i)\}$.

Define $h : V(CS(G)) \to \{1,2,3,...\}$ such that $h(v_i) = f(v_i)$, $1 \leq i \leq n$; $h(v_1') = R(G)+2$; $h(v_2') = R(G)+1$; $h(v_i') = R(G)+3$, $3 \leq i \leq n$. Here, we have $|h(v_1) - h(v_2')| = 1$ and $|h(v_i') - h(v_j')| \geq 1$. Thus h is an R – labeling of $CS(G)$. Also, $h_R = R(G) + n$ and hence $R(CS(G)) \leq R(G) + n$. Since $d_{CS(G)}(v_i', v_j') \geq 2$, distinct n positive integers $R(G)+1$, $R(G)+2, ..., R(G)+n$ are enough to label the vertices of $V(CS(G)) - V(G)$. Hence $R(CS(G)) \geq R(G) + n$. This completes the proof. ∎

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