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A Special Type Of Minimal Dominating Fuzzy Graph

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Abstract— In this paper we introduce a new type of Fuzzy Dominating Graph such as Minimal Dominating Fuzzy Graph. The minimal dominating Fuzzy graph is denoted by MDF(G): (σ_m,μ_m) and is defined to be the intersection fuzzy graph on the minimal dominating sets of vertices in Fuzzy Graph. And characterizations are given for fuzzy graphs whose dominating fuzzy graph is connected and complete. Some other results are established relating to this new Fuzzy Graph.

Keywords—connectivity, connectedness, minimal dominating set, dominating fuzzy graph, intersection fuzzy graph.
2010 Mathematics subject classification: 03E72, 05C72

1. INTRODUCTION

The study of domination set was initiated by Ore & Berge. The domination number and the independent domination number were introduced by Cockayne and Hedetniemi. Rosenfeld introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. Somasundaram and somasundaram discussed domination in fuzzy graphs. They defined domination using effective edges in fuzzy graphs. NagoorGani and Chandrasekaran discussed domination in fuzzy graph using strong arcs. NagoorGani and Vadivel discussed domination, independent domination and irredundance in fuzzy graphs using strong arcs. V.R. Kulli Et.al introduced various type of dominating graphs which are graph valued functions in the field of domination theory. B. Basavanagoud and S.M. Hosamani introduced a new classes of intersection graph in the field of domination theory. In this paper we introduce a new type of fuzzy graph in the field of fuzzy domination theory.

2. PRELIMINARIES

Definition 2.1
The order p and size q of the fuzzy graph G=(σ,μ) are defined by p=$\sum_{v\in V}\sigma(v)$ and q=$\sum_{(u,v)\in E}\mu(u,v)$.

Definition 2.2

A path ρ in a fuzzy graph is a sequence of distinct nodes $x_0,x_1,x_2,\ldots.x_n$ such that $\mu(x_{i-1},x_i)>0$, $1 \leq i \leq n$; here $n \geq 0$ is called the length of the path.

Definition 2.3

An arc (u,v) in a fuzzy graph G=(σ,μ) is said to be strong if $\mu^{\infty}(u,v)= \mu(u,v)$ and the node v is said to be strong neighbor of u. If μ(u,v)=0 for every v ∈ V, then u is called isolated node.

Definition 2.4

Two nodes of a fuzzy graph are said to be fuzzy independent if there is no strong arc between them. A subset S of V is said to be a fuzzy independent set of G if any two nodes of S are fuzzy independent.

Definition 2.5

The strength of connectedness between two nodes u, v in a fuzzy graph G is $\mu^{\infty}(u,v)=\sup\{ \mu^k(u,v); k=1,2,3,\ldots.\}$ where $\mu^k(u,v)= \sup \{ \mu(u,u_1)\Lambda\, \mu(u,u_2)\Lambda \ldots\ldots \Lambda\, \mu(u_{k-1},v)\}$.

Definition 2.6

Let G=(σ,μ) be a fuzzy graph. A subset D of V is said to be fuzzy dominating set of G if for every v ∈ V-D there exists u ∈ D such that (u,v) is a strong arc. A dominating set D is called a minimal dominating set if no proper subset of

D is a dominating set.

Definition 2.7

A fuzzy graph G: (σ,μ) with underlying crisp graph G^*: (σ^*,μ^*) be given. Let G^* be (V,E) , S is the collection of all minimal dominating set of G. The dominating fuzzy graph of G is denoted by D(G): (σ_D,μ_D) with node set the disjoint union

of V ∪ S, where

$\sigma_D(u) = \sigma(u)$ if $u \in \sigma^*$

$= \mu^{\infty}(u,v)$ if $u,v \in \mu^*$

= 0 otherwise.

$\mu_D(v_i,v_j) = 0$ if $v_i,v_j \in \sigma *$

$\mu_D(v_i,e_j) = \mu(e_j)$ if $v_i \in \sigma *$, $e_j \in \mu *$

= 0 otherwise.

As σ_D is defined only through the values of σ & μ, σ_D: V ∪ E →[0,1] is a well- defined fuzzy subset on V ∪ E. Also μ_D is a fuzzy relation on σ_D &

$\mu_D(u,v) \leq \sigma_D(u)\ \Lambda\ \sigma_D(v)$ ∀ u,v in V ∪ E.

3. MINIMAL DOMINATING FUZZY GRAPH

Definition : 3.1

The minimal dominating fuzzy graph of G is denoted by MDF(G): (σ_m,μ_m) with the collection of all minimal dominating set S of G and whose vertices are minimal dominating sets in G for any two nodes U, V in S, (U,V) is a strong arc if U ∩V ≠ φ.where

$\sigma_m(U) = \mu^{\infty}(u,v)$ if u,v∈ Uand ∀ U ∈ S

= 0 otherwise.

$\mu_m(U,V) = \sigma_m(U)\ \Lambda\ \sigma_m(V)$ if U ∩ V ≠ φ

= 0 if U ∩ V = φ

Example:3.2

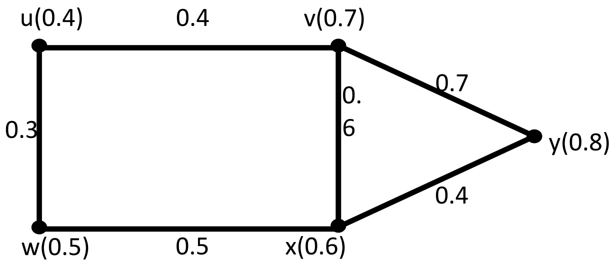

Fig 1.1- Fuzzy Graph G

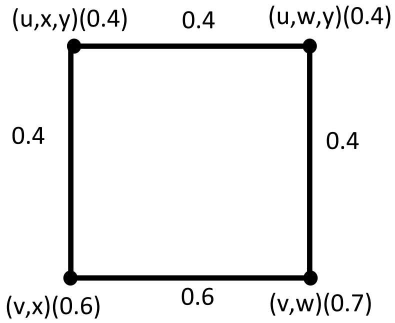

Fig 1.2 - Minimal dominating fuzzy graph MDF(G)

Properties of Minimal Dominating Fuzzy Graph
Proposition: 3.3
The Minimal Dominating Fuzzy Graph MDF(G) is a strong fuzzy graph.

Proof:
Consider an edge (U,V) in MDF(G).
By the definition of MDF(G), (U,V) is a strong arc
if $U \cap V \neq \varphi$.where
$\mu_m(U,V) = \sigma_m(U) \wedge \sigma_m(V)$
Thus every arc in MDF(G) is a strong arc.
Hence MDF(G) is a strong fuzzy graph.

Theorem: 3.4
For any fuzzy graph G with at least two vertices, The minimal dominating fuzzy graph is connected
if $\Delta(G) < p-1$.
Proof:
Assume that, $\Delta(G) < p-1$.
Let S_1 and S_2 be two disjoint minimal dominating sets of G.
We consider the following cases.

Case (i) suppose there exist two vertices u ∈ S_1 and v ∈ S_2 such that u and v are not strong neighbor. Then there exists another dominating set S_3 containing u and v. since S_3 is also a minimal dominating set, this implies that S_1 and S_2 are connected in fuzzy minimal dominating graph via S_3.
Case (ii) Suppose every vertex in S_1 is strong neighbor to every vertex in S_2. We consider the following subcases:

(a) Suppose there exist two vertices u ∈ S_1 and v ∈ S_2, such that every vertex not in S_1 U S_2 is strong neighbor to u or v. Then {u,v} is a minimal dominating set of G and hence S_1 and S_2 are connected in fuzzy minimal dominating graph over {u,v}

(b) Suppose for any two vertices u ∈ S_1 and v ∈ S_2, there exist a vertex w not belongs to S_1 U S_2 such that w is strong neighbor to neither u nor v. Then there exist two maximal independent sets S_3 and S_4 containing u,w and v respectively. Thus as the above S_1 and S_2 are connected in fuzzy minimal dominating graph via S_3 and S_4.

Thus, every two vertices in fuzzy minimal dominating graph are connected.
Hence fuzzy minimal dominating graph is connected.
Converse part need not be true.
Suppose fuzzy minimal dominating graph is connected. We consider the following cases.
Case (i)Let $\Delta(G)$ = p-1 and u is a vertex of degree p-1and also strong neighbor for all vertices in G .Then {u} is a minimum dominating set of G and V-{u} also contains minimal dominating set of G. This implies that fuzzy minimal dominating graph has at least two components, which is a contradiction to our assumption fuzzy minimal dominating graph is connected. Hence $\Delta(G) < p-1$.
Case (ii)Let $\Delta(G)$ = p-1 and u is a vertex of degree p-1and also u is not a strong neighbor for at least one vertex w in G. Then the singleton set {u} is not a minimal dominating set of G. Otherwise (u,w) is a minimal dominating set of G. The minimal dominating fuzzy graph is connected since the minimal dominating set (u,w) is intersect with some other minimal dominating set contained in u or w. Hence the converse part need not be true in this case.

Theorem: 3.5
Order [MDF(G)] = $\sum_{(u,v)\in S} \mu^{\infty}(u,v)$
Proof:
Order [MDF(G)] = $\sum_{(u,v)\in S} \sigma_m$(S)
= $\sum_{(u,v)\in S} \mu^{\infty}(u,v)$
i.e) The node set of MDF(G) is taken as the weight of strongest arc for all minimal dominating set of G.
Hence Order [MDF(G)] = $\sum_{(u,v)\in S} \mu^{\infty}(u,v)$.

Theorem: 3.6
Size [MDF(G)] = $\sum_{U\cap V\neq\emptyset} \mu^{\infty}(U)\Lambda\, \mu^{\infty}(V)$
Proof:
w.k.t. Size [MDF(G)] = $\sum_{U,V\in S} \mu(U,V)$ if $U \cap V \neq \emptyset$
By the definition of MDF(G)
$\mu_m(U,V) = \sigma_m(U)\ \Lambda\ \sigma_m(V)$ if $U \cap V \neq \varphi$
Size [MDF(G)] = $\sum_{U,V\in S} \sigma_m(U)\Lambda\, \sigma_m(V)$
= $\sum_{U,V\in S} \mu^{\infty}(U)\Lambda\, \mu^{\infty}(V)$if $U \cap V \neq \emptyset$
[By definition 3.1]
Hence Size [MDF(G)] = $\sum_{U\cap V\neq\emptyset} \mu^{\infty}(U)\Lambda\, \mu^{\infty}(V)$

Theorem 3.7
If G is a complete fuzzy graph then the following conditions hold
(i) MDF(G) will be disconnected graph.
(ii)MDF(G) has an isolated vertices only.
(iii)|E[MDF(G)]| = 0.
Proof:
let G be a complete fuzzy graph.
Since every node of G is a strong neighbor to all vertex in G,
Then every vertex of G is a minimal dominating set.
There is no intersection between the minimal dominating setssince each minimal dominating set has a distinct vertex.
This implies that MDF(G) will be disconnected graph.
And also MDF(G) is totally disconnected hence has an isolated vertices only.
There is no edge between any two vertices in MDF(G) since $U \cap V = \varphi$.
Hence |E(MDF(G))| = 0.
Theorem:3.8
The minimal dominating fuzzy graph MDF(G) of a fuzzy graph G is
(i)connected fuzzy graph (or)
(ii)totally disconnected graph(or)
(ii)MDF(G) has at most one component that is not K_1
Proof:
We consider the following cases:
Case(i): If $\Delta(G) < p-1$, then by theorem 2.4 fuzzy minimal dominating graph is connected.
Case(ii): If $\Delta(G) = \delta(G) = p-1$ and G be a complete fuzzy graph.
Then each vertex is a minimum dominating set of G.
There is no intersection between the minimal dominating set of G. And hence each component of minimal dominating fuzzy graph is K_1. Hence MDF(G) is totally disconnected graph.
Case(iii) If $\Delta(G) = \delta(G) = p-1$ and G is a semi complete fuzzy graph.
Then some arc in G may not be a strong arc and hence minimal dominating graph has at most one component that is not K_1.
Theorem:3.9
For any fuzzy graph G, the clique number $\omega(G) \leq |V(MDF(G))|$.

Proof:
W.k.t. The clique number is the cardinality of complete induced sub graph in the fuzzy graph G.
i.e) $\omega(G) = |H|$, where H be the set of vertices of complete induced sub graph in G.
Then for each vertex $u \in H$, there exist a minimal dominating set containing u.
Since every minimal dominating set is the vertices of minimal dominating fuzzy graph,
Hence $\omega(G) \leq |V(MDF(G))|$.

3. PLANARITY

Observation:4.1
If P_n is a fuzzy path with n vertices, then the characterization of minimal dominating fuzzy graph is,
(i)If $n \leq 3$, then MDF(G) is totally disconnected graph
(ii)If $3 < n \leq 5$, then MDF(G) is a planar graph
(iii)If n = 6 then MDF(G) is an outer planar graph

(iv)If n > 6, then MDF(G) is a non-outer planar graph.

Observation:4.2

If C_n is a fuzzy cycle with n vertices, then the characterization of minimal dominating fuzzy graph is,

(i)If n =3, then MDF(G) is totally disconnected graph.

(ii)If n = 4, then MDF(G) is a planar graph.

(iii)If n = 5, then MDF(G) is an outer planar graph.

(iv)If n ≥ 6, then MDF(G) is a non-outer planar graph.

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