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Isolate Detour Eccentric Domination In Graphs

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Abstract—In this paper, the isolate detour eccentric point set and isolate detour eccentric dominating sets and their numbers are defined with examples. Also the isolate detour eccentric domination number of a graph is obtained for some standard graphs. Some theorems related to isolate detour eccentric dominating sets are stated and proved.

Keywords— Isolate Domination Number, Isolate Eccentric number, Isolate Eccentric Domination Number, Isolate Detour Eccentric Domination Number.

1. INTRODUCTION

O. Ore [3] introduced the dominating set and domination number of a graph in the year 1962 and T.N. Janakiraman et al. [6] discussed about the concepts of eccentric domination in graphs in the year 2010. I. S. Hamid and S. Balamurugan [11] initiated a study on isolated domination in graph in the year 2016 and M. Bhanumathi and R. Niroja [4] were introduced the concept of isolate eccentric domination in graphs in the year 2018. In 2019, A. M. Ismayil and R. Priyadharshini [7] introduced the new concept of detour eccentric domination number of a graph. In this paper, the isolate detour eccentric dominating sets and its numbers are defined with examples. The isolate detour eccentric domination number of a graph is obtained for some standard graphs. Some theorems related to isolate detour eccentric dominating sets are stated and proved.

2. PRELIMINARIES

In this section, some standard preliminary definitions which are used in this paper are given. Undefined terminologies and notations can refer [1.2]. Here after the graph G is a simple and connected graph.

Definition 2.1 [5]: Let $G = (V, E)$ be any graph, where $V(G)$the set of vertices and $E(G)$ is the set of edges of G. The number of elements in a vertex set V is called the *order* of $G$and is denoted by p and the number of element in an edge set E is called the *size* of G and is denoted by q. The *distance* between the vertices u and v are in V is the length of the shortest path joining between u and v and is denoted by $d(u,v)$. If $d(u, v) = 1$, then u and v are said to be *adjacent*.

Definition 2.2 [6]: Let u and v are any two vertices in a connected graph G, the *eccentricity* of u is $e(u) = \max\{d(u,v): v \in V\}$. A vertex v of G is called an *eccentric vertex* of u if $d(u,v) = e(u)$. The *radius* R and the *diameter* D of G are denoted and defined by $R = rad(G) = \min\{e(v): v \in V$ and $D = diam(G) = \max\{e(v): v \in V\}$. For any connected graph G, $rad(G) \leq diam(G) \leq 2rad(G)$. If $e(v) = rad(G)$ then the vertex v in G is called *central vertex.* The set of all central vertices are called the center $C(G)$. The central sub graph $\langle C(G) \rangle$ is induced by the center. If $e(v) = diam(G)$ then the vertex v is called *peripheral vertex.* The periphery $P(G)$ is the set of all peripheral vertices. The peripheral sub graph $\langle P(G) \rangle$ is induced by the periphery.

Definition 2.3 [7]: Let u and v are any two vertices in a connected graph G, the distance $D(u,v)$ is the length of the longest $u - v$ path in G is called *detour distance*. For any vertex u of G, the *detour eccentricity* of u is $e_D(u) = \max\{D(u,v): v \in V\}$. A vertex v of G is called a *detour eccentric vertex* of u such that $D(u,v) = e_D(u)$. The *detour radius* R_D and *detour diameter* D_D of G are defined by $R_D = rad_D(G) = \min\{e_D(v): v \in V\}$ and $D_D = diam_D(G) = \max\{e_D(v): v \in V\}$ respectively. If $e_D(v) = rad_D(G)$ then the vertex v is called *detour central vertex*. The *detour center* $C_D(G)$ is the set of all detour central vertices. The detour central sub graph $\langle C_D(G) \rangle$ is induced by the center. If $e_D(v) = diam_D(G)$ then the vertex v is called *detour peripheral vertex*. The *detour periphery* $P_D(G)$ is the set of all detour peripheral vertices. The detour peripheral sub graph $\langle P_D(G) \rangle$is induced by the detour periphery.

Definition 2.4 [9]: A set $D \subseteq V$ in a graph G is said to be a *dominating set* (D-set) if every vertex in $V - D$ is adjacent to some vertex in D. The *domination number* is denoted and defined by $\gamma(G) = \min\{|D_i| \,/\, D_i$ is a dominating set$\}$.

Definition 2.5[12]: A vertex u in V is a detour neighbor of v if $\overline{D}(u) = D(u,v)$ where$\overline{D}(u) = \min\{D(u,v)/u \in V - \{v\}\}$. A vertex u is said to detour dominate a vertex v if u is a detour neighbor of v.

Definition 2.6 [8]: A set $D \subseteq V$ in a graph G is said to be a *detour dominating set (DD-set)* in G, if every vertex of $V - D$ is detour dominated by some vertex of D. The *detour domination number* is denoted and defined by $\gamma_D(G) = \min\{|D_i| \,/\, D_i$ is a detour dominating set$\}$.

Definition 2.7 [7]: A dominating set $D \subseteq V$ of a graph G is a *detour eccentric dominating set (DED-set),* if for every$v \in V - D$, there exists at least a detour eccentric vertex v of u in D. A detour eccentric dominating set D is a *minimal detour eccentric dominating set* if there exists a subset $D' \subset D$ is not a detour eccentric domination set. The *detour eccentric domination number* is denoted and defined by $\gamma_{Ded}(G) = \min\{|D_i| \,/ D_i$ is a minimal detour eccentric dominating set$\}$. The *upper detour eccentric domination number* is denoted and defined by $\Gamma_{Ded}(G) = \max\{|D_i| \,/ D_i$ is a minimal detour eccentric dominating set$\}$.

Definition 2.8 [10]: A dominating $D \subseteq V(G)$ is called an *isolate dominating set* (ID-set) if the induced subgraph $\langle D \rangle$has an isolate vertex. The *isolate domination number* is denoted and defined by $\gamma(G) = \min\{|D_i| \,/\, D_i$ is an isolate dominating set$\}$.

Definition 2.9 [4]: An eccentric dominating set $D \subseteq V(G)$ is an *isolate eccentric dominating set (IED-set)* if the induced sub graph $\langle D \rangle$ has atleast one isolated vertex. The minimum cardinality of the isolate eccentric dominating set of G is called the isolate eccentric domination number $\gamma_{oed}(G)$.

Theorem 2.1 [7]:

$$\gamma_{Ded}(P_n) = \begin{cases} \lceil n/3 \rceil \quad , \quad if\ n = 3k+1 \\ \lceil n/3 \rceil + 1 \ , \ if\ n = 3k\ or\ 3k+2 \end{cases}$$

3. ISOLATE DETOUR ECCENTRIC POINT SET

In this chapter, the isolate detour eccentric point set and its numbers are defined with illustrative examples.

Definition 3.1: A set $H \subset V$ in a graph G is an eccentric point set (detour eccentric point set) if for every vertex $v \in V - H$ there exists at least one eccentric point (detour eccentric point) u of v in V.

Definition 3.2: A detour eccentric point set $H \subset V$ in a graph G is called an *isolate detour eccentric point set* (IDEP-set), if the induced sub graph $\langle H \rangle$ has at least an isolate vertex. The IDEP-set H is a minimal IDEP-set if a subset $H' \subset H$ is not an IDEP-set. The *isolate detour eccentric number* is denoted and defined by $e_{oDe}(G) = \min\{|D_i| \,/ D_i$ is a minimal IDEP-set}.The *upper detour eccentric number* is denoted and defined by $E_{oDe}(G) = \max\{|D_i| \,/ D_i$is a minimal IDEP-set}.

Definition 3.3: A vertex $v \in V$ of a graph G is a detour eccentric neighbor of u if $D(u,v) = e_D(u)$ and in this case $N_{De}(u) = \{v \in V \,/\, D(u,v) = e_D(u)\}$.

Example 3.1: Consider the graph G given in Figure 1,
$N_{De}(x_1) = \{x_2, x_6\}$ since$D(x_1,x_6) = D(x_1,x_2) = e_D(u_1) = 5$.

Example 3.2: Consider the graph G given in Figure 1.

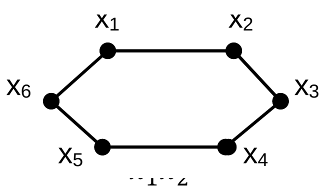

Figure 1

The IDEP-sets are $H_1=\{x_1, x_4\}$, $H_2=\{x_2, x_5\}$, $H_3=\{x_3, x_6\}$, $H_4=\{x_1, x_3, x_5\}$, $H_5=\{x_2, x_4, x_6\}$. The isolate detour eccentric number is $e_{oD}(G) = 2$ and the upper isolate detour eccentric number is $E_{oD}(G) = 3$.

Observations 3.1:

(i) For any tree T, if IDEP-set has at least one pendent vertex.

(ii) For any tree T, IEP-set and IDEP-set are the same. Since for any Tree, $d(u,v) = D(u,v)$.

(iii) For any graph G, where $e_{oD}(G) \leq E_{oD}(G)$.

(iv) Let G be a graph and H is a minimal IDEP-set, then $H' \subset H$ need not be an IDEP-set.

(v) Let G be a graph, then the complement of an isolate detour eccentric point set need not be an IDEP-set.

Example 3.3: In a path P_p, all the end vertices forms an IDED-set. But the complement of the end vertices in P_p not an IDED-set.

Observation3.2:

(i) $e_{oD}(K_p) = 1.$

(ii) $e_{oD}(W_p) = 1.$

(iii) $e_{oD}(P_p) = \begin{cases} 1, & if\ p = 2 \\ 2, & if\ p \geq 3 \end{cases}$

Theorem 3.1: let G be a graph and let H be an isolate detour eccentric point set. Then H is a minimal isolate detour eccentric point set if and only if for each vertex $u \in H$, satisfies one of the following conditions:

(a) $N_{De}(u) = \phi$

(b) there exists some $v \in V - H$ such that $E_{oD}(v) \cap H = \{u\}$.

Proof: Consider H is a minimal IDEP-set of a graph G. Then $\forall\, u \in H, H - \{u\}$ is not an IDEP-set. Then $\exists$some $v \in (V - H) \cup \{u\}$ which is not isolate detour eccentric by any vertex in $H - \{u\}$ or $\exists\ v \in (V - H) \cup \{u\}$ such that v has no isolate detour eccentric point in $H - \{u\}$.

Case(a): Assume that $u = v$, then $N_{De}(u) = \phi$ in V of graph G or u has no isolate detour eccentric point in H.

Case(b): Assume that u$\neq v$. Suppose that v has no isolate detour eccentric point in $H - \{u\}$, but v has an isolate detour eccentric point in H. Then u is the only isolate detour eccentric point of v in H. That is, $E_{oD}(v) \cap H = \{u\}$.

Conversely, suppose that H is an IDEP-set and for each $u \in H$ one of the conditions holds.

Assume that H is not a minimal IDEP-set, then $\exists$ a vertex $u \in H$ such that $H - \{u\}$ is an IDEP-set. Therefore, u is detour eccentric neighbor to at least one vertex v in $H - \{u\}$ and u has an isolate detour eccentric point in $H - \{u\}$. Hence, condition (a) does not hold.

Also, if $H - \{u\}$ is an IDEP-set, then every element v in $V - H$ is detour eccentric neighbor to at least one vertex in $H - \{u\}$ and v has an isolate detour eccentric point in $H - \{u\}$. Therefore, condition (b) does not hold. This is contradiction to our assumption that for each $u \in H$ satisfies one of the conditions. Hence, H is a minimal IDEP-set.

Observation 3.3: Every maximal independent set and its complement are D-set but need not be an IDEP-set.

Example 3.4: Consider the graph given in Figure 2.

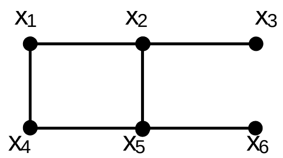

Figure 2

The some maximal independent sets { x_1, x_3, x_6} and { x_3, x_4, x_6}these are D-sets but not IDEP-sets. The sets { x_1, x_5, x_6} and { x_2, x_3, x_4}are IDEP-sets but not D-sets.

4. ISOLATE DETOUR ECCENTRIC DOMINATION IN GRAPHS

In this chapter, the isolate detour eccentric dominating set and its numbers are defined with suitable example. Isolate detour eccentric domination numbers are obtained for some well known graphs.

Definition 4.1: A DED-set $D \subset V$ in a graph G is called an *isolate detour eccentric dominating set* (IDEP-set), if the induced sub graph $\langle D \rangle$ has at least an isolated vertex. The IDED-set D is a minimal IDED-set if a subset $D' \subset D$ is not an IDED-set. The *isolate detour eccentric domination number* is denoted and defined by $\gamma_{oDe}(G) = \min\{|D_i| \,/\, D_i$ is a minimal IDED-set$\}$.The *upper detour eccentric domination number* is denoted and defined by $\Gamma_{oDe}(G) = \max\{|D_i| \,/ D_i$ is a minimal IDED-set$\}$.

Example 4.1: Consider a graph G given in Figure 3.

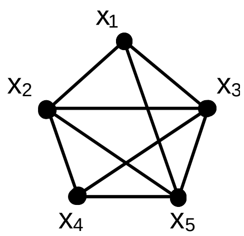

Figure 3

The D-set of G is D$_1$={x$_2$} and the domination number is $\gamma(G) = 1$.The DED-set and IDED-set of G is D$_2$={x$_1$} and the detour domination number and the isolate detour domination number is $\gamma_{Ded}(G) = \gamma_{oDed}(G) = 1$. The minimal IDED-set of G is D$_3$={x$_1$, x$_4$} then the upper isolate detour domination number is $\Gamma_{oDed}(G) = 2$.

Observation4.1:

(i) $\gamma_{oDed}(K_p) = 1$.

(ii) $\gamma_{oDed}(W_p) = 1$.

(iii) $\gamma_{oDed}(K_{1,p}) = p$.

Theorem 4.1:

$$\gamma_{oDed}(P_p) = \begin{cases} \left\lceil \frac{P}{3} \right\rceil , & if\ p = 3n+1 \\ \left\lceil \frac{p}{3} \right\rceil + 1 , & if\ p = 3n\ or\ p = 3n+2 \end{cases}$$

Proof:

The eccentricity and the detour eccentricity are same in a path graph P_p. Then the eccentric dominating set and the detour eccentric dominating set are the same. Then by theorem 2.1 the result follows.

Theorem 4.2:

$\gamma_{oDed}(C_p) = \left\lfloor \frac{p+2}{3} \right\rfloor$, if $p \geq 3$.

Proof:

Let the cycle C_P be odd or even vertices and let $C_P = x_1x_2x_3 \dots x_px_1$ be the sequence of vertices in C_P. Since C_P is 2-regular then every vertex dominates neighboring vertices and itself. Let us take $p = 3q + r,\ 0 \leq r \leq 2$.

Case(i) If $r = 0$, then q vertices eccentric dominates all the $2q$ vertices and induced subgraph of the q vertices are isolated. Therefore $\gamma_{oDed}(C_p) = q = \frac{3q}{3} = \frac{p}{3} = \left\lfloor \frac{p+2}{3} \right\rfloor$.
Case (ii) if $r = 1$ or $= 2$, then $q + 1$ vertices eccentric dominates all the $2q$ or $2q + 1$ vertices respectively and induced subgraph of the $q + 1$ vertices has an isolate vertex. Therefore $\gamma_{oDed}(C_p) = \frac{3q+1}{3} = \frac{p+1}{3} = \left\lfloor \frac{p+2}{3} \right\rfloor$ or $\gamma_{oDed}(C_p) = \frac{3q+2}{3} = \frac{p+2}{3} = \left\lfloor \frac{p+2}{3} \right\rfloor$.
Hence from the cases (i) and (ii) $\gamma_{oDed}(C_p) = \left\lfloor \frac{p+2}{3} \right\rfloor$, if $p \geq 3$.

Theorem 4.3:
For any graph G with radius two and diameter three, if G has a pendant vertex and detour eccentricity three, then $\gamma_{oDed} \leq \Delta(G)$.
Proof:
Let G be a graph has a pendant vertex and detour eccentricity three, then its support vertex v has detour eccentricity two. Therefore $N(v)$ is an IED-set, since the pendent vertex is an isolated vertex in the induced subgraph $\langle N(v) \rangle$. Thus, $\gamma_{oDed} \leq \deg(v) \leq \Delta(G)$.

Theorem 4.4:
Let T be a tree of order p with unique central vertex w and radius two then $\gamma_{oDed}(T) < p - \deg(w)$.
Proof:
Let T be a tree with unique central vertex w and radius two, then w dominates $N[w]$ and the vertices in $V - N[w]$ dominate themselves and each vertex in $N[w]$ has detour eccentric vertices in $V - N[w]$ only. Therefore, $D = V - N(w)$ is a DED-set and w is an isolated vertex of $\langle D \rangle$. Thus, $\gamma_{oDed(T)} < p - \deg(w)$.

Theorem 4.5:
Let W be a wheel graph of order p with a unique central vertex w, then $\gamma_{oDed} = p - \deg(w)$.
Proof:
If W is a wheel graph with unique central vertex w, then w dominates $N[w]$ and the vertices in $V - N[w]$ dominate themselves and each vertex in $N[w]$ has eccentric vertices in $V - N[w]$ only. Therefore, $D = V - N(w)$ is a DED-set and w is an isolated vertex of $\langle D \rangle$. Thus, $\gamma_{oDed}(G) = p - \deg(w)$.

Theorem 4.6:
Let G be a graph of order p with $rad(G) \geq 2$, then $\gamma_{oDed}(G) \leq p - \Delta(G)$.
Proof:
Let G be a graph and v be a vertex of maximum degree $\Delta(G)$. Then v dominates $N[v]$ and the vertices in $V - N[v]$ dominate themselves. Also, since $diam(G) > 2$, each vertex in $N(v)$ has detour eccentric vertices in $V - N[v]$ only. Therefore, $V - N(v)$ is a DED-set of cardinality $p - \Delta(G)$ and v is an isolated vertex in $\langle V - N(v) \rangle$, so that $\gamma_{oDed}(G) \leq p - \Delta(G)$.

Theorem 4.7:
Let G be a connected graph of order p, then $\gamma_{oDed}(G^\circ K_1) = p$.
Proof:
Let $V(G) = \{x_1, x_2, x_3, \ldots, x_p\}$. Consider $\{x_{s_1}, x_{s_2}, x_{s_3}, \ldots, x_{s_p}\}$ be the pendent vertices adjacent to $\{x_1, x_2, x_3, \ldots, x_p\}$ in $G^\circ K_1$. Then $\{x_{s_1}, x_{s_2}, x_{s_3}, \ldots, x_{s_p}\}$ is an IDED-set for $G^\circ K_1$ and is also a minimum D-set for $G^\circ K_1$. Hence, $\gamma_{oDed}(G^\circ K_1) = p$.

Observation4.2:

(i) If $= \overline{K_2} + K_1 + K_1 + \overline{K_2}$, then $\gamma(G) = 2, \gamma_o(G) = 3, \gamma_{oDed}(G) = \gamma_{Ded}(G) = 4.$

(ii) If $G = \overline{K_q} + K_1 + K_1 + \overline{K_p}$ with $\leq p$, then $\gamma(G) = 2$, $\gamma_o(G) = \min(q,p) + 1$, $\gamma_{oDed}(G) = \gamma_{Ded}(G) = \min(q,p) + 2.$

(iii) If $= K_p + K_1 + K_1 + K_p$, $p > 2$ then $\gamma(G) = \gamma_o(G) = 2$ and $\gamma_{oDed}(G) = \gamma_{Ded}(G) = 3.$

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