International Journal of Aquatic Science
ISSN: 2008-8019
Vol 12, Issue 02, 2021

An Economic Order Quantity Model With Imperfect Quality Items And Karush Khun Tucker Approach

I.Francina Nishandhi

Assistant Professor
Department of Mathematics, Holy Cross College (Autonomous)
Trichy -620 002

Abstract: This paper examines an inventory system for growing items, so that the new-born items are fed to reach the ideal weight for consumers after which they are slaughtered. All the slaughtered items are gone through the screening process so that we could identify the good quality items from those of imperfect quality. So the shortage is occurring and the shortages are fully backordered. And also this paper develops the model under the concept of proper disposal of animal mortality during their growth period. Due to the certain circumstances the animals are fall into the sick and dead, and for the environmental concern they are properly disposed from their places. To encourage the purchasing companies the supplier offers the incremental quantity discounts but it will leads to increase the holding cost of the purchasing company and this problem is sort out by using budget capacity constraint. So that the main aim of this proposed model is to reduce the total cost of the system.

Keywords: Inventory management, Economic order quantity, Growing items, imperfect quality, back order, waste disposal cost

1. INTRODUCTION

Inventory management is essential for smooth progressing of every business scenario to fulfill the customer's desire at the right time. The first EOQ model was developed by Harris in the year 1913 with the base of this model Taft established an economic production quantity model in the year 1929. Based on the EOQ and EPQ model many researchers have developed their model under various assumptions. Rosenblatt et al., (1986) he was explained the inventory model under the concept of imperfect quality items. Salameh and Jaber (2000) developed an EOQ model with defective items in which the products are sold in a single batch at the end of 100% screening process. Wee et al., (2007) added a shortage backordering assumption to Salameh and Jaber's (2000) model. Razaei (2014) was the first researcher who has developed his inventory model under the concept of growing items. In this model he explained the inventory system had distinct periods, namely growth and consumption periods. Various researchers have started to extend the work on inventory control for growing items into diverse other areas. For instance, Zhang et al., (2016) incorporated environmental sustainability to Rezaei (2014)'s work by developing an EOQ model for growing items in a carbon constrained environment. Nobil et al., (2018) extended the growing items inventory. Sabatjane M. et al., (2018) had explained the Economic order quantity model for growing items with imperfect quality. This proposed model is the extension work Sebatjane. M. et al., (2019). In this paper he explained an EOQ model for growing items with incremental quantity discounts. To attract the purchasing companies if the suppler offers the incremental discount but this will intend to increase the holding cost of the purchasing company. By using budget capacity to limited the purchasing quantities of the product. Due to the certain circumstances the animals are fall into the sick and dead, and for the environmental concern they are properly disposed from their places. This proposed work also considers the concept of imperfect quality items. Before it reaches the customers the slaughtered items are screened and the imperfect quality items are found so that the shortages occur and it is fully backordered. The main objective of this proposed work is to reduce the total cost of the purchasing company.

The remainder of the session organized as follows, section 2 develops the notation and assumptions of the proposed model. Section 3 illustrates the mathematical model of the proposed model, section 4 explains the numerical example and section 5 concludes the proposed model.

2. NOTATIONS AND ASSUMPTIONS:

The following notations and assumptions is used for this proposed model

Y : Number of ordered newborn items per cycle

w_o : Approximated weight of each newborn item

w_1 : Approximated weight of each grown item at the time of slaughtering

Q_t : Total weight of inventory at time t

α : Asymptotic weight of the items

β : Integration constant

λ : Exponential growth rate of the items

p_j : Purchasing cost per weight unit at the j^{th} break point

m: Number of break points

y_j : Lower bound for the order quantity for price j

h: Holding cost per weight unit per unit per time

K: Setup cost per cycle

D: Demand in weight units per unit time

c: Feeding cost per weight unit per unit time

x : Percentage of slaughtered items that are poorer quality

Z: screening cost per weight unit

r: screening rate

t : Growing period

T: Cycle length

γ : cost to dispose waste to the environment

θ : Proportion of waste produced per lot Y

B : Buyer's maximum available budget to purchase products

b : backorder cost rate

S: Maximum backorder inventory

α' : Proportion of demand returned

Assumptions:

1. The ordered items are capable of growing prior to being slaughtered
2. A single type of item is considered
3. Feeding cost is incurred for feeding and growing the items during the growth period. These costs are proportional to the weight gained by the items.
4. Holding costs are incurred for the duration of the consumption period
5. The supplier of the live newborn items offers incremental quantity discounts.
6. Demand is a deterministic constant
7. A random fraction of the slaughtered items is of poorer quality.
8. The screening process is 100 percent effective
9. Shortage is completely backordered
10. The purchasing cost for all products is limited , mathematically,

$$pY \le B$$

3. MATHEMATICAL MODEL:

The Cost functions are defined as follows:

Purchasing cost per unit time is PCU = $D\left[\frac{R_j}{Yw_1(1-E(x))}+\frac{p_j w_0}{w_1}-\frac{p_j w_0 y_j}{Yw_1}\right]$

Food Procurement cost per unit time is FCU = $\frac{cD\alpha}{w_1(1-E(x))}\left\{t+\frac{1}{\lambda}\left[\ln(1+\beta e^{-\lambda t})-\ln(1+\beta)\right]\right\}$

Setup Cost per unit time is SCU = $\frac{KD}{yw_1(1-E(x))}$

Screening cost per unit time is ZCU = $\frac{ZD}{w_1(1-E(x))}$

Holding cost per unit time is HCU= $h\left[\frac{Yw_1(1-E(x))}{2}+\frac{Yw_1DE(x)}{r(1-E(x))}\right]$

Backordering cost per unit time is: $\frac{bS^2}{2Yw_1(1-E(x))}$

Waste disposal cost: $\frac{\gamma_o D}{Yw_1(1-E(x))}+\frac{\gamma\left(\theta+\alpha'\right)D}{w_1\left(1-E(x)\right)}$

The expected total annual cost of the system is ***ETC (Y)*** :

$$D\left[\frac{R_j}{Yw_1(1-E(x))}+\frac{p_j w_0}{w_1}-\frac{p_j w_0 y_j}{Yw_1}\right]+\frac{cD\alpha}{w_1(1-E(x))}\left\{t+\frac{1}{\lambda}\left[\ln(1+\beta e^{-\lambda t})-\ln(1+\beta)\right]\right\}$$
$$+\frac{KD}{yw_1(1-E(x))}+\frac{ZD}{w_1(1-E(x))}+h\left[\frac{Yw_1(1-E(x))}{2}+\frac{Yw_1DE(x)}{r(1-E(x))}\right]+\frac{bS^2}{2Yw_1(1-E(x))}$$
$$+\frac{\gamma_o D}{Yw_1(1-E(x))}+\frac{\gamma\left(\theta+\alpha^{'}\right)D}{w_1\left(1-E(x)\right)}$$
(1)

The purchasing cost for all products is limited, mathematically $pY \le B$.The vendor buyer inventory model with budget capacity constraint is

$$D\left[\frac{R_j}{Yw_1(1-E(x))}+\frac{p_j w_0}{w_1}-\frac{p_j w_0 y_j}{Yw_1}\right]+\frac{cD\alpha}{w_1(1-E(x))}\left\{t+\frac{1}{\lambda}\left[\ln(1+\beta e^{-\lambda t})-\ln(1+\beta)\right]\right\}$$
$$+\frac{KD}{yw_1(1-E(x))}+\frac{ZD}{w_1(1-E(x))}+h\left[\frac{Yw_1(1-E(x))}{2}+\frac{Yw_1DE(x)}{r(1-E(x))}\right]+\frac{bS^2}{2Yw_1(1-E(x))}$$
$$+\frac{\gamma_o D}{Yw_1(1-E(x))}+\frac{\gamma\left(\theta+\alpha^{'}\right)D}{w_1\left(1-E(x)\right)}$$

Subject to $pY \le B$. (2)

3.1 Solution Technique:

The development of Karush Khun Tucker conditions is based on the Lagrangian method. The expected total cost of equation (1) can be written as follows,

Minimize f(z) = *ETC (Y)*

Subject to g(z)= $pY - B \le 0$

A new function i.e. the Lagrangian function $ETC\left(Y,\lambda\right)$ is formed by introducing Lagrangian multiplier λ then we have

$$ETC\left(z,\lambda\right)=f(z)-\lambda g(z)$$

$$D\left[\frac{R_j}{Yw_1(1-E(x))}+\frac{p_j w_0}{w_1}-\frac{p_j w_0 y_j}{Yw_1}\right]+\frac{cD\alpha}{w_1(1-E(x))}\left\{t+\frac{1}{\lambda}\left[\ln(1+\beta e^{-\lambda t})-\ln(1+\beta)\right]\right\}$$
$$+\frac{KD}{yw_1(1-E(x))}+\frac{ZD}{w_1(1-E(x))}+h\left[\frac{Yw_1(1-E(x))}{2}+\frac{Yw_1DE(x)}{r(1-E(x))}\right]+\frac{bS^2}{2Yw_1(1-E(x))}$$
$$+\frac{\gamma_o D}{Yw_1(1-E(x))}+\frac{\gamma\left(\theta+\alpha^{'}\right)D}{w_1\left(1-E(x)\right)}-\lambda\left(pY-B\right)$$

(3)

The Khun –Tucker conditions need z and λ to be stationary point of minimization problem which can be summarized as following:

$$\begin{cases}\nabla f(z)-\lambda g(z)=0\\ \lambda g(z)=0\\ g(z)\le 0\\ \lambda\ge 0\end{cases}$$

By the method of Khun- Tucker conditions, consider the two cases λ =0 and $\lambda \neq 0$

For λ =0 the optimal order quantity of case (1) as follows

$$Y^* = \sqrt{\frac{2r\left\{D\left[R_j - p_j y_j w_0 + K + \gamma_0\right] + bS^2\right\}}{w_1^2\left\{r\left(1-E(x)\right)^2 + 2DE(x)\right\}}}$$

$\lambda \neq 0$, the optimal value of Y and λ expressed as follows,

$$\lambda = \frac{p_j D}{pY^2 w_1(1-E(x))} - \frac{y_j w_0 p_i}{pY^2 w_1(1-E(x))} + \frac{KD}{pY^2 w_1(1-E(x))} + \frac{hw_1\left(1-E\left(x\right)\right)}{2p} + \frac{hw_1 DE(x)}{rp\left(1-E\left(x\right)\right)}$$
$$+ \frac{bS^2}{2Y^2 w_1\left(1-E\left(x\right)\right)p} + \frac{\gamma_0 D}{Y^2 w_1\left(1-E\left(x\right)\right)p}$$

$$Y^* = \sqrt{\frac{2r\left\{D\left[R_j - p_j y_j w_0 + K + \gamma_0\right] + bS^2\right\}}{w_1^2\left\{r\left(1-E(x)\right)^2 + 2DE(x)\right\} - \lambda p}}$$

3. NUMERICAL EXAMPLE:

D = 1,000000g/year, K=1 000 ZAR/cycle, h=0.04 ZAR/g/year, c =2.5 ZAR/g/year; w_1 = 1 500 g; p = 0.025 ZAR/g, h= 0.00025ZAR/g , r=5,256,000 g/year, B =3000/year, f = 2 S = 4000 /year , γ = 1, θ =0.5 , α' = 0.02, α =41kg , β = 5 , λ = 7.3 /year

λ = 0, the optimum order quantity Y = 140 and Expected Total cost is = 53140

$\lambda \neq 0$, the optimum order quantity Y = 140 and Expected Total cost is = 48140

4. CONCLUSION:

In this proposed model we have developed the inventory model for the live stocks along with the budget capacity constraints. The supplier offer the incremental discount for the purchasing company but during the screening process the buyer able to find the poorest quality items so that the shortages are fully backordered. Ordering the larger quantity items it would increase the holding cost of the buyer, so using budget capacity constraint the buyer will limit his purchases and also it will reduces his total cost.

5. REFERENCES:

[1]. Harris, F.W, 1913 , How many parts to make at once. Factoty The Maazine of Management 10(2), 135–136, 152

[2].Covert, R.P. Philip, G.C, 1973, An EOQ model for items with Weibull distribution deterioration. AIIE Trans. 5(4), 323–326.

[3]. Dobson,G, Pinker,E.J, Yildiz,O, 2017, An EOQ model for perishable goods with age-dependent demand rate. European Journal of Operations Research. 257(1), 84–88

[4]. Ghare, P.M. Schrader, G.F, 1963, A model for exponentially decaying inventory, Journal of Industrial Engineering , 14(5), 238–243
[5]. Goyal,S.K, Giri,B.C, 2001, Recent trends in modeling of deteriorating inventory. European Journal of Operations Research, 134(1), 1–16.
[6]. Nobil A, Sedigh A, Cardenas Barron L , A generalized economic order quantity inventory model with shortage ; case study of a poultry framer. Arabian Journal of science Engineering, http :// doi.org/10.1007/s13369-018-3322-z
[7]. Rezaei,J,2014 Economic order quantity for growing items. International Journal of Production Economics, 155, 109–113.
[8]. Rosenblatt MJ, Lee HL, 1986, Economic production cycles with imperfect production process, IIE Transcactions, 18 : 48 -55.
[9]. M. Salameh , MY. Jaber, 2000, Economic production quantity model for items with imperfect quality. International Journal of Production Economics, 64: 59 - 64
[10]. Sebatjane .M., Adetunji .O., 2019 , Economc order quanity model for growing items with incremental quantity discounts, Journal of Industrial Engineering International. http:// doi.org/10.1007/s40092 -019-0311-0
[11]. Sebatjane .M, 2018, Economic order quantity model for growing items with imperfect quality, Operations Research Perspectives, doi: https://doi.org/10.1016/j.orp.2018.11.004
[12]. Taft, E.W. 1918, The most economical production lot. Iron Age 101, 1410–1412.
[13]. Wee.H.M, Lo.S.T, Huang.W.C, 2007, An Integrated production inventory model with imperfect production processes and Weibull distribution deterioration under inflation, International Journal of Production Economics 106 (1) , 248 -260.
[14]. Whitin, T.M, 1953 ,The Theory of Inventory Management. Princeton University Press, Princeton.
[15]. Yan.X,Wang,Y, 2013, An EOQ model for perishable items with supply uncertainty.Math.Probl.Eng.,ArticleID739425,pp.1–13.
[16]. Zhang.Y, Li, L.Y.Tian, X.Q, Feng,C. 2016, Inventory management research for growing items with carbon-constrained. In: Control Conference(CCC),35thChinese,pp.9588–9593.IEEE.