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Intuitionistic Fuzzy Local Function In Topology

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Abstract: In this paper we discuss the basic definitions of intuitionistic fuzzy set and intuitionistic fuzzy topology. Closure of a Intuitionistic fuzzy sets in a Intuitionistic fuzzy topological space are defined and some of its basic observations are given.

Keywords: Intuitionistic fuzzy sets, Intuitionistic fuzzy topology, Intuitionistic fuzzy topological spaces

1. INTRODUCTION

The application of the set theory had their own limitations. It is with this point of view as the background, Zadeh[6] in 1965 enunciated a new branch of study known as fuzzy mathematics. He defined in his preliminary work [7], the concept of fuzzy sets as a function from an ordinary set X into the closed interval [0,1].
According to George J. Klir an important point in the evaluation of the modern concept of uncertainty was the publication of the paper by Zadeh. In his paper, L.A. Zadeh introduced a theory whose objects-fuzzy sets- are sets with boundaries that are not precise. The membership in a fuzzy is not just a matter of affirmation or denial, but rather a matter of a degree.

After the introduction of the concept of fuzzy sets by Zadeh[6], in 1968, Chang[2] became the pioneer to present the idea of fuzzy topology. Fundamentally, he replaced the classical notion of open sets by a notion called fuzzy open sets.

However, later in 1976, Lowen found that many of the well known results in general topology cannot be obtained if one follows Chang's definition fuzzy topology. So, he redefined fuzzy topology by including constant fuzzy sets in it. This implies that in Chang's definition of fuzzy topology one more axiom was included. It should be mentioned that those who follow Chang's definition of fuzzy topology, call Lowen's definition of it as "fully stratified" fuzzy topology.

In 1985, Sostak[4] independently studies I-fuzzy topology, where I is the usual unit interval. In 1991, Ying introduced the concept of fuzzifying topology.

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was first introduced by Atanassov [1] in 1986. Coker introduced intuitionistic fuzzy topology. In fact Coker constructed the fundamental theory on intuitionistic fuzzy topological spaces (IFTSs).

Coker and others studied compactness, connectedness, continuity, separation, convergence and paracompactness in intuitionistic fuzzy topological spaces.In the year 2000, G.J. Wang and Y.Y. He[5] established that every intuitionistic fuzzy set may be regarded as an L-fuzzy set for some appropriate lattice L.

2. PRELIMINARIES

Definition1[3]: LetXbeanon-emptyfixedset.Anintuitionisticfuzzyset(IFSfor short)Aisan objecthavingtheformA={(x,μ_A(x),ν_A(x)):x∈X}wherethefunctions μ_A: X → I and ν_A: X → I denote the degree of membership (namely μ_A(x)) and the degreeofnon-membership(namelyν_A(x))ofeachelementx∈XtothesetA,respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$, for each x ∈X.

Definition 2 [3]: Let X be a non empty set, and the IFSs A and B be in the form
A = {(x, μ_A(x), γ_A(x)) : x ∈ X} and B = {(x, μ_B(x), γ_B(x)) : x ∈ X}
1. A ⊆ B iff $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for all x ∈X.

2. $\overline{A}$ = B iff A ⊆ B and B ⊆A.
3.A = {(x, γ_A(x), μ_A(x)) : x ∈X}. 4.A∩ B = {(x, μ_A(x) ∩ μ_B(x), γ_A(x) ∪ γ_B(x)) : x ∈X}
5.A ∪ B = {(x, μ_A(x) ∪ μ_B(x), γ_A(x) ∩ γ_B(x)) : x ∈X}
6.[ ]A={(x,μ_A(x),1−μ_A(x)):x∈X
7. ( )A = {(x, 1 −γ_A(x), γ_A(x)) : x ∈X}

Definition 3[3]:
Let {A_α/α ∈ J} be a family of IFSs in X.
Then $\wedge A_\alpha = (\wedge\mu_{A_\alpha}, \vee\gamma_{\mu A_\alpha})$, and $\vee A_\alpha = (\vee\mu_{A_\alpha}, \wedge\gamma_{\mu A_\alpha})$.

Definition 4: For every (a, b) ∈ Δ, $C_{a,b}$ denotes the IFS defined by $C_{a,b}(x) = (a, b)$, for all x ∈ X.
Note that $C_{1,0} = 1_\sim$ and $C_{0,1} = 0_\sim$.

Definition 5 [3]
Let X be a non empty set and τ be a family of IFSs in X satisfying the following axioms:
(T1) $0_\sim$, $1_\sim$ ∈ τ
(T2) f , g ∈ τ ⇒ f ∧ g ∈ τ
(T3) If { f_α : α ∈ J} is a subfamily of τ, then $\vee G_i$∈ τ.

Thepair(X,τ)iscalledanintuitionisticfuzzytopologicalspace(IFTS)andiff∈τ,thenf issaidtobeanintuitionisticfuzzyopenset(IFOS)inX,andf^{C}issaidtobeanintuitionistic fuzzy closed set (IFCS) in the IFT space (X,τ).

Results1[3]: If (X, τ_0) is a fuzzy topological space in the sense ofChang, then (X, τ_0) produces the followingIFTS.
$\tau = \{A = (\mu_A, 1 - \mu_A) \in S^X / A = \mu_A \in \tau_0\}$.

Results 2 : Let (X, τ) be an intuitionistic fuzzy topological space. If A ∈ IFS (X), we denote A by $A = (\mu_A, \gamma_A)$. Then
(i) $\tau_1 = \{\mu_A / A \in \tau\}$
(ii) $\tau_2 = \{1 - \gamma_A / A \in \tau\}$

Definition 6
Lest (X, τ) be an IFTS and A ∈ IFS (X). Then the intuitionistic fuzzy interior, (in short, IFI), int A and the intuitionistic fuzzy closure (IFC) clA of A are defined by:
int A = $\vee\{G \in \tau / G \leq A\}$
cl A = $\wedge\{F / F^C \in \tau \text{ and } A \leq F\}$
Remarks1:
1. cl(A) is an IFCS and int A is an IFOS in (X,τ)
2. clA is the smallest IFCS containing A and intA is the largest IFOS contained inA.
3. for all A ∈IFS (X), $cl(A^C) = (intA)^C$ and $int(A^C) = (clA)^C$.
4. A is an IFCS in X, if and only if clA =A.
5.A is an IFOS in X if and only if int(A) =A.

Definition 7: Let X be a non - empty set and let $\tau \subset S^X$. Then τ is called an intuitionistic fuzzy topology (IFT) on X, if it satisfies the followingaxioms:
(T 1) $0_\sim$, $1_\sim \in \tau$.
(T 2) $G_1, G_2 \in \tau \Rightarrow G_1 \wedge G_2 \in \tau$.
(T 3) for every family $\{G_\alpha\}_{\alpha \in J}$ of members of τ, $\vee_\alpha G_\alpha \in \tau$.

Thepair(X,τ)iscalledanintuitionisticfuzzytopologicalspace(IFTS).
EachmemberofGofτiscalledanintuitionisticfuzzyopenset(IFOS)inX.
$A \in \mathbf{I}^X$iscalledanintuitionistic fuzzy closed set (IFCS) in X, if $A^C \in \tau$.

Definition 8 (IFTS in the sense of Lowen): Let (X, τ) be an IFTS. Then (X, τ) is called anIFTSinthesenseofLowenifforevery(a,b)∈Δ,theIFS$C_{a,b}$∈τ,where$C_{a,b}$isdefined by $\mu_{C_{a,b}}(x) = a$ and $\mu_{C_{a,b}}(x) = b$, for all x ∈X.

Closure of an IFS in an IFTS
D. Coker (1997) introduced the concept of closure and interior of an IFS and IFTS.

Let (X, τ) be an IFTS. If $A \in \mathbf{I}^X$, then the closure $\bar{A}$ and the interior A° of A in (X, τ) are defined as follows:
$\bar{A} = \wedge\{K / K^c \in \tau \text{ and } A \leq K\}$ and
$A^\circ = \vee\{G / G \in \tau \text{ and } G \leq A\}$.

Result 3 : $\bar{A} = \wedge\{G^c / A \leq G^c \text{ and } G \in \tau\} = \wedge\{G^c / G \in \tau \text{ and } A \cap G \in 0^\sim\}$

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Remark 2: If (X, σ) is a crisp topology on X and A is a crisp subset of X, then the closure $\bar{A}$ of A in (X, σ) is also defined as

$\bar{A}$ = {x $\in$X / G $\in\tau$ and x $\in$ G $\Rightarrow$G$\cap$ A $\neq\phi$}

Definition 9 : Let (X, τ) be an IFTS and A $\in\mathfrak{I}^X$. Then
$A^* \in \mathfrak{I}^X$ is defined for each x $\in$X, $A^*(x)=\vee\{(a,b) \in\Delta/G \in\tau$ and $(a,b) \leq G^c(x) \Rightarrow A\wedge G\in 0^\sim\}$.

The following observations are made

1. Let $\wedge_x = \{(a,b) \in\Delta/G \in\tau$ and $(a,b) \leq G^c(x) \Rightarrow A\wedge G\in 0^\sim\}$.

Then $A^*(x)= (\mu,\lambda) = \vee\{(a,b)/(a,b) \in\wedge_x \}$.
Let G $\in\tau$ and $(a,b) \leq G^c(x)$.
Then either $\mu>\gamma_G(x)$ or $\lambda<\mu_G(x)$.
Find $(a,b) \in\wedge_x$ such that either $\mu\geq a >\gamma_G(x)$ or $\lambda\geq b >\mu_G(x)$.
Then $(a,b) \leq G^c(x)$. As $(a,b) \in\wedge_x$, we get $A\wedge G\in 0^\sim$.
Hence G $\in\tau$, and $A^*(x) \leq G^c(x) \Rightarrow A\wedge G\in 0^\sim$. So $A^*(x)\in\wedge_x$.

2. $A^* = cl(A^*)$. Clearly $A^*\leq cl(A^*)$. Conversely, let G $\in\tau$ and $cl(A^*) \leq G^c$.

Then $G \cap A^*\in 0^\sim$.
So there is a point y $\in$ X such that either $\mu_G(y) >\gamma_{A*}(y)$ or $\mu_{A*}(y) >\gamma_G(y)$.
Thus $A^*(y) \leq G^c(x)$ for some y $\in$ X.

3. $(A^*)^*\leq A^*$.

4. A $\leq$ B $\Rightarrow A^X\leq B^*$. It follows from the fact $G \cap A^*\in 0^\sim$ if A $\leq$ B.

5. For all A,B $\in\mathfrak{I}^X$. $(A\vee B)^* = A^*\vee B^*$.

Assume that for some x$\in$X, $(A^*\vee B^*)(x) < (A\vee B)^*(x)$.
Then $A^*(x) < (A\vee B)^*(x)$ and $B^*(x) < (A\vee B)^*(x)$.
We can find $G_1, G_2\in\tau$ such that $(A\vee B)^*(x)$ □ $G_1^c(x)$ and $(A\vee B)^*(x)$ □ $G_2^c(x)$ with $G_1\wedge A\in 0^\sim$ and $G_2\wedge B\in 0^\sim$.
But $G_i\wedge (A \vee B) \notin 0^\sim$ for i=1,2.
As $(A\vee B)^*(x)$ □ $G_1^c(x) \vee G_2^c(x) = (G_1\wedge G_2)^c(x)$, we have $(G_1\wedge G_2)\wedge(A\vee B) \notin 0^\sim$.
But $(G_1\wedge G_2)\wedge(A\vee B) = ((G_1\wedge G_2)\wedge(A)) \vee ((G_1\wedge G_2)\wedge(B))\leq (G_1\wedge A) \vee (G_2\wedge B) \in 0^\sim$ which is a contraction.
Thus, $(A\vee B)^* = A^*\vee B^*$, for all A,B $\in\mathfrak{I}^X$.

6. If $(a,b) \in\Delta$, then $C^*_{a,b}\leq C_{a,b}$.

As $A^* = cl(A^*) \leq cl(A)$ for all A $\in\mathfrak{I}^X$, it follows that $A^*\leq A$ for all intuitionistic fuzzy closed sets.
As $C_{a,b}$ is closed, for all $(a,b) \in\Delta$,we get $C^*_{a,b}\leq C_{a,b}$.

Definition 10: A map f : $\mathfrak{I}^X \rightarrow\mathfrak{I}^X$ is called intuitionistic fuzzy closure operator on a set X, if it satisfies the following axioms:

1. $f(C_{a,b}) = C_{a,b}$ for all $(a,b) \in\Delta$.
2. A $\leq$ f(A), for all A $\in\mathfrak{I}^X$.

3. $f(f(A)) = f(A)$, for all $A \in \Im^X$.
4. $f(A \vee B) = f(A) \vee f(B)$ for $A,B \in \Im^X$.

Intuitionistic fuzzy ideal

Definition 11: A family $\Im \subset \Im^X$ of IFTS, is called an intuitionistic fuzzy ideal (IFI) on X if

1. $0^{\sim} \subset \Im$
2. $f \in \Im$, $g \leq f \Rightarrow g \in \Im$
3. $f, g \in \Im, \Rightarrow f \vee g \in \Im$

Example 1 : $0^{\sim}$ is an IFI on X and $\Im^X$ itself is an IFI.

Example 2: Let $0<a<1$. Fix a point $x_0 \in X$. Then
$\Im = \{f \in \Im^X / \mu_f(x_0) \leq a\}$ is an IFI on X.

Example 3: Let X= R the real line. Let m be the Lebegque measure on R.
$f_m = \{ f \in \Im^X$ / Supp(A) has Lebegque measure zero$\}$ is a intuitionistic fuzzy ideal on R.

Result 4: Let (X, τ) be a fuzzy topological space in the sense of Lowen.
Let $f = \{f \in \Im^X$ / Int $(cl(\mu_f)) = \bar{\mathbf{0}}$ in $(X, \sigma)\}$.

Result 5: Let(X, σ) be a crisp topological space.
Let $f = \{f \in \Im^X$ / int $(\overline{supp(f)}) = \phi$ in $(X, \sigma)\}$.
Then f is an intuitionistic fuzzy ideal in X.

Intuitionistic fuzzy local function

Let (X, τ) be an IFTS and $\Im$ be an IFI in X. For every $A \in \Im^X$,
define $A^*(\Im)$ as follows:
$A^*(\Im)(x) = \vee\{(a,b) \in \Delta / G \in \tau$ and $(a,b) \leq G^c(x) \Rightarrow A \wedge^{\circ} G \notin \Im\}$.
Then $A^*(\Im)$ is called local function associated with A and $\Im$.

Remark 3 : If$f = 0^{\sim}$, then $A^*(f) = A^*$.

Remark 4 : If $A \in f$then
$A^*(f)(x) = \vee\{(a,b) \in \Delta / G \in \tau$ and $(a,b) \leq G^c(x) \Rightarrow A \wedge^{\circ} G \notin f \} = 0$ (as $A \in f \Rightarrow A \wedge^{\circ} G \in f$

Theorem 1: Let f be an intuitionistic fuzzy closure operator on a set X. Let $\Im = \{A \in \Im^X /f(A)\}$ and let $\tau = \{A^c / A \in \Im\}$. Then τ is an IFT on X such that $f(A) = cl_t(A)$ for all $A \in \Im^X$. This IFT τ is called the IFT induced by the Kuratowski closure operator f.

Theorem 2 : Let (X, τ) be an IFTS. Define the map $\phi : \Im^X \rightarrow \Im^X$ as $\phi(A) = A \vee A^*$, for all $A \in \Im^X$. Then the map ϕ is a kuratowski closure operator on $\Im^X$.

Proof: As for all $(a,b) \in \Delta$, $C^*_{a,b} \leq C_{a,b}$ we get $\phi(C_{a,b}) = C_{a,b}$
By the definition of ϕ, $A \leq \phi(A)$ for all $A \in \Im^X$.
For $A \in \Im^X$,
$\phi\phi(A) = \phi(A \vee A^*) = (A \vee A^*) \vee (A \vee A^*)^*$

$=(A\vee A^*)\vee A^*\vee (A^*)^*$
$= A\vee A^*\vee A^*\vee A^*$ as $(A^*)^* = A^*$
$=A\vee A^* = \phi(A)$
For A, B $\in\mathfrak{I}^X$,
$\phi(A\vee B) = (A\vee B) \vee (A\vee B)^*$
$= (A\vee B) \vee (A^*\vee B)^*$
$= (A\vee A^*)\vee (B\vee B^*)$
$= \phi(A)\vee\phi(B)$.

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