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1 – Quasi Total Single Valued Neutrosophic Graph And Its Properties

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Abstract —. In this paper we construct the 1 – Quasi Total Single valued Neutrosophic Graph of the given Single valued Neutrosophic Graph. Some properties and relationships are observed. Also the Isomorphic property in Single Valued Neutrosophic Line graph is observed.

Keywords – ***Single valued Neutrosophic Graph, Total Single valued Neutrosophic Graph, 1 – Quasi Total Single valued Neutrosophic Graph.***

1. INTRODUCTION

Fuzzy set theory and intuitionistic fuzzy sets theory are useful models for dealing with uncertainty and incomplete information. But they may not be sufficient in modeling of indeterminate and inconsistent information encountered in real world. In order to cope with this issue, neutrosophic set theory was proposed by Smarandache as a generalization of fuzzy sets and intuitionistic fuzzy sets.

Neutrosophic set is a powerful tool to deal with incomplete, indeterminate and inconsistent information in real world. It is a generalization of the theory of fuzzy set , intuitionistic fuzzy sets , interval-valued fuzzy sets and interval-valued intuitionistic fuzzy sets , then the neutrosophic set is characterized by a truth-membershipdegree (T), an indeterminacy-membership degree (I) and a falsity-membership degree (F)independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$.

Properties and isomorphism of total and middle fuzzy graphs was given by Nagoorgani and Malarvizhi. Here, in this paper some properties of 1 – Quasi total Single valued Neutrosophic graphs is defined and isomorphic relation is discussed. Also the Isomorphic property in Single Valued Neutrosophic Line graph is observed.

2. PRELIMINARIES

A Single-Valued Neutrosophic graph(SVN graph) is a pair G = (A,B) of the crisp graph G* = (V, E)(i.e., with underlying set V) , where A : V → [0, 1] is single-valued neutrosophic set in V and B : V × V → [0, 1] is single-valued neutrosophic relation on V such that
$T_B(xy) \leq \min\{T_A(x), T_A(y)\}$,
$I_B(xy) \leq \min\{I_A(x), I_A(y)\}$,
$F_B(xy) \leq \max\{F_A(x), F_A(y)\}$
for all x, y ∈ V . A is called single-valued neutrosophic vertex set of G and B is called single-valued neutrosophic edge set of G, respectively.

Given a single-valued neutrosophic graph G = (A,B) of a crisp graph $G^* = (V, E)$, the order of G is defined as Order (G) = $(O_T(G), O_I(G), O_F(G))$,where $O_T(G) = \sum_{v\in V} T_A(v)$, $O_I(G) = \sum_{v\in V} I_A(v)$, $O_F(G) = \sum_{v\in V} F_A(v)$.

Given a single-valued neutrosophic graph G = (A,B) of a crisp graph $G^* = (V, E)$, the size of G is defined as Size(G) = $\left(S_T(G), S_I(G), S_F(G)\right)$, where $S_T(G) = \sum_{u\neq v} T_B(u, v)$, $S_I(G) = \sum_{u\neq v} I_B(u, v)$, $S_F(G) = \sum_{u\neq v} F_B(u, v)$.

The degree of a vertex x in an SVNG, G = (A, B) is defined to be sum of the weights of the edges incident at x. It is denoted by $d_G(u)$ and is equal to $(\sum_{u\neq v} T_B(u, v), \sum_{u\neq v} I_B(u, v), \sum_{u\neq v} F_B(u, v))$ for all v adjacent to u in G^*.

Two vertices x and y are said to be neighbors in SVNG if either one of the following conditions hold

$$T_B(x, y) > 0, I_B(x, y) > 0, F_B(x, y) > 0$$
$$T_B(x, y) = 0, I_B(x, y) > 0, F_B(x, y) > 0$$
$$T_B(x, y) > 0, I_B(x, y) = 0, F_B(x, y) > 0$$
$$T_B(x, y) > 0, I_B(x, y) > 0, F_B(x, y) = 0$$
$$T_B(x, y) = 0, I_B(x, y) = 0, F_B(x, y) > 0$$
$$T_B(x, y) = 0, I_B(x, y) > 0, F_B(x, y) = 0$$

$T_B(x, y) > 0, I_B(x, y) = 0, F_B(x, y) = 0$ for x, y ∈ A

Let G and G′ be single valued neutrosophic graphs with underlying sets V and V′ respectively. A homomorphism of single valued neutrosophic graphs, $h : G \to G'$ is a map $h : V \to V'$ which satisfies

$T_A(u) \leq T_{A'}(h(u))$, $I_A(u) \leq I_{A'}(h(u))$, $F_A(u) \leq F_{A'}(h(u))$ for all u ∈ V

$T_B(u, v) \leq T_{A'}(h(u), h(v))$, $I_B(u, v) \leq I_{B'}(h(u), h(v))$, $F_B(u, v) \leq F_{B'}(h(u), h(v))$ for all $u, v \in V$.

Let G and G′ be single valued neutrosophic graphs with underlying sets V and V′ respectively. An isomorphism of single valued neutrosophic graphs, $h : G \to G'$ is a bijective map $h : V \to V'$ which satisfies

$T_A(u) = T_{A'}(h(u))$, $I_A(u) = I_{A'}(h(u))$, $F_A(u) = F_{A'}(h(u))$ for all u ∈ V

$T_B(u, v) = T_{B'}(h(u), h(v))$, $I_B(u, v) = I_{B'}(h(u), h(v))$, $F_B(u, v) = F_{B'}(h(u), h(v))$ for all $u, v \in V$.Then G is said to be isomorphic to G′.Two isomorphic graphs are given below

A weak isomorphism of single valued neutrosophic graphs, $h : G \to G'$ is a map $h : V \to V'$ which is a bijective homomorphism that satisfies

$T_A(u) = T_{A'}(h(u))$, $I_A(u) = I_{A'}(h(u))$, $F_A(u) = F_{A'}(h(u))$ for all u ∈ V

A co-weak isomorphism of single valued neutrosophic graphs, $h : G \to G'$ is a map $h : V \to V'$ which is a bijective homomorphism that satisfies

$T_B(u, v) = T_{B'}(h(u), h(v))$, $I_B(u, v) = I_{B'}(h(u), h(v))$, $F_B(u, v) = F_{B'}(h(u), h(v))$ for all $u, v \in V$.

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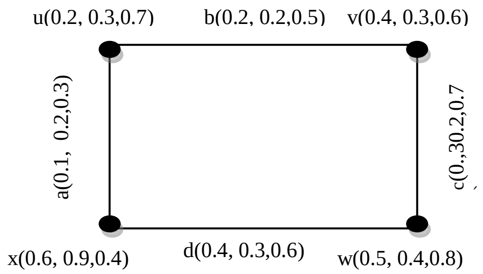
The busy value of the vertex x in G is $BV(x) = (BV_{T_A}(x), BV_{I_A}(x), BV_{F_A}(x)) = (\sum_i T_A(x) \wedge T_A(x_i), \sum_i I_A(x) \wedge I_A(x_i), \sum_i F_A(x) \vee F_A(x_i))$ where x_i are the neighbours of x and the busy value of G is $BV(G) = \sum_i BV(x_i)$ where x_i are the vertices of G.
A vertex in a G is a busy vertex if $(T_A, I_A, F_A)(x) \leq d_G(x)$.
Let G be a graph with vertex set V(G) and edge set E(G). The **1–quasitotal graph**, (denoted by $Q_1(G)$) of G is defined as follows:
The vertex set of $Q_1(G)$, that is $V(Q_1(G)) = V(G) \cup E(G)$.
Two vertices x, y in $V(Q_1(G))$ are adjacent if they satisfy one of the following conditions:
(i). x, y are in V(G) and (x,y)∈ E(G).
(ii). x, y are in E(G) and x, y are incident in G.
Let G : (A,B) be a SVN graph with the underlying crisp graph $G^* = (V, E)$. The vertices and edges of G are taken together as vertex set of $sd(G) = (A_{sd}, B_{sd})$, each edge 'e' in G is replaced by a new vertex and that vertex is made as a adjacent of those vertices which lie on 'e' in G. Here A_{sd} is a SVN subset defined on V ∪ E as

$$(T_A, I_A, F_A)_{sd}(x) = (T_A, I_A, F_A)(x) \qquad \text{if } x \in V$$
$$= (T_B, I_B, F_B)(x) \qquad \text{if } x \in E$$

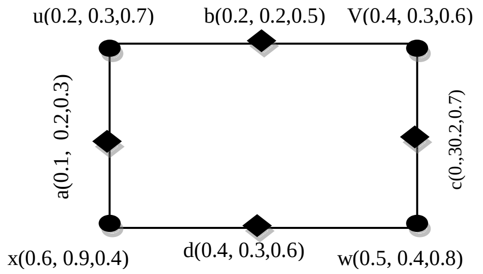
The SVN relation B_{sd} on V ∪ E is defined as

$$T_{B_{sd}}(x, y) = T_A(x) \wedge T_B(y) \qquad \text{if } x \in V \text{ and } y \in E$$
$= 0$ otherwise
$$I_{B_{sd}}(x, y) = I_A(x) \wedge I_B(y) \qquad \text{if } x \in V \text{ and } y \in E$$
$= 0$ otherwise
$$F_{B_{sd}}(x, y) = F_A(x) \vee F_B(y) \qquad \text{if } x \in V \text{ and } y \in E$$
$= 0$ otherwise

$(T_{B_{sd}}, I_{B_{sd}}, F_{B_{sd}})(x, y)$ is a SVN relation on $(T_{A_{sd}}, I_{A_{sd}}, F_{A_{sd}})$ and hence the pair $sd(G) = (A_{sd}, B_{sd})$, is a SVN graph. This pair is said as subdivision SVN graph of G.



SVN Graph - G



Subdivision Graph - sd(G)

In the above sd(G) , (a,u) =(0.1,0.2,0.7), (u,b) = (0.2,0.2,0.7), (b,v) = (0.2,0.2,0.6), (v,c) = (0.3,0.2,0.7), (c,w) = (0.3,0.2,0.8), (w,d) = (0.4,0.3,0.8), (d,x) = (0.4,0.3,0.6), (x,a) = (0.1,0.2,0.4)

Let G=(A,B) be a SVN graph with its underlying crisp graph $G^* = (V, E)$. The pair $tl(G) = (A_{tl}, B_{tl})$ of G is defined as follows. The vertex set of tl(G) is $V \cup E$. The SVN subset A_{tl} is defined on $V \cup E$ as,

$$(T_A, I_A, F_A)_{tl}(x) = (T_A, I_A, F_A)(x) \qquad \text{if } x \in V$$
$$= (T_B, I_B, F_B)(x) \qquad \text{if } x \in E$$

The SVN relation B_{tl} on $V \cup E$ is defined as

$$T_{B_{tl}}(x,y) = T_B(x,y), I_{B_{tl}}(x,y) = I_B(x,y), F_{B_{tl}}(x,y) = F_B(x,y) \text{ if } (x,y) \in E$$
$$T_{B_{tl}}(x,y) = T_A(x) \wedge T_B(y) \qquad \text{if } x \in V \text{ and } y \in E$$
= 0 otherwise
$$I_{B_{tl}}(x,y) = I_A(x) \wedge I_B(y) \qquad \text{if } x \in V \text{ and } y \in E$$
= 0 otherwise
$$F_{B_{tl}}(x,y) = F_A(x) \vee F_B(y) \qquad \text{if } x \in V \text{ and } y \in E$$
= 0 otherwise
$$T_{B_{tl}}(e,f) = T_B(e) \wedge T_B(f) \qquad \text{if } e,f \in E \ \& \ \text{they have a vertex in common}$$
= 0 otherwise
$$I_{B_{tl}}(e,f) = I_B(e) \wedge I_B(f) \qquad \text{if } e,f \in E \ \& \ \text{they have a vertex in common}$$
= 0 otherwise
$$F_{B_{tl}}(e,f) = F_B(e) \vee F_B(f) \qquad \text{if } e,f \in E \ \& \ \text{they have a vertex in common}$$
= 0 otherwise

Thus by the definition B_{tl} is a single valued neutrosophic relation on A_{tl}. Hence the pair $tl(G) = (A_{tl}, B_{tl})$ is a SVN graph and is termed as Total Single Valued Neutrosophic Graph.

I. 1 – QUASI TOTAL SINGLE VALUED NEUTROSOPHIC GRAPH

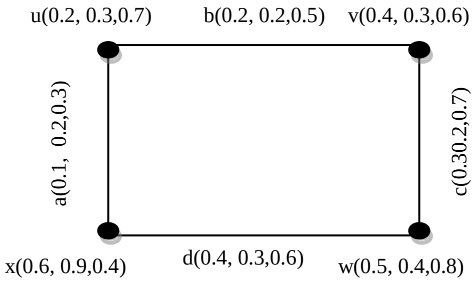
Definition 3.1 Let G=(A,B) be a SVN graph with its underlying crisp graph $G^* = (V, E)$. The pair $Q_1tl(G) = (A_{Q_1tl}, B_{Q_1tll})$ of G is defined as follows. The vertex set of Q_1tl(G) is $V \cup E$. The SVN subset A_{Q1tl} is defined on $V \cup E$ as,

$$(T_A, I_A, F_A)_{Q_1tl}(x) = (T_A, I_A, F_A)(x) \qquad \text{if } x \in V$$
$$= (T_B, I_B, F_B)(x) \qquad \text{if } x \in E$$

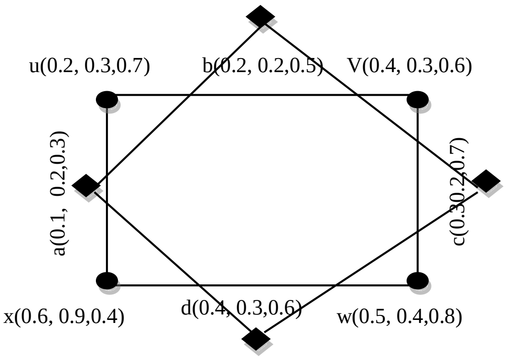
The SVN relation B_{Q_1tl} on $V \cup E$ is defined as

$$T_{B_{Q_1tl}}(x,y) = T_B(x,y), I_{B_{Q_1tll}}(x,y) = I_B(x,y), F_{B_{Q_1tl}}(x,y) = F_B(x,y) \text{ if } (x,y) \in E$$
$$T_{B_{Q_1tl}}(e,f) = T_B(e) \wedge T_B(f) \qquad \text{if } e,f \in E \ \& \ \text{they have a vertex in common}$$
= 0 otherwise
$$I_{B_{Q_1tl}}(e,f) = I_B(e) \wedge I_B(f) \qquad \text{if } e,f \in E \ \& \ \text{they have a vertex in common}$$
= 0 otherwise
$$F_{B_{Q_1tl}}(e,f) = F_B(e) \vee F_B(f) \qquad \text{if } e,f \in E \ \& \ \text{they have a vertex in common}$$
= 0 otherwise

Thus by the definition B_{Q1tl} is a single valued neutrosophic relation on A_{Q1tl}. Hence the pair $Q_1tl(G) = \left(A_{Q_1tll}, B_{Q_1tl}\right)$ is a SVN graph and is termed as 1 – Quasi Total Single Valued Neutrosophic Graph.



SVN Graph - G



1 – Quasi Total SVN Graph - Q_1tl(G)

In the above Q_1tl(G), (u, v) = (0.2, 0.2, 0.5), (v, w) = (0.3, 0.2, 0.7), (w, x) = (0.4, 0.3, 0.6), (x, u) = (0.1,0.2, 0.3), (a, b) = (0.1, 0.2, 0.5), (b, c) = (0.2, 0.2, 0.7), (c, d) = (0.3, 0.2, 0.7), (d, a) = (0.1, 0.2, 0.6)

Properties of 1 – Quasi Total SVN Graph

Theorem 3.2

Let G=(A,B) be SVN graph and tl(G) is its Total SVN graph, order of tl(G) = order(G) + size(G).

Proof : By definition of Q_1tl(G), vertex set of Q_1tl(G) is V ∪ E.

Order of Q_1tl(G) = $\left(O_T(\,Q_1 tl(G)), O_I(\,Q_1 tl(G)), O_F(Q_1 tl(G))\right)$

$= \left(\sum_{x\in V\cup E} T_{A_{Q_1 tl}}(x)\,, \sum_{x\in V\cup E} I_{A_{Q_1 tl}}(x)\,, \sum_{x\in V\cup E} F_{A_{Q_1 tl}}(x)\right)$

$= \left(\sum_{x\in V} T_{A_{Q_1 tl}}(x) + \sum_{x\in E} T_{A_{Q_1 tl}}(x)\,, \sum_{x\in V} I_{A_{Q_1 tl}}(x) + \sum_{x\in E} I_{A_{Q_1 tll}}(x)\,, \sum_{x\in V} F_{A_{Q_1 tl}}(x) + \sum_{x\in E} F_{A_{Q_1 tl}}(x)\right)$ $=\left(\sum_{x\in V} T_{A_{Q_1 tl}}(x)\,, \sum_{x\in V} I_{A_{Q_1 tl}}(x)\,, \sum_{x\in V} F_{A_{Q_1 tl}}(x)\right) + \left(\sum_{x\in E} T_{A_{Q_1 tl}}(x)\,, \sum_{x\in E} I_{A_{Q_1 tl}}(x)\,, \sum_{x\in E} F_{A_{Q_1 tl}}(x)\right)$

= order(G) + size (G) .

Theorem 3.3 : Let G=(A,B) be SVN graph and tl(G) is its Total SVN graph, size ofQ_1 tl(G) = size(G) +$\left(\sum_{x,y\in E} T_B(x) \wedge T_B(y)\,, \sum_{x,y\in E} I_B(x) \wedge I_B(y)\,, \sum_{x,y\in E} F_B(x) \vee F_B(y)\right)$

Proof : size of Q_1tl(G)= $\left(S_T(Q_1 tl(G)), S_I(Q_1 tl(G)), S_F(Q_1 tl(G))\right)$

$= \left(\sum_{x,y\in V\cup E} T_{B_{Q_1 tl}}(x,y)\,, \sum_{x,y\in V\cup E} I_{B_{Q_1 tl}}(x,y)\,, \sum_{x,y\in V\cup E} F_{B_{Q_1 tl}}(x,y)\right)$

$$= \left(\left(\sum_{x,y\in V} T_{B_{Q_1tl}}(x,y), \sum_{x,y\in V} I_{B_{Q_1tl}}(x,y), \sum_{x,y\in V} F_{B_{Q_1tl}}(x,y)\right) + \left(\sum_{x,y\in E} T_{B_{Q_1tl}}(x,y), \sum_{x,y\in E} I_{B_{Q_1tl}}(x,y), \sum_{x,y\in E} F_{B_{Q_1tl}}(x,y)\right)\right)$$

$$= \left(\sum_{x,y\in V} T_{B_{Q_1tl}}(x,y), \sum_{x,y\in V} I_{B_{Q_1tl}}(x,y), \sum_{x,y\in V} F_{B_{Q_1tl}}(x,y)\right) + \left(\sum_{x,y\in E} T_B(x)\wedge T_B(y), \sum_{x,y\in E} I_B(x)\wedge I_B(y), \sum_{x,y\in E} F_B(x)\vee F_B(y)\right)$$

$$= \mathrm{size}(G) + \left(\sum_{x,y\in E} T_B(x)\wedge T_B(y), \sum_{x,y\in E} I_B(x)\wedge I_B(y), \sum_{x,y\in E} F_B(x)\vee F_B(y)\right)$$

Theorem 3.4 : $d_{Q1tl(G)}(u) = d_G(u)$ if $u \in V$, $d_{Q1tl(G)}(y_i)$ = busy value of y_i in $Q_1tl(G)$ if $y_i \in E$.
Proof : By the definition of degree of a vertex given
Case 1: Let $x \in V$,

$$d_{Q_1tl(G)}(x) = \left(\sum_{a\in V} T_{B_{Q_1tl}}(x,a), \sum_{a\in V} I_{B_{Q_1tl}}(x,a), \sum_{a\in V} F_{B_{Q_1tl}}(x,a)\right)$$

$$= \left(\sum_{y\in E} T_B(y), \sum_{y\in E} I_B(y), \sum_{y\in E} F_B(y)\right)$$

$=d_G(x)$
Case 2: If $y_i \in E$,

$$d_{Q_1tl(G)}(y_i) = \left(\sum_{b\in E} T_{B_{tl}}(y_i,b), \sum_{b\in E} I_{B_{tl}}(y_i,b), \sum_{b\in E} F_{B_{tl}}(y_i,b)\right)$$

$$= + \left(\sum_{b\in E} T_B(y_i)\wedge T_B(b), \sum_{b\in E} I_B(y_i)\wedge I_B(b), \sum_{b\in E} F_B(y_i)\vee F_B(b)\right)$$

= busy value of y_i in $Q_1tl(G)$.

Theorem 3.5 : 1- Quasi Total Single Valued Neutrosophic of any Single Valued Neutrosophic is disconnected.
Proof: Let G = (A, B) be a SVN graph. The SVN vertex set of $Q_1tl(G)$ is $V \cup E$ where V and E are vertex set and Edge set of G respectively, and the SVN relation is only defined between $x, y \in V$and $e, f \in E$. As there is no SVN relation between $x \in V$and $e \in E$ of elements in the vertex set of $Q_1tl(G)$, there is no path that connects u and e in $Q_1tl(G)$. Hence, $Q_1tl(G)$ is disconnected graph.
Theorem 3.6 : If G is a SVN graph then sd(G) is weak isomorphic toQ_1 tl(G).
Proof : Let G = (A,B) be a SVN graph with its underlying crisp graph G^*= (V,E). By the definition of sd(G), A_{sd} is a SVN subset defined on $V \cup E$ as

$$(T_A, I_A, F_A)_{sd}(x) = (T_A, I_A, F_A)(x) \qquad \text{if } x \in V$$

$= (T_B, I_B, F_B)(x) \quad \text{if } x \in E \quad (1)$

The SVN relation B_{sd} on $V \cup E$ is defined as

$$T_{B_{sd}}(x, y) = T_A(x) \wedge T_B(y) \quad \text{if } x \in V \text{ and } y \in E$$

$= 0$ otherwise

$$I_{B_{sd}}(x, y) = I_A(x) \wedge I_B(y) \quad \text{if } x \in V \text{ and } y \in E$$

$= 0$ otherwise

$$F_{B_{sd}}(x, y) = F_A(x) \vee F_B(y) \quad \text{if } x \in V \text{ and } y \in E$$

$= 0$ otherwise

Using (1) in the above equation,

$$T_{B_{sd}}(x, y) = T_{A_{sd}}(x) \wedge T_{A_{sd}}(y) \quad \text{if } x \in V \text{ and } y \in E$$

$= 0$ otherwise

$$I_{B_{sd}}(x, y) = I_{A_{sd}}(x) \wedge I_{A_{sd}}(y) \quad \text{if } x \in V \text{ and } y \in E$$

$= 0$ otherwise

$$F_{B_{sd}}(x, y) = F_{A_{sd}}(x) \vee F_{A_{sd}}(y) \quad \text{if } x \in V \text{ and } y \in E$$

$= 0$ otherwise

Define a map 'g' from sd(G) to Q_1tl(G) as identity map $g: V \cup E \to V \cup E$, g be bijection satisfying

$$(T_A, I_A, F_A)_{Q_1tl}(g(x)) = (T_A, I_A, F_A)_{Q_1tl}(x) = (T_A, I_A, F_A)(x) = (T_A, I_A, F_A)_{sd}(x) \text{ if } x \in V$$

$$(T_A, I_A, F_A)_{Q_1tl}(g(x)) = (T_A, I_A, F_A)_{Q_1tl}(x) = (T_B, I_B, F_B)(x) = (T_A, I_A, F_A)_{sd}(x) \text{ if } x \in E$$

That is $(T_A, I_A, F_A)_{Q_1tl}(g(x)) = (T_A, I_A, F_A)_{sd}(x)$ if $x \in V \cup E$

Case 1:

If $x, y \in V$, $(T_B, I_B, F_B)_{tl}\big(g(x), g(y)\big) = (T_B, I_B, F_B)_{Q_1tl}(x, y) = (T_B, I_B, F_B)(x, y)$ if $x, y \in V$.

By the definition of sd(G), $(T_B, I_B, F_B)_{sd}(x, y) = 0$ if $x, y \in V$

That implies $(T_B, I_B, F_B)_{sd}(x, y) \leq (T_B, I_B, F_B)_{tl}(g(x), g(y))$ if $x, y \in V$

Case 2:

If $x = e_i, y = e_j \in E$ then

$$T_{B_{Q_1tl}}(e_i, e_j) = \min\{T_B(e_i), T_B(e_j)\} \text{ if } e_i, e_j \text{ have a vertex in common}$$

$$I_{B_{Q_1tl}}(e_i, e_j) = \min\{I_B(e_i), I_B(e_j)\} \text{ if } e_i, e_j \text{ have a vertex in common}$$

$$F_{B_{Q_1tl}}(e_i, e_j) = \max\{F_B(e_i), F_B(e_j)\} \text{ if } e_i, e_j \text{ have a vertex in common}$$

$= 0$ otherwise

$$T_{B_{sd}}(e_i, e_j) \leq T_{B_{tl}}(e_i, e_j) \text{ if } e_i, e_j \in E$$

$$I_{B_{sd}}(e_i, e_j) \leq I_{B_{tl}}(e_i, e_j) \text{ if } e_i, e_j \in E$$

$$F_{B_{sd}}(e_i, e_j) \leq F_{B_{tl}}(e_i, e_j) \text{ if } e_i, e_j \in E$$

Thus from the cases we get

$$T_{B_{sd}}(x, y) \leq T_{B_{tl}}(x, y) \text{ if } x, y \in V \cup E$$

$$I_{B_{sd}}(x, y) \leq I_{B_{tl}}(x, y) \text{ if } x, y \in V \cup E$$

$$F_{B_{sd}}(x, y) \leq F_{B_{tl}}(x, y) \text{ if } x, y \in V \cup E$$

Therefore $g: sd(G) \to Q_1tl(G)$ is a weak isomorphism.

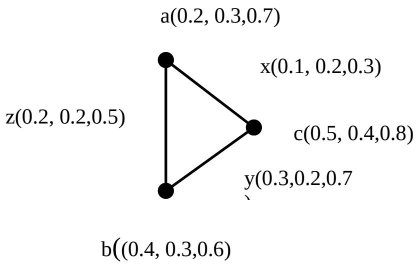
IV. SINGLE VALUED NEUTROSOPHIC LINE GRAPH

Definition 4.1 : Let G = (A, B) be a SVN graph with the underlying graph G* = (V, E). The SVN line graph of G is L(G) = (P, Q) with the underlying graph (Z, W) where the vertex set is $Z = \{S_x = \{x\} \cup \{u_x, v_x\}: x \in E, u_x, v_x \in V, x = (u_x, v_x)\}$
and $W = \{(S_x, S_y) : S_x \cap S_y \neq \emptyset, x, y \in E, x \neq y\}$

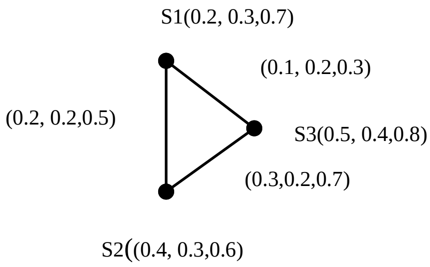
$P(S_x) = (T_B(x), I_B(x), F_B(x))$

$Q(S_x, S_y) = (\mathrm{T_B}(\mathrm{x}) \wedge \mathrm{T_B}(\mathrm{y}), \mathrm{I_B}(\mathrm{x}) \wedge \mathrm{I_B}(\mathrm{y}), \mathrm{F_B}(\mathrm{x}) \vee F_{\mathrm{B}}(\mathrm{y}))$for all $(S_x, S_y) \in W$.

Example:



SVN Graph - G



Line Graph – L(G)

Theorem 4.2: if G1 = (A1, B1) and G2 = (A2, B2) are the two isomorphic SVN graphs then their SVN line graphs are also isomorphic.

Proof: Give G1 and G2 are the two isomorphic SVN graphs with the underlying set S1 and S2 respectively, i.e., there exists a bijective map h : S1 → S2 satisfying

$T_{A_1}(x) = T_{A_2}(h(x));\ I_{A_1}(x) = I_{A_2}(h(x));\ F_{A_1}(x) = F_{A_2}(h(x))\ for\ all\ x \in S_1$

$T_{B_1}(x) = T_{B_2}(h(x), h(y));\ T_{B_1}(x) = T_{B_2};\ T_{B_1}(x) = T_{B_2}(h(x), h(y))\ for\ all\ x, y \in S_1$.

Let L(G1) = (P1,Q1) and L(G2) = (P2, Q2) be the line graphs of G1 and G2 respectively.

Consider an $x \in E_1$. Let $x = (u_x, v_x)$. As h : S1 → S2 is one to one, onto, $h(x) = (h(u_x), h(v_x)) \in E_2$

Define : Z1 → Z2 as $g(S_x) = S_{h(x)}$

As h is one to one and onto, g is well defined and one to one onto mapping.

Consider$T_{P_1}(S_x) = T_{B_1}(x) = T_{B_1}(u_x, v_x) = T_{B_2}(h(u_x), h(v_x)) = T_{B_2}(h(x))$

$T_{P_1}(S_x) = T_{P_2}(S_{h(x)}) = T_{P_2}(g(S_x))\ for\ all\ x \in Z_1$

Similarly, $I_{P_1}(S_x) = I_{P_2}(g(S_x))$ and $F_{P_1}(S_x) = F_{P_2}(g(S_x))\ for\ all\ x \in Z_1$ (1)

$T_{Q_1}(S_x, S_y) = T_{B_1}(x) \wedge T_{B_1}(y)$ for all $(S_x, S_y) \in W_1$

= $T_{B_1}(u_x, v_x) \wedge T_{B_1}(u_y, v_y)$

= $T_{B_2}(h(u_x), h(v_x)) \wedge T_{B_2}(h(u_y), h(v_y))$

= $T_{B_2}(h(x)) \wedge T_{B_2}(h(y))$

= $T_{Q_2}(S_{h(x)}, S_{h(y)})$

= $T_{Q_2}(g(S_x), g(S_y))$ for all $x, y \in E_1$
$T_{Q_1}(S_x, S_y) = T_{Q_2}(g(S_x), g(S_y))$ for all $S_x, S_y \in Z_1$
Similarly,
$I_{Q_1}(S_x, S_y) = I_{Q_2}(g(S_x), g(S_y))$, $F_{Q_1}(S_x, S_y) = F_{Q_2}(g(S_x), g(S_y))$ for all $S_x, S_y \in Z_1$ (2)
From equations (1) and (2), L(G1) and L(G2) are isomorphic SVN line graphs when G1 and G2 are the two isomorphic SVN graphs.

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