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On Nano Regular B-Connectedness In Nano Topological Spaces

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Abstract--- In this paper, we introduce the concepts of Nrb-neighbourhood, Nrb-limit point and Nrb-seperated sets in nano topological spaces and also we study the Nano regular b-connectedness (Nrb-connectedness) in nano topological spaces.

Keywords--- Nrb-neighbourhood, Nrb-limit point, Nrb-seperated sets, Nrb-connected space and Nrb-disconnected space.

1. INTRODUCTION

In 1970, Levine [9] introduced the concept of generalized closed sets in topological space. A.V.Arhangel'skii [2] introduced the concept of connectedness and disconnectedness in topology in 1975. Later Andrijevic [1] introduced a new class of generalized open sets namely, b-open sets in 1996. In 2012 regular b-closed sets was introduced by A.Narmadha and N.Nagaveni[10]. A.narmadha et.al [11] introduced regular b-open sets in 2013. S.S.Benchali and Priyanka M.Bansali [3] studied gb-compactness and gb-connectedness in topological spaces in 2011. In 2013 b-connectedness and b-disconnectedness and their applications was introduced by A.A.El-Atik, H.M.Abu Donia and A.S.Salama [5]. The notion of nano topology was introduced by Lellis Thivagar [7] in 2013. And also Lellis Thivagar and S.P.R. Priyalatha [8] introduced the different types of neighbourhoods in nano topological spaces in 2013. In 2018 P.Sathishmohan et. al [14] studied nano semi pre neighbourhoods in nano topological spaces. Some generalization of neighbourhoods in nano topological spaces was introduced by K.Chitrakala, A.Vadivel and G.Saravanakumar [4] in 2019. S. Krishnaprakash , R.Ramesh and R.Suresh [6] introduced the nano-compactness and nano-connectedness in nano topological space in 2018. In 2021 P.Srividhya and T.Indira introduced the Nrb-closed sets and Nrb-open sets in nano topological spaces [15]. And also they studied the concepts of Nrb-continuous functions. In this paper, we have introduced the concepts of Nrb-neighbourhood, Nrb-limit point, Nrb-seperated sets and Nrb-connectedness in nano topological space.

2. PRELIMINARIES

Definition: 2.1 [7]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$. Then

- The lower approximation of X with respect to R is the set of all objects which can be for certainly classified as X with respect to R and is denoted by $L_R(X)$.

$L_R(X) = \bigcup_{x \in U} \{R(x):R(x) \subseteq X\}$

- The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.

$U_R(X) = \bigcup_{x \in U} \{R(x):R(x) \cap X \neq \phi\}$

- The boundary region of X with respect to R is the set of all objects which can be classified neither as X

nor as not –X with respect to R and is denoted by $B_R(X)$.

$B_R(X) = U_R(X) - L_R(X)$.

Example: 2.2

Let U={1,2,3,4}, X={1,2}, U/R={{1},{3}{2,4}}.

$L_R(X)=\{1\}$, $U_R(X)=\{1\} \cup \{2,4\}=\{1,2,4\}$,

$B_R(X) = U_R(X) - L_R(X) =\{1,2,4\}-\{1\}=\{2,4\}$

Definition: 2.3 [7]

Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

- U and $\phi \in \tau_R(X)$.
- The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is called the nano topology on U with respect to X, $(U,\tau_R(X))$ is called the nano topological space.

Elements of the nano topology are known as nano open sets.

The complement of elements of the nano open sets are called as nano closed sets.

Definition: 2.4 [7]

If $(U,\tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

- The nano interior of a set A is defined as the union of all nano open sets contained in A and it is denoted by Nint(A).
- The nano closure of a set A is defined as the intersection of all nano closed sets containing A and it is denoted by Ncl(A).

Example: 2.5

Let U={1,2,3,4}, X={1,2}, U/R={{1},{3}{2,4}},

$\tau_R(X)=\{U, \phi,\{1\},\{1,2,4\},\{2,4\}\}$.

Nano open sets=$\{U, \phi,\{1\},\{1,2,4\},\{2,4\}\}$,

Nano closed sets=$\{U, \phi,\{2,3,4\},\{3\},\{1,3\}\}$.

Let A={1,2}, Nint(A) $=\{1\} \cup \phi =\{1\}$, Ncl(A)=U.

Definition: 2.6 [7]

A subset A of a nano topological space $(U,(\tau_R(X))$ is called

a) Nano pre closed if Ncl(Nint(A)) $\subseteq$ A [7]
b) Nano semi closed if Nint(Ncl(A)) $\subseteq$ A [7]
c) Nano regular closed if A= Ncl(Nint(A)) [7]
d) Nano α (pre semi) closed if Ncl(Nint(Ncl(A))) $\subseteq$ A [7]
e) Nano β (semi pre) closed if Nint(Ncl(Nint(A))) $\subseteq$ A [13]
f) Nano b-closed if Nint(Ncl(A)) $\cap$ Ncl(Nint(A)) $\subseteq$ A [12]

Example : 2.7

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}}.
τ_R(X) = {U, ϕ,{1},{1,2,4},{2,4}}.
Nano open sets={U, ϕ,{1},{1,2,4},{2,4}},
Nano closed sets={U, ϕ,{2,3,4},{3},{1,3}}.
Nano pre closed sets= {U, ϕ,{2},{3},{4},{1,3},{2,3}, {3,4}{1,2,3},{1,3,4},{2,3,4}}.
Nano semi closed sets = {U, ϕ,{1},{3},{1,3},{2,4}, {2,3,4}}.
Nano regular closed sets= {U, ϕ,{1,3},{2,3,4}.
Nano α-closed sets={U, ϕ,{3},{1,3},{2,3,4}}.
Nano β-closed sets = {U, ϕ,{1},{2},{3},{4},{1,2}, {1,3},{1,4},{2,3},{2,4},{3,4},{1,2,3},{1,3,4},{2,3,4}}
Nano b-closed sets= { U, ϕ,{1},{2},{3},{4},{1,3}, {2,3},{2,4},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Definition: 2.8 [15]

A subset A of a nano topological spaces (U, τ_R(X)) is said to be "nano regular b-closed" (briefly nano rb-closed) if Nrcl(A) $\subset$ G whenever A $\subset$ G and G is nano b-open in U.

Example: 2.9

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}},
τ_R(X) = {U, ϕ,{1},{1,2,4},{2,4}}.
Nrb-closed sets={ U, ϕ,{3},{1,3},{2,3,4}}.

Definition: 2.10 [15]

The union of all nano regular b-open sets contained in A is nano regular b-interior of A and it is denoted by Nrbint(A).
The intersection of all nano regular b-closed sets containing A is called the nano regular b-closure of A and it is denoted by Nrbcl(A).
The family of all Nrb-open sets and Nrb-closed sets of (U, τ_R(X)) is denoted by NRBO(U,X) and NRBC(U,X).

Example: 2.11

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}},
τ_R(X) = {U, ϕ,{1},{1,2,4},{2,4}}.
Nrb-closed sets={ U, ϕ,{3},{1,3},{2,3,4}},
Nrb-open sets= {U, ϕ {1},{2,4},{1,2,4}}.

Definition: 2.12 [8]

Let (U, τ_R(X)) be a nano topological space and let $x \in U$. A subset N of U is said to be a nano neighbourhood of x if there exists a nano open set G such that $x \in G \subset N$.

The collection of all neighbourhoods of x is called the nano neighbourhood system at x it is denoted by N(x).

Example: 2.13

Let U={1,2,3,4}, X={1,2}, U/R={1,3},{2},{4}.

$\tau_R(X)$ = {U, ϕ,{2},{1,2,3},{1,3}}.

Here x = 1 $\in$ {1,2,3,4} = U.

N(x) = {{1},{1,2,3},{1,3,4}}.

But {1,3} and {1,2,3} are nano open neighbourhoods.

Definition: 2.14 [6]

A nano topological space (U, $\tau_R(X)$) is said to be nano connected if (U, $\tau_R(X)$) cannot be expressed as a union of two non-empty disjoint nano open sets.

A subset A of (U, $\tau_R(X)$) is nano connected as a subspace of (U, $\tau_R(X)$).

Definition: 2.15 [ ]

Let U = {a,b,c,d}, U/R = {a},{b},{c},{d}

$\tau_R(X)$ = {U, ϕ,{a,d}}.

Here (U, $\tau_R(X)$) cannot be expressed as a disjoint union of two non-empty nano open sets.

$\therefore$ (U, $\tau_R(X)$) is nano connected.

Definition: 2.16 [6]

A nano topological space (U, $\tau_R(X)$) is said to be nano disconnected iff it is the union of two non-empty nano separated sets.

Example: 2.17

Let U={a,b,c,d}, X = {a,d}, U/R = {{a},{b},{c},{d}}.

$\tau_R(X)$ = {U, ϕ,{a,d},{b,c}}.

Nano closed sets = {U, ϕ,{b,c},{a,d}}.

A = {a,d}, B = {b,c}

Ncl(A) = {a,d}, Ncl(B) = {b,c}.

A $\cap$ Ncl(B) = {a,d} $\cap$ {b,c} = ϕ ,

Ncl(A) $\cap$ B = {a,d} $\cap$ {b,c} = ϕ.

$\therefore$ A and B are nano separated.

Here {a,d} $\cup$ {b,c} = {a,b,c,d} = U.

$\therefore$ (U, $\tau_R(X)$) is nano disconnected.

3. NANO REGULAR b-CONNECTEDNESS

(briefly Nrb-connectedness)

Definition: 3.1

Let (U, $\tau_R(X)$) be a nano topological space and let A be a subset of U, a point x $\in$ U is called a nano limit point (or) nano cluster point (or) a nano accumulation point of A if every neighbourhood of x contains a point of A other than x.

The set of all nano limit points of A is called the nano derived set of A and it is denoted by $ND_x(A)$ or ND(A).

Example: 3.2

Let U={a,b,c,d}, X={a,c}, U/R={a,b},{c},{d}.

$\tau_R(X)$ = {U, ϕ,{c},{a,b,c},{a,b}}.

Let A = {b,d}.

a) If x=a, N(a) = {{a,b},{a,b,c},{a,b,d}}.
$\therefore$ x= a is a nano limit point, since every neighbourhood of x contains a point of A other than a.
b) If x=b, N(b) = {{a,b},{a,b,c},{a,b,d}}.
$\therefore$ x=b is not a nano limit point, since N(b) ={{a,b}, {a,b,c}} contains no point of A other than b.
c) If x=c, N(c) = {{c},{a,c},{a,b,c},{a,c,d},{b,c,d}}
$\therefore$ x = c is not a nano limit point, since N(c) = {{c}, {a,c}} contain no point of A.
d) If x=d, N(d) = {U}.
$\therefore$ x= d is a limit point, since N(d) = {U} contains a point of A other than d.
$\therefore$ a and d are nano limit points.

Definition: 3.3

Let (U, τ_R(X)) be a nano topological space and let A be a subset of U, a point $x \in U$ is called a nano adherent point (or) nano contact point of A if every nano neighbourhood of x contains a point of A.

The set of all nano adherent points of A is called the nano adherence of A and it is denoted by Adh A.

Example: 3.4

In the above example 3.2 a,b and d are nano adherent points since every nano neighbourhood of these points contain a point of A.

Definition: 3.5

Let (U, τ_R(X)) be a nanp topological space. Two non-empty subsets A and B of U are said to be nano separated if
$A \cap Ncl(B) = \phi$ and $B \cap Ncl(A) = \phi$.

It is equivalent to $(A \cap Ncl(B)) \cup (Ncl(A) \cap (B)) = \phi$.

$\therefore$ A and B are nano separated iff A and B are disjoint and neither of them contains nano limit point of the other.

$A \cap Ncl(B) = \phi \Leftrightarrow A \cap (B \cup ND(B)) = \phi$
$\Leftrightarrow (A \cap B) \cup (A \cap ND(B)) = \phi$
$\Leftrightarrow A \cap D(B) = \phi$
($\because$ A and B are disjoint)
$\Leftrightarrow$ A contains no nano limit point of B

Example: 3.6

Let U={a,b,c,d}, X = {a,d}, U/R = {{a},{b},{c},{d}}.
τ_R(X) = {U, ϕ,{a,d},{b,c}}.
Nano closed sets = {U, ϕ,{b,c},{a,d}}.
A = {a}, B = {c}
Ncl(A) = {a,d}, Ncl(B) = {b,c}.
$A \cap Ncl(B) = \{a\} \cap \{b,c\} = \phi$,
$Ncl(A) \cap B = \{a,d\} \cap \{c\} = \phi$.
$\therefore$ A and B are nano separated.

Result: 3.7

Any two nano separated sets are disjoint. But two disjoint sets are not necessarily nano separated.

Example: 3.8

Let U = {1,2,34}, X = {1,2}, U/R = {1,3},{2},{4}.

$\tau_R(X)$ = {U, ϕ,{2},{1,2,3},{1,3}}.
Nano closed = {U,ϕ,{1,3,4},{2,4},{4}}.
Let A = {1,2}, B = {3,4} where A and B are disjoint.
Ncl(A) = U, Ncl(B) = {1,3,4}.
$\therefore$ Ncl(A)$\cap$B = U$\cap${3,4} = {3,4}$\neq \phi$
A$\cap$Ncl(B) = {1,2}$\cap${1,3,4} = {1}$\neq \phi$
$\therefore$ A and B are not nano separated.
From this example result 3.6 is proved.

Definition: 3.9

Let (U,$\tau_R(X)$) be a nano topological space and let $x \in U$. A subset N of U is said to be a Nrb-neighbourhood of x if there exists an Nrb-open set G such that $x \in G \subset N$.

Example: 3.10

Let U = {1,2,3,4,5}, X = {1,3,5},
U/R = {{1},{2},{3,4},{5}}.
$\tau_R(X)$ = {U,ϕ,{1,5},{1,3,4,5},{3,4}}
which are Nano open sets.
Nano closed sets = { U,ϕ,{2},{1,2,5},{2,3,4}}.
Nrb-open sets = { U,ϕ,{1,2,5},{2,3,4}}.
Nrb-closed sets = {U,ϕ,{1,5},{3,4}}.
Here x = 1$\in${1,2,3,4,5} = U.
$\therefore$ N(x) = {{1,2,5},{1,2,3,5},{1,2,4,5}}.
Here {1,2,5} is open Nrb-neighbourhood for 1.

Definition: 3.11

Let (U,$\tau_R(X)$) be a nano topological space and let A be a subset of U, a point $x \in U$ is called a Nrb-limit point (or) Nrb-cluster point (or) Nrb-accumulation point of A if every nrb-neighbourhood of x contains a point of A other than x.

The set of all Nrb-limit points of A is called the Nrb-derived set of A and it is denoted by Nrb D(A).

Theorem: 3.12

Let U = {1,2,3,4,5}, X = {1,3,5},
U/R = {{1},{2},{3,4},{5}}.
$\tau_R(X)$ = {U,ϕ,{1,5},{1,3,4,5},{3,4}}.
Nrb-open sets = { U,ϕ,{1,2,5},{2,3,4}}.
Let A = {2,5}

a) x = 1, N(1) = {{1,2,5},{1,2,3,5},{1,2,4,5}}.
1 is a limit point, since every neighbourhood of 1 contains a point of A other than 1.
b) x = 2, N(2) = {{1,2,5},{2,3,4},{1,2,3,5},{1,2,4,5}}.
2 is not a limit point since N(2) ={2,3,4} and {1,2,3,4} contain no point of A other than 2.
c) x = 3, N(3) = {{2,3,4},{1,2,,3,4},{2,3,4,5}}.
3 is a limit point since every neighbourhood of 3 contains a point of A other than 3.
d) x = 4, N(4) = {{2,3,4},{1,2,3,4},{2,3,4,5}}.
4 is a limit point since every neighbourhood of 4 contains a point of A other than 4.
e) x = 5, N(5) = {{1,2,5},{{1,2,3,5},{1,2,4,5}}.
5 is a limit point since every neighbourhood of 5 contains a point of A other than 5.

Definition: 3.13

Let $(U, \tau_R(X))$ be a nano topological space and let A be a subset of U, a point $x \in U$ is called a Nrb- adherent point (or) Nrb - contact point of A if every Nrb - neighbourhood of x contains a point of A.

The set of all Nrb-adherent points of A is called the Nrb- adherence of A and it is denoted by Nrb- Adh A.

Example: 3.14

In the above example 3.11 a,b,c,d and e are Nrb- adherent points since every Nrb - neighbourhood of these points contain a point of A.

Definition: 3.15

Let $(U, \tau_R(X))$ be a topological space. Two non-empty subsets A and B of U are said to be Nrb-seperated if $A \cap$ Nrbcl(A) = ϕ and Nrbcl(A) $\cap$ B = ϕ .

Example: 3.16

Let U = {1,2,3,4,5}, X = {1,3,5},
U/R = {1},{2},{3,4},{5}.
$\tau_R(X)$ = {U, ϕ ,{1,5},{1,3,4,5},{3,4}}.
Nrb-open sets = { U, ϕ ,{1,2,5},{2,3,4}}.
Nrb-closed sets = {U, ϕ ,{1,5},{3,4}}.
Here A = {1}, B = {3}.
Nrbcl(A) = {1,5}, Nrbcl(B) = {3,4}.
$A \cap$ Nrbcl(B) = $\{1\} \cap \{3,4\} = \phi$,
Nrbcl(A) $\cap$ B = $\{1,5\} \cap \{3\} = \phi$

$\therefore$ A and B are Nrb-separated.

Result: 3.17

Any two Nrb-seperated sets are disjoint. But two disjoint sets need not be Nrb-seperated.

Example: 3.18

From the above example 3.6, we take A = {1,2}, B = {3,5} where A and B are disjoint.

Nrbcl(A) = U, Nrbcl(B) = U.

$A \cap$ Nrbcl(B) = $\{1,2\} \cap U = \{1,2\} \neq \phi$,
Nrbcl(A) $\cap$ B = $\{3,5\} \cap U = \{3,5\} \neq \phi$.

$\therefore$ A and B are not Nrb-separated.

Definition: 3.19

A nano topological space $(U, \tau_R(X))$ is said to be Nrb-connected if $(U, \tau_R(X))$ cannot be expressed as a disjoint union of two non-empty Nrb-open sets.

A subset A of $(U, \tau_R(X))$ is Nrb-connected as a subspace of $(U, \tau_R(X))$.

Example: 3.20

Let U = {1,2,3,4,5}, X = {1,3,5},
U/R = {{1},{2],{3,4},{5}}.
Nano closed sets = { U, ϕ ,{2},{1,2,5},{2,3,4}}.
Nrb-open sets = { U, ϕ ,{1,2,5},{2,3,4}}.
Nrb-closed sets = {U, ϕ ,{1,5},{3,4}}.
Here $(U, \tau_R(X))$ cannot be expressed as a disjoint union of Nrb-open sets.
$\therefore$ $(U, \tau_R(X))$ is Nrb- connected.

Definition: 3.21

A nano topological space $(U, \tau_R(X))$ is said to be Nrb-disconnected if it is the union of two non-empty Nrb-separated sets.

Example: 3.22

Let U = {1,2,3,4}, U/R = {1,2},{3,4}, X = {1,2,3}.

$\tau_R(X) = \{U, \phi, \{1,2\}, \{3,4\}\}$.

Nrb-closed sets = $\{U, \phi, \{3,4\}, \{1,2\}\}$.

A = {1,2}, B = {3,4}.

Nrbcl(A) = {1,2}, Nrbcl(B) = {3,4}.

$A \cap$ Nrbcl(B) = $\{1,2\} \cap \{3,4\} = \phi$,

Nrbcl(A) $\cap$ B = $\{1,2\} \cap \{3,4\} = \phi$.

Therfore A and B are Nrb-seperated.

Here $\{1,2\} \cup \{3,4\} = \{1,2,3,4\}$ = U.

$\therefore (U, \tau_R(X))$ is Nrb-disconnected.

Theorem: 3.23

A subset A of a nano topological space is nano open if and only if it is a nano neighbourhood of each of its points.

Proof:

Let A be a nano open subset of a nano topological space.

Then for every $x \in A$, there exists a nano neighbourhood of x namely A.

(i.e) $x \in A \subset A$.

$\therefore$ A is a neighbourhood of each of its points.

Conversely, let A be a nano neighbourhood of each of its points.

Let A = ϕ, then A is nano open.

Suppose $A \neq \phi$, then for each $x \in A$ there exists a nano open set G_x such that $x \in G_x \subset$ A.

Then we get $A = \cup \{G_x : x \in A\}$.

$\therefore$ A is the union of nano open sets.

Theorem: 3.24

An arbitrary union of Nrb-neighbourhood of a point x is again a Nrb-neighbourhood of U.

Proof:

Let $\{A_\lambda\}$ $\lambda \epsilon I$ be an arbitrary collection of Nrb-neighbourhoods of a point $x \epsilon U$.

To prove that $\{\cup A_\lambda\}_{\lambda \epsilon I}$ is also a Nrb-neighbourhood of x.

Here I denotes the indexed set.

For all $\lambda \epsilon I$ there exists Nrb-open set G_x such that $x \in G_x \subset A_\lambda \subset \cup A_\lambda$.

(i.e) $x \in \cup G_x \subset \cup A_\lambda$.

$\therefore \cup A_\lambda$ is a Nrb- neighbourhood of x.

$\therefore$ Arbitrary union of Nrb-neighbourhoods of x is again a Nrb-neighbourhoods of x.

Theorem: 3.25

Let $(U, \tau_R(X))$ be a nano topological space and let A be a subset of U. Then A is nano closed if and only if $ND(A) \subset A$.

Proof:

Let A be nano closed. Then A^c is open and so for each $x \in A^c$ there exists a nano neighbourhood N_x of x such that $N_x \subset A^c$.

Since $A \cap A^c = \phi$, then nano neighbourhood N_x contains no point of A and also x is not a nano limit point of A.

$\therefore$ No point of A^c can be a nano limit point of A.

(i.e) A contains all its nano limit points.

Hence $ND(A) \subset A$.

Conversely let $ND(A) \subset A$ and let $x \in A^c$. Then $x \notin A$.

Since $ND(A) \subset A$, $x \in ND(A)$.

There exists a nano neighbourhood N_x of x such that $N_x \cap A = \phi$ so that $N_x \subset A^c$.

Thus A^c contains a nano neighbourhood of each of its point and so A^c is nano open.

(i.e) A is nano closed.

Hence the theorem.

Theorem: 3.26

If A and B are Nrb separated subsets of a space U and $C \subset A$ and $D \subset B$, then C and D are also Nrb-separated.

Proof:

To prove that C and D Nrb – separated.

Given that A and B are Nrb-separated.

$\therefore A \cap Nrbcl(B) = \phi$ and $Nrbcl(A) \cap B = \phi \rightarrow (1)$

Also $C \subset A \Rightarrow Nrbcl(C) \subset Nrbcl(A)$

and $D \subset B \Rightarrow Nrbcl(D) \subset Nrbcl(B) \rightarrow (2)$

From (1) and (2), we get

$C \cap Nrbcl(D) = \phi$ and $Nrbcl(C) \cap D = \phi$.

Hence C and D are Nrb-separated.

Theorem: 3.27

Two Nrb-open subsets of a nano topological space are Nrb-sepaprated if and only if they are disjoint.

Proof:

Let A, B be Nrb – open sets of a nano topological space $(U, \tau_R(X))$.

Suppose that A,B are disjoint so that $A \cap B = \phi$.

To prove that A and B are Nrb- separated we have to prove that $Nrbcl(A) \cap B = \phi$, $A \cap Nrbcl(A) = \phi$.

Suppose if possible, $Nrbcl(A) \cap B \neq \phi$.

Then for all $x \in Nrbcl(A) \cap B$ so that $x \in Nrbcl(A)$ and $x \in B$.

$\therefore x \in Nrbcl(A) \Rightarrow$ x is an Nrb-adherent point of A.

Which implies every Nrb-neighbourhood of x must contain atleast one point of A.

Also $x \in A$, A is Nrb-open.

Hence A is a Nrb-neighbourhood of each f its points.

This prove that $A \cap B \neq \phi$ contradicting $A \cap B = \phi$.

Hence our assumption $Nrbcl(A) \cap B \neq \phi$ is wrong so that $Nrbcl(A) \cap B = \phi$.

In the same way $A \cap Nrcl(B) = \phi$ is proved.

Hence A and B are Nrb-separated.

Conversely assume that A and B are Nrb-separated.

To prove that $A \cap B = \phi$.

Since A and B are Nrb-separated, we have

$A \cap Nrcl(B) = \phi$ and $Nrbcl(A) \cap B = \phi$.

Let $A \subset Nrbcl(A)$, $B \subset B$.
$\Rightarrow A \cap B \subset Nrbcl(A) \cap B$
$\Rightarrow A \cap B \subset \phi$ $\rightarrow (1)$
Since ϕ is a subset of every set, then
$\phi \subset A \cap B$ $\rightarrow (2)$
From (1) and (2) we get $A \cap B = \phi$.
Hence A and B are disjoint.

Theorem: 3.28

Let $(U, \tau_R(X))$ be a nano topological space. If there exists a non-empty proper subset of U which is both Nrb-open and Nrb-closed in U. Then U is Nrb-disconnected.

Proof:

Let A be a non-empty proper subset of U which is both Nrb-open and Nrb-closed.
To prove that U is Nrb-disconnected.
Let $B = A^{|}$.
Then B is non-empty since A is proper subset of U.
And also $A \cup B = U$ and $A \cap B = \phi$.
Since A is both Nrb-closed and Nrb-open, B is also Nrb-closed and Nrb-open.
Hence Nrbcl(A) = A and Nrbcl(B) = B.
$A \cap Nrbcl(B) = \phi$. ($\because A \cap B = \phi$, B = Nrbcl(B)).
U has been expresses as union of two Nrb-separated sets.
U is Nrb-disconnected.

Theorem: 3.29

Let $(U, \tau_R(X))$ be a nano topological space and let E be a Nrb-connected subset of U such that $E \subset A \cup B$ where A and B are Nrb-separated sets. Then prove that $E \subset A$ or $E \subset B$, (i.e.) E cannot intersect both A and B.

Proof:

Let E be Nrb-connected.
Since A,B are Nrb-separated,
$A \cap Nrbcl(B) = \phi$, $Nrbcl(A) \cap B = \phi$.
Now $E \subset (A \cup B)$, $E = E \cap (A \cup B)$
$E = (E \cap A) \cup (E \cap B) \rightarrow (1)$
We prove that at least one of the sets $E \cap A$ and $E \cap B$ is empty.
Suppose that none of these sets is empty.
(i.e.) $E \cap A \neq \phi$ and $E \cap B \neq \phi$.
Then $(E \cap A) \cap Nrbcl(E \cap B)$
$= (E \cap A) \cap (Nrbcl(E) \cap Nrbcl(B))$
$= (E \cap Nrbcl(E) \cap (A \cap Nrbcl(B))$
$= E \cap Nrbcl(E) \cap \phi$
(since A and B are Nrb-separated sets).
$(E \cap A) \cap (Nrbcl(E \cap B)) = \phi \rightarrow (1)$
Similarly, $Nrbcl(E \cap A) \cap (E \cap B)$
$= ((Nrbcl(E) \cap Nrbcl(A)) \cap (E \cap B)$
$= (Nrbcl(E) \cap E \cap (Nrbcl(A) \cap B))$
$= Nrbcl(E) \cap E \cap \phi$
(since A and B are Nrb-separated sets).

(Nrbcl(E$\cap$A)$\cap$(E$\cap$B)) = $\phi \rightarrow$ (2)
From (1) and (2) E$\cap$A and E$\cap$B are Nrb-seperated sets.
$\therefore$ E has been expressed as the union of two non-empty Nrb-separated sets.
E is Nrb-disconnected.
This is contradiction to our hypothesis.
$\therefore$ Atleast one of the sets E$\cap$A and E$\cap$B is empty.
If E$\cap$A = ϕ, (1) E = E$\cap$B.
which implies E$\subset$B.
Similarly if E$\cap$B = ϕ , then from (2) E = E$\cap$A.
which implies E$\subset$A
Hence either E$\subset$A or E$\subset$B.

4. CONCLUSION

In this paper, we introduced the concepts of Nrb-neighbourhood, Nrb-limit point and Nrb-seperated sets in nano topological spaces and also we studied the Nano regular b-connectedness (Nrb-connectedness) in nano topological spaces. In future this work will be extended with some real life applications.

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