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Shoddy Pro – Offering Technique Using Lagrangian Method

W.Ritha[1], R.Saarumathi[2]

[1,2]Department of Mathematics,Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirappalli – 620002 Tamilnadu, India

ABSTRACT: This paper signifies the manufacture of a product in a single stage manufacturing system which may generate imperfect quality products. Such defective products are reworked using fuzzy optimization and trapezoidal numbers to find the total cost so as to reduce the overall production cost significantly. To achieve this objective, two inventory models are developed. The first model gives a unique solution for imperfect production system with rework and the second model highlights on imperfect production system with rework and shortage.

KEYWORDS: Fuzzy inventory, rework, shortage, Graded mean integration and Lagrangian method

1. INTRODUCTION:

The principle strategy of a manufacturing firm is to satisfy the customer's demand to their fullest expectation at a low-cost. To pursue the above goal, improvement goal like TQM (Total Quantity Management), JIT (Just-in-time production), EPQ (Economic Production Quantity) are to be used by the practitioners in the field of production and inventory management to assist rapid product development at low cost-relatively quick and with minimal resources.

However, generation of defective items is inevitable in real life production environment. At the same time, defective items cannot be ignored in the production process.

EPQ determines the company to minimize the total inventory cost by balancing the inventory holding cost and average fixed ordering cost. In February 1913, Harris first introduced (EOQ) Economic Order Quantity and a few years later Economic Production Quantity (EPQ) inventory model was proposed by E. W. Taft in 1918. These models assist the manufacturers to minimize the total inventory cost by balancing the inventory holding cost and average fixed ordering cost.

The primary goal of this paper is to keep right quantity of every material in order to satisfy the customer's demand and rework to avoid shortage and excess inventory. With this view, this paper focusses on imperfect production system with rework and shortages using trapezoidal fuzzy number.

2. PRELIMINARIES:

FUZZY SET:

If X is an universe of discourse and x is a particular element of X then the fuzzy set A defined on X can be written as the collection of ordered pairs

$$A = \{ X, \mu_i(x); x \in X \}$$

FUZZY NUMBER:

A fuzzy number is a fuzzy subset in the universe of discourse X that is both convex and normal.

TRAPEZOIDAL FUZZY NUMBER:

A trapezoidal fuzzy number $\tilde{A} = (a,b,c,d)$ is represented with membership function $\mu_{\tilde{A}}(x)$ as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, when\, a \le x \le b \\ 1, when\, b \le x \le c \\ \frac{d-x}{d-c}, when\, c \le x \le d \\ 0, otherwise \end{cases}$$

3. ASSUMPTIONS:

1. Holding cost is constant.
2. Proportion of defective item in the manufacturing process during the production cycle is taken into account as a constant one.
3. Shortages are not allowed.

4. NOTATIONS:

P - Production rate
D - Demand rate
d - Deteriorating defective item
Q - Optimal extent of production run
A - Setup cost
H - Holding cost
R - Reworking cost
S - Shortage cost
x - Proportion of defective item at the time of production
t - Time period in units
T - Cycle time
B - Considerable shortage in units
C - Production cost

5. MATHEMATICAL MODEL:

5.1 AN INVENTORY MODEL FOR SHODDY PRO – OFFERING TECHNIQUE WITH REWORK:

We consider the model with rework in fuzzy environment since the holding cost, production cost, reworking cost are fuzzy in nature. We fuzzify them with trapezoidal fuzzy numbers using Lagrangian method.

International Journal of Aquatic Science
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Setup cost: $\frac{D}{Q}A$

Production cost: DC

Inventory carrying cost: $\frac{QH}{2P}[P - D(1+x+x^2)]$

Reworking cost per unit time: DxR

Total cost is given by

$$\tilde{TC}(Q) = \frac{1}{6}\left\{\left(\frac{D_1A_1}{Q_1} + D_1C_1 + \frac{Q_1H}{2P_1}[P_1 - D_1(1+x+x^2)] + D_1xR_1\right) + 2\left(\frac{D_2A_2}{Q_2} + D_2C_2 + \frac{Q_2H}{2P_2}[P_2 - D_2(1+x+x^2)] + D_2xR_2\right) + 2\left(\frac{D_3A_3}{Q_3} + D_3C_3 + \frac{Q_3H}{2P_3}[P_3 - D_3(1+x+x^2)] + D_3xR_3\right) + \left(\frac{D_4A_4}{Q_4} + D_4C_4 + \frac{Q_4H}{2P_4}[P_4 - D_4(1+x+x^2)] + D_4xR_4\right)\right\}$$

partially differentiating with respect to $'Q'$

$$\tilde{TC}(Q) = \frac{1}{6}\left\{\left(\frac{-D_1A_1}{Q_1^2} + \frac{H}{2P_1}[P_1 - D_1(1+x+x^2)]\right) + 2\left(\frac{-D_2A_2}{Q_2^2} + \frac{H}{2P_2}[P_2 - D_2(1+x+x^2)]\right) + 2\left(\frac{-D_3A_3}{Q_3^2} + \frac{H}{2P_3}[P_3 - D_3(1+x+x^2)]\right) + \left(\frac{-D_4A_4}{Q_4^2} + \frac{H}{2P_4}[P_4 - D_4(1+x+x^2)]\right)\right\}$$

$$Q = \sqrt{\frac{2P_1D_1A_1 + 2(2P_2D_2A_2) + 2(2P_3D_3A_3) + 2P_4D_4A_4}{H\{[P_1 - D_1(1+x+x^2)] + [P_2 - D_2(1+x+x^2)] + [P_3 - D_3(1+x+x^2)] + [P_4 - D_4(1+x+x^2)]\}}}$$

Step 1:

$$\frac{\partial P}{\partial Q_1} = \frac{1}{6}\left\{\frac{-D_1A_1}{Q_1^2} + \frac{H}{2P_1}[P_1 - D_1(1+x+x^2)]\right\}$$

Let $\frac{\partial P}{\partial Q_1} = 0$

$$Q_1 = \sqrt{\frac{2P_4D_4A_4}{H[P_1 - D_1(1+x+x^2)]}}$$

$$\frac{\partial P}{\partial Q_2} = \frac{1}{6}\left\{\frac{-D_2A_2}{Q_2^2} + \frac{H}{2P_2}[P_2 - D_2(1+x+x^2)]\right\}$$

Let $\frac{\partial P}{\partial Q_2} = 0$

$$Q_2 = \sqrt{\frac{2P_3D_3A_3}{H[P_2 - D_2(1+x+x^2)]}}$$

$$\frac{\partial P}{\partial Q_3} = \frac{1}{6}\left\{\frac{-D_3A_3}{Q_3^{\ 2}} + \frac{H}{2P_3}[P_3 - D_3(1+x+x^2)]\right\} \quad \text{Let } \frac{\partial P}{\partial Q_3} = 0$$

$$Q_3 = \sqrt{\frac{2P_2D_2A_2}{H[P_3 - D_3(1+x+x^2)]}}$$

$$\frac{\partial P}{\partial Q_4} = \frac{1}{6}\left\{\frac{-D_4A_4}{Q_4^{\ 2}} + \frac{H}{2P_4}[P_4 - D_4(1+x+x^2)]\right\} \quad \text{Let } \frac{\partial P}{\partial Q_4} = 0$$

$$Q_4 = \sqrt{\frac{2P_1D_1A_1}{H[P_4 - D_4(1+x+x^2)]}}$$

The above result shows that $Q_1 \succ Q_2 \succ Q_3 \succ Q_4$, it does not satisfy the constraint $0 \le Q_1 \le Q_2 \le Q_3 \le Q_4$. Set $A = 1$ and go to step 2.

Step 2:

Convert the inequality constraint $Q_2 - Q_1 \ge 0$ into equality constraint $Q_2 - Q_1 = 0$. The Lagrangian function is given as

$$l(Q_1, Q_2, Q_3, Q_4, \lambda) = P[TC - \lambda(Q_2 - Q_1)]$$

$$Q_1 = Q_2 = \sqrt{\frac{2(P_4D_4A_4 + P_3D_3A_3)}{H\{[P_1 - D_1(1+x+x^2)] + [P_2 - D_2(1+x+x^2)]\}}}$$

$$Q_3 = \sqrt{\frac{2P_2D_2A_2}{H[P_3 - D_3(1+x+x^2)]}}$$

$$Q_4 = \sqrt{\frac{2P_1D_1A_1}{H[P_4 - D_4(1+x+x^2)]}}$$

The above result shows that $Q_3 \succ Q_4$ and it does not satisfy the constraint $0 \le Q_1 \le Q_2 \le Q_3 \le Q_4$. Set $A = 2$ and go to step 3.

Step 3:

Convert the inequality constraint $Q_2 - Q_1 \ge 0; Q_3 - Q_2 \ge 0$ into equality constraint $Q_2 - Q_1 = 0; Q_3 - Q_2 = 0$. The Lagrangian function is given by

$$l(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2) = P[TC - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2)]$$

$$Q_1 = Q_2 = Q_3 = \sqrt{\frac{2(P_4D_4A_4 + P_3D_3A_3 + P_2D_2A_2)}{H\{[P_1 - D_1(1+x+x^2)] + [P_2 - D_2(1+x+x^2)] + [P_3 - D_3(1+x+x^2)]\}}}$$

$$Q_4 = \sqrt{\frac{2P_1D_1A_1}{H[P_4 - D_4(1+x+x^2)]}}$$

The above result shows that $Q_1 \succ Q_4$ and it does not satisfy the constraint $0 \le Q_1 \le Q_2 \le Q_3 \le Q_4$. Set $A = 3$ and go to step 4.

Step 4:

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Convert the inequality constraint $Q_2 - Q_1 \geq 0; Q_3 - Q_2 \geq 0; Q_4 - Q_3 \geq 0$ into equality constraint $Q_2 - Q_1 = 0; Q_3 - Q_2 = 0; Q_4 - Q_3 = 0$. The Lagrangian function is given by

$$l(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2, \lambda_3) = P[TC - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2) - \lambda_3(Q_4 - Q_3)]$$

The minimization of $l(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2, \lambda_3)$ is given by

$$Q_1 = Q_2 = Q_3 = Q_4 = \sqrt{\frac{2(P_1D_1A_1 + P_2D_2A_2 + P_3D_3A_3 + P_4D_4A_4)}{H\{[P_1 - D_1(1+x+x^2)] + [P_2 - D_2(1+x+x^2)] + [P_3 - D_3(1+x+x^2)] + [P_4 - D_4(1+x+x^2)]\}}}$$

The solution $(\tilde{Q})^* = (Q_1, Q_2, Q_3, Q_4)$ satisfies all the inequality constraints.

Let $Q_1 = Q_2 = Q_3 = Q_4 = (\tilde{Q})^*$ then the optimal fuzzy production quantity is given by

$$(\tilde{Q})^* = \sqrt{\frac{2(P_1D_1A_1 + P_2D_2A_2 + P_3D_3A_3 + P_4D_4A_4)}{H\{[P_1 - D_1(1+x+x^2)] + [P_2 - D_2(1+x+x^2)] + [P_3 - D_3(1+x+x^2)] + [P_4 - D_4(1+x+x^2)]\}}}$$

NUMERICAL EXAMPLE:

Consider the following data to illustrate the proposed model.

$\tilde{D} = (5550, 5850, 6150, 6450); \tilde{P} = (6550, 6850, 7150, 7450); \tilde{A} = (25, 75, 125, 175); \tilde{C} = (35, 65, 135, 165)$

x	Q	T	SETUP COST	HOLDING COST	REWORK COST	TOTAL COST
0.01	973.8982	0.1565	638.9776	638.9776	594	615111.8177
0.02	1007.6055	0.1619	617.6652	617.6652	1198.67	615674.5584
0.03	1045.8434	0.1678	595.9476	595.9476	1798	616230.3337
0.04	1089.6636	0.1747	572.4098	572.4098	2397.33	616783.0269
0.05	1140.4955	0.1826	547.6451	547.6451	2996.67	617341.6044
0.06	1200.3395	0.1920	520.8300	520.8300	3596	617879.3976
0.07	1272.0985	0.2032	492.1300	492.1300	4195.33	618416.9391
0.08	1360.1776	0.2169	461.0400	461.0400	4794.67	618959.5812
0.09	1471.6574	0.2342	426.9900	426.9900	5394	619488.5106
0.10	1618.8231	0.2555	391.3900	391.3900	5993.33	620018.5025

5.2 AN INVENTORY MODEL FOR SHODDY PRO – OFFERING SYSTEM WITH REWORK AND SHORTAGE:

In this case we consider a model with rework and shortage in fuzzy environment since the holding cost, production cost, inventory carrying cost, shortage cost and reworking cost are fuzzy in nature. We fuzzify them with trapezoidal fuzzy numbers using Lagrangian method.

Setup cost: $\frac{D}{Q}A$

Production cost: DC

Inventory holding cost: $\frac{QH}{2P}[P - D(1+x+x^2)] - \frac{DBH}{2P}[1+2x]$

Shortage cost: $\frac{PB^2S(1-x)}{2Q(P-D-d)}$

Reworking cost: DxR

Total cost is given by

$$\tilde{TC}(Q)=\frac{1}{6}\left\{\left(\frac{D_1A_1}{Q_1}+D_1C_1+\frac{Q_1H}{2P_1}[P_1-D_1(1+x+x^2)-\frac{D_1BH}{2P_1}(1+2x)+\frac{P_1B^2S_1(1-x)}{2Q_1(P_1-D_1-d_1)}+D_1xR_1\right)+\right.$$
$$2\left(\frac{D_2A_2}{Q_2}+D_2C_2+\frac{Q_2H}{2P_2}[P_2-D_2(1+x+x^2)]-\frac{D_2BH}{2P_2}(1+2x)+\frac{P_2B^2S_2(1-x)}{2Q_2(P_2-D_2-d_2)}+D_2xR_2\right)+$$
$$2\left(\frac{D_3A_3}{Q_3}+D_3C_3+\frac{Q_3H}{2P_3}[P_3-D_3(1+x+x^2)]-\frac{D_3BH}{2P_3}(1+2x)+\frac{P_3B^2S_3(1-x)}{2Q_3(P_3-D_3-d_3)}+D_3xR_3\right)+$$
$$\left.\left(\frac{D_4A_4}{Q_4}+D_4C_4+\frac{Q_4H}{2P_4}[P_4-D_4(1+x+x^2)]-\frac{D_4BH}{2P_4}(1+2x)+\frac{P_4B^2S_4(1-x)}{2Q_4(P_4-D_4-d_4)}+D_4xR_4\right)\right\}$$

partially differentiating with respect to $'Q'$

$$\tilde{TC}(Q)=\frac{1}{6}\left\{\left(\frac{-D_1A_1}{Q_1^{\,2}}+\frac{H}{2P_1}[P_1-D_1(1+x+x^2)]-\frac{P_1B^2S_1(1-x)}{2Q_1^{\,2}(P_1-D_1-d_1)}\right)+\right.$$
$$2\left(\frac{-D_2A_2}{Q_2^{\,2}}+\frac{H}{2P_2}[P_2-D_2(1+x+x^2)]-\frac{P_2B^2S_2(1-x)}{2Q_2^{\,2}(P_2-D_2-d_2)}\right)+$$
$$2\left(\frac{-D_3A_3}{Q_3^{\,2}}+\frac{H}{2P_3}[P_3-D_3(1+x+x^2)]-\frac{P_3B^2S_3(1-x)}{2Q_3^{\,2}(P_3-D_3-d_3)}\right)+$$
$$\left.\left(\frac{-D_4A_4}{Q_4^{\,2}}+\frac{H}{2P_4}[P_4-D_4(1+x+x^2)]-\frac{P_4B^2S_4(1-x)}{2Q_4^{\,2}(P_4-D_4-d_4)}\right)\right\}$$

$$Q=\sqrt{\frac{8P_1^{\,3}D_1(1-x)S_1A_1+2\left(8P_2^{\,3}D_2(1-x)S_2A_2\right)+2\left(8P_3^{\,3}D_3(1-x)S_3A_3\right)+8P_4^{\,3}D_4(1-x)S_4A_4}{\begin{array}{c}[(4P_1^{\,2}(1-x)S_1H[P_1-D_1(1+x+x^2)]-D_1^{\,2}H^2(P_1-D_1-d_1)(1+2x)^2)+\\(4P_2^{\,2}(1-x)S_2H[P_2-D_2(1+x+x^2)]-D_2^{\,2}H^2(P_2-D_2-d_2)(1+2x)^2)+\\(4P_3^{\,2}(1-x)S_3H[P_3-D_3(1+x+x^2)]-D_3^{\,2}H^3(P_3-D_3-d_3)(1+2x)^2)+\\(4P_4^{\,2}(1-x)S_4H[P_4-D_4(1+x+x^2)]-D_4^{\,2}H^4(P_4-D_4-d_4)(1+2x)^2)]\end{array}}}$$

Step 1:

$$\frac{\partial P}{\partial Q_1}=\frac{1}{6}\left\{\frac{-D_1A_1}{Q_1^{\,2}}+\frac{H}{2P_1}[P_1-D_1(1+x+x^2)]-\frac{P_1B^2S_1(1-x)}{2Q_1^{\,2}(P_1-D_1-d_1)}\right\}$$

Let $\frac{\partial P}{\partial Q_1}=0$

$$Q_1=\sqrt{\frac{8P_4^{\,3}D_4(1-x)S_4A_4}{4P_1^{\,2}(1-x)S_1H[P_1-D_1(1+x+x^2)]-D_1^{\,2}H(P_1-D_1-d_1)(1+2x)^2}}$$

$$\frac{\partial P}{\partial Q_2}=\frac{1}{6}\left\{\frac{-D_2A_2}{Q_2{}^2}+\frac{H}{2P_2}[P_2-D_2(1+x+x^2)]-\frac{P_2B^2S_2(1-x)}{2Q_2{}^2(P_2-D_2-d_2)}\right\}$$

Let $\frac{\partial P}{\partial Q_2}=0$

$$Q_2=\sqrt{\frac{8P_3{}^3D_3(1-x)S_3A_3}{4P_2{}^2(1-x)S_2H[P_2-D_2(1+x+x^2)]-D_2{}^2H(P_2-D_2-d_2)(1+2x)^2}}$$

$$\frac{\partial P}{\partial Q_3}=\frac{1}{6}\left\{\frac{-D_3A_3}{Q_3{}^2}+\frac{H}{2P_3}[P_3-D_3(1+x+x^2)]-\frac{P_3B^2S_3(1-x)}{2Q_3{}^2(P_3-D_3-d_3)}\right\}$$

Let $\frac{\partial P}{\partial Q_3}=0$

$$Q_3=\sqrt{\frac{8P_2{}^3D_2(1-x)S_2A_2}{4P_3{}^2(1-x)S_3H[P_3-D_3(1+x+x^2)]-D_3{}^2H(P_3-D_3-d_3)(1+2x)^2}}$$

$$\frac{\partial P}{\partial Q_4}=\frac{1}{6}\left\{\frac{-D_4A_4}{Q_4{}^2}+\frac{H}{2P_4}[P_4-D_4(1+x+x^2)]-\frac{P_4B^2S_4(1-x)}{2Q_4{}^2(P_4-D_4-d_4)}\right\}$$

Let $\frac{\partial P}{\partial Q_4}=0$

$$Q_4=\sqrt{\frac{8P_1{}^3D_1(1-x)S_1A_1}{4P_4{}^2(1-x)S_4H[P_4-D_4(1+x+x^2)]-D_4{}^2H(P_4-D_4-d_4)(1+2x)^2}}$$

The above result shows that $Q_1 \succ Q_2 \succ Q_3 \succ Q_4$ and it does not satisfy the constraint $0 \le Q_1 \le Q_2 \le Q_3 \le Q_4$. Set A=1 and go to step 2.

Step 2:

Convert the inequality constraint $Q_2 - Q_1 \ge 0$ into equality constraint $Q_2 - Q_1 = 0$. The Lagrangian function is given by

$$l(Q_1,Q_2,Q_3,Q_4,\lambda)=P[TC-\lambda(Q_2-Q_1)]$$

$$Q_1=Q_2=\sqrt{\frac{8\left(P_4{}^3D_4(1-x)S_4A_4+P_3{}^3D_3(1-x)S_3A_3\right)}{\begin{array}{l}H\{(4P_1{}^2(1-x)S_1[P_1-D_1(1+x+x^2)])+(4P_2{}^2(1-x)S_2[P_2-D_2(1+x+x^2)])\}-\\H^2\{(D_1{}^2(P_1-D_1-d_1)(1+2x)^2)+(D_2{}^2(P_2-D_2-d_2)(1+2x)^2)\}\end{array}}}$$

$$Q_3=\sqrt{\frac{8P_2{}^2D_2(1-x)S_2A_2}{4P_3{}^2(1-x)S_3H[P_3-D_3(1+x+x^2)]-D_3{}^2H^2(P_3-D_3-d_3)(1+2x)^2}}$$

$$Q_4=\sqrt{\frac{8P_1{}^3D_1(1-x)S_1A_1}{4P_4{}^2(1-x)S_4H[P_4-D_4(1+x+x^2)]-D_4{}^2H^2(P_4-D_4-d_4)(1+2x)^2}}$$

The above result shows that $Q_3 \succ Q_4$ it does not satisfy the constraint $0 \le Q_1 \le Q_2 \le Q_3 \le Q_4$. Set $A=2$ and go to step 3.

Step 3:

Convert the inequality constraint $Q_2 - Q_1 \geq 0; Q_3 - Q_2 \geq 0$ into equality constraint $Q_2 - Q_1 = 0$ and $Q_3 - Q_2 = 0.$ The Lagrangian function is given by

$$l(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2) = P[TC - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2)]$$

$$Q_1 = Q_2 = Q_3 = \sqrt{\frac{8\left(P_4^{\,3} D_4(1-x)S_4A_4 + P_3^{\,3} D_3(1-x)S_3A_3 + P_2^{\,3} D_2(1-x)S_2A_2\right)}{\begin{array}{l} H\{(4P_1^{\,2}(1-x)S_1[P_1 - D_1(1+x+x^2)]) + (4P_2^{\,2}(1-x)S_2[P_2 - D_2(1+x+x^2)]) + \\ (4P_3^{\,2}(1-x)S_3[P_3 - D_3(1+x+x^2)])\} - H^2\{(D_1^{\,2}(P_1 - D_1 - d_1)(1+2x)^2) + \\ (D_2^{\,2}(P_2 - D_2 - d_2)(1+2x)^2) + (D_3^{\,2}(P_3 - D_3 - d_3)(1+2x)^2)\} \end{array}}}$$

$$Q_4 = \sqrt{\frac{8P_1^{\,3} D_1(1-x)S_1A_1}{4P_4^{\,2}(1-x)S_4H[P_4 - D_4(1+x+x^2)] - D_4^{\,2}H^2(P_4 - D_4 - d_4)(1+2x)^2}}$$

The above result shows that $Q_1 \succ Q_4$ and it does not satisfy the constraint $0 \leq Q_1 \leq Q_2 \leq Q_3 \leq Q_4.$ Set $A = 3$ and go to step 4.

Step 4:

Convert the inequality constraint $Q_2 - Q_1 \geq 0; Q_3 - Q_2 \geq 0; Q_4 - Q_3 \geq 0$ into equality constraint $Q_2 - Q_1 = 0; Q_3 - Q_2 = 0; Q_4 - Q_3 = 0.$ The Lagrangian function is given by

$$l(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2, \lambda_3) = P[TC - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2) - \lambda_3(Q_4 - Q_3)]$$

The minimization of $(l, Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2, \lambda_3)$ is given by

$$Q_1 = Q_2 = Q_3 = Q_4 \sqrt{\frac{8\left(P_4^{\,3} D_4(1-x)S_4A_4 + P_3^{\,3} D_3(1-x)S_3A_3 + P_2^{\,3} D_2(1-x)S_2A_2 + P_1^{\,3} D_1(1-x)S_1A_1\right)}{\begin{array}{l} H\{(4P_1^{\,2}(1-x)S_1[P_1 - D_1(1+x+x^2)]) + (4P_2^{\,2}(1-x)S_2[P_2 - D_2(1+x+x^2)]) + \\ (4P_3^{\,2}(1-x)S_3[P_3 - D_3(1+x+x^2)]) + (4P_4^{\,2}(1-x)S_4[P_4 - D_4(1+x+x^2)])\} - \\ H^2\{(D_1^{\,2}(P_1 - D_1 - d_1)(1+2x)^2) + (D_2^{\,2}(P_2 - D_2 - d_2)(1+2x)^2) + \\ (D_3^{\,2}(P_3 - D_3 - d_3)(1+2x)^2) + (D_4^{\,2}(P_4 - D_4 - d_4)(1+2x)^2)\} \end{array}}}$$

The solution $(\tilde{Q})^* = (Q_1, Q_2, Q_3, Q_4)$ satisfies all the inequality constraints.

Let $Q_1 = Q_2 = Q_3 = Q_4 = (\tilde{Q})^*$ then the optimal fuzzy production quantity is given by

$$(\tilde{Q}) = \sqrt{\frac{8\left(P_4^{\,3} D_4(1-x)S_4A_4 + P_3^{\,3} D_3(1-x)S_3A_3 + P_2^{\,3} D_2(1-x)S_2A_2 + P_1^{\,3} D_1(1-x)S_1A_1\right)}{\begin{array}{l} H\{(4P_1^{\,2}(1-x)S_1[P_1 - D_1(1+x+x^2)]) + (4P_2^{\,2}(1-x)S_2[P_2 - D_2(1+x+x^2)]) + \\ (4P_3^{\,2}(1-x)S_3[P_3 - D_3(1+x+x^2)]) + (4P_4^{\,2}(1-x)S_4[P_4 - D_4(1+x+x^2)])\} - \\ H^2\{(D_1^{\,2}(P_1 - D_1 - d_1)(1+2x)^2) + (D_2^{\,2}(P_2 - D_2 - d_2)(1+2x)^2) + \\ (D_3^{\,2}(P_3 - D_3 - d_3)(1+2x)^2) + (D_4^{\,2}(P_4 - D_4 - d_4)(1+2x)^2)\} \end{array}}}$$

NUMERICAL EXAMPLE:

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Consider the following data to illustrate the above proposed model. $\tilde{D}=(5550,5850,6150,6450);\tilde{P}=(6550,6850,7150,7450);\tilde{A}=(25,75,125,175);\tilde{C}=(35,65,135,165);$ $\tilde{S}=(11,14,6,9);\tilde{R}=(12,13,7,8)$

x	Q	T	SETUP COST	HOLDING COST	REWORK COST	SHORTAGE COST	TOTAL COST
0.01	1073.0835	0.1806	553.7099	553.7099	594	263.2619	614941.6829
0.02	1103.1008	0.1880	531.8206	531.8206	1188	268.8323	615503.6043
0.03	1145.3195	0.1992	502.0080	502.0080	1782	271.3349	616041.8181
0.04	1203.4821	0.2090	478.4307	478.4307	2376	275.5604	616595.3795
0.05	1267.1357	0.2214	451.6712	451.6712	2970	280.1846	617188.6978
0.06	1337.1882	0.2339	427.5941	427.5941	3564	281.7766	617683.0246
0.07	1486.0597	0.2501	399.8880	399.8880	4158	292.2453	618224.2753
0.08	1526.7119	0.2711	368.8676	368.8676	4752	274.1362	618760.0911
0.09	1656.4996	0.2967	337.0959	337.0959	5346	300.7978	619305.8736
0.10	1824.5699	0.3308	302.2975	302.2975	5940	310.1668	619848.7678

6. CONCLUSION:

Thus in the current paper, our goal is to propose the fuzzy inventory model with defective items considering rework and shortages. In this model, the input parameter like setup cost, demand, rework cost, production cost, inventory cost are considered as trapezoidal fuzzy numbers and are defuzzified by using Graded mean integration method and the total cost for both the models are calculated.

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