International Journal of Aquatic Science
ISSN: 2008-8019
Vol 12, Issue 02, 2021

Cost And Profit Analysis Of M/E_k/1 Queueing System By Triangular Fuzzy Numbers

M.Bhuvaneshwari[1], W.Ritha[2]

[1,2]PG and Research Department of Mathematics Holy Cross College (Autonomous), Trichirapalli – 620 002 Affliated to Bharathidasan university

ABSTRACT: In this work, we have ventured to explore the cost and profit analysis of an M/E_k/1 queueing model using triangular fuzzy numbers. We analyse the total expected cost of the system under the condition when the system is turned off and turned on and in second case,we discuss the total expected cost of the system with utilization of idle time and finally total optimal costs in both of cases have been obtained.

KEYWORDS: Cost and Profit, L-R method, fuzzy queue, total expected cost, triangular fuzzy number, total expected profit, total expected revenue.

1. INTRODUCTION:

Customers always expects convenient service without any call waiting and with unlimited agent availability. Our main goal is to have the correct number of agents on duty at all times, with no under or over staying.Erlang is a dimensionless unit that is mainly used in telephony as a measure of offered load or carried load on service providing dements such as telephone circuits or telephone switching equipment. Erlang model has the unit system ITU. It has the notation E.

The Queueing theory had its origin in 1909, when A.K. Erlang published his fundamental paper relating to the study of congestion in telephone traffic. The literature on the theory of queues and on the diverse areas of its applications have grown tremendously over the years. Queueing theory was developed to provide models to predict the behaviour of systems that attempt to provide service for randomly arising demands. It originated as a very practical subject, but much of the literature up to the middle1980s was of little direct practical value.

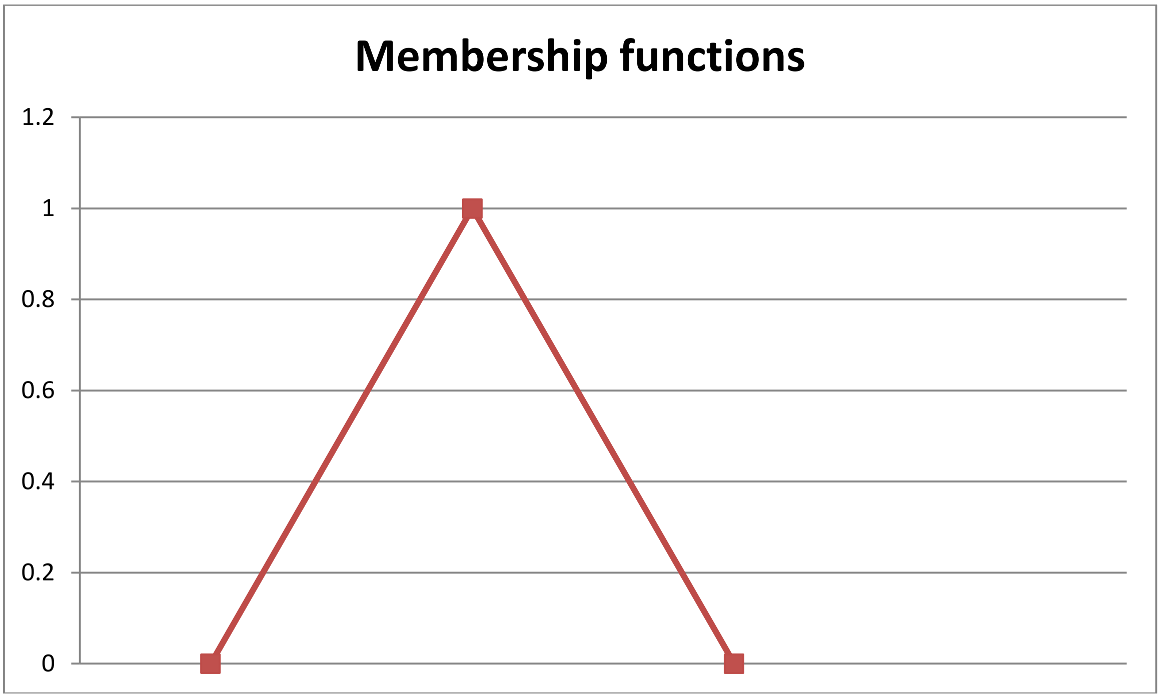
A queue or waiting line is formed when units or customers needing some kind of service arrive at a service channel that offers such facility. A queueing system can be described by the flow of units for service, forming or joining the queue, if service is not immediately available and leaving the system after being served or sometimes without being served. The basic features which characterize a system are:

(i) the input
(ii) the service mechanism
(iii) the queue discipline
(iv) the number of service channels.

By units we mean those demanding service, e.g. customers at a bank counter or at a reservation counter, calls arriving at a telephone exchange, vehicular traffic at a traffic intersection, machines for repair before a repairman, airplanes waiting for take-off or landing at a busy airport, merchandise waiting for shipment at a yard, computer programmes waiting to be run on a time sharing basis etc.

In usual queueing situations, the process of arrivals is stochastic and it is thus necessary to know the probability distribution describing the times between successive customer arrivals. The Cost and Profit analysis of M/E_k/1 queueing model is very useful in furnishing the structure for analysis of varied practical situations including telephony. Xadin and Naor launched the concept of an N-policy which switch on the system when the number of customers in the system reaches a particular number (N≥ 1) and switches off the server when there are zero customers in the system.

The input describes the manner in which units arrive and join the system. The system is known as a delay or loss system depending on whether a unit who, on arrival, finds the service facility occupied, joins or leaves the system. The system may have either a limited or an unlimited capacity for holding units. The source from which the units come may be finite or infinite. A unit may arrive either singly or in a group. The interval between two consecutive arrivals is the interarrival time or interval.



We have quantified the Total Expected Cost, Total Expected Revenue and Total Expected Profit in utilization of L-R method on triangular fuzzy numbers. In triangular fuzzy number, the membership functions are taken in a triangular form is shown in the above figure.

2. DESCRIPTION OF THE MODEL:

We consider an M/E_k/1 queueing system in which idle time of the server is optimized by shifting the server to another service station in the turned off state of the system. It is assumed that the customer arrival follows Poisson process with parameter ϕ and with service time according to an Erlang distribution with mean $\frac{1}{\theta}$ and stage parameter k.
The Erlang type k distribution is made up to k independent and identical exponential stages, each with mean $\frac{1}{k\theta}$. The service operates when N customers have accumulatedand is shutdown when no customers are there for service. Additionally, it is assumed that there is no case of reneging and balking in the system.

3. LITERATURE REVIEW:

Gross and Harris [1] exchanged their views about an ordinary M/E_k/1 queueing system. In this series of investigations, Wang and Huang [3] have made a study on optimal operation of an M/E_k/1 queueing system with a removable service station. They have defined total expected cost function with respect to N and finally they have calculated the number of customers N in the system when system is in the state of turned on and turned off. Additionally, service level is the function of θ and hence by optimizing θ, we can have a better service level which in turn, leads to optimization of total expected cost by reducing customers' waiting time. Wang[2] confered about optimal control of M/E_k/1 queueing system.

Pearn and Chang [5] contemplated a sensitivity investigation of the N-policy M/E_k/1 queueing system with removable service station.

Yadin and Naor [4] examined around queueing system with a removable service station and introduced the concept of an N policy which turns the server on when the number of customers in the system reaches a certain number N (N≥ 1)and turns the server off when there are zero customers in the system.

J.P. Mukeba Kanyinda [9] presented L-R method for solving the triangular numbers. R. J. Li, E.S. Lee [6] [7] made an analysis on fuzzy queues. Moreover they issued ideas based on their applications in computer research.

D. S. Negi, E.S. Lee, elaborated about the simulation of fuzzy queues. Further they had a deliberation about fuzzy sets and systems.

J.P.Mukeba[13] engrossed the Applications of L-R method when there is a single server using fuzzy queue. He also applied this method for the patient customers who complete the service even if there is any delay in the service. Furthermore he investigated the Journal of Pure and Applied Mathematics.

B. Kalpana and N. Anusheela[12] worked on the Analysis of a single server non-preemtive fuzzy queues with the assistance of L-R method. They also found out enormous applications of L-R method in Triangular fuzzy numbers. Shanmugasundaram, Thamotharan and Ragapriya elucidated about the study on fuzzy queueing model with single server on their International Journal of Latest Trends in Engineering and Technology.

4. COST AND PROFIT ANALYSIS OF THE SYSTEM :

For an M/E_k/1 queueing system following results in steady states are given

$H_{off} = \sum_{n=1}^{N-1} nR_{nk}^{0} = \frac{(N-1)(1-\eta)}{2}$

$H_{on} = \sum_{n=1}^{r}\sum_{i=1}^{k} nQ'_{ni} = \eta\left[\frac{(N+1)-\eta N+\frac{\eta}{k}}{2(1-\eta)}\right]$ --------- (1)

$H_N = H_{off} + H_{on}$

$= \frac{(N-1)(1-\eta)}{2} + \eta\left[\frac{(N+1)-\eta N+\frac{\eta}{k}}{2(1-\eta)}\right]$

Consider,

$H_{on} = \sum_{n=1}^{r}\sum_{i=1}^{k} nQ'_{ni} = \eta\left[\frac{(N+1)-\eta N+\frac{\eta}{k}}{2(1-\eta)}\right]$

$= \frac{\eta(N+1)-\eta^2 N+\frac{\eta^2}{k}}{2(1-\eta)}$

$= \frac{k\eta(N+1)-\eta^2 Nk+\eta^2}{2k(1-\eta)}$

N=1,

$= \frac{2k\eta+\eta^2(1-k)}{2k(1-\eta)}$

$H_N = H_{off} + H_{on}$

$= \frac{N-1}{2} + \frac{2k\eta+\eta^2(1-k)}{2k(1-\eta)}$, $\frac{E(I)}{E(C)}$ = 1-q

Since there is no traffic intensity in H_{off} , $\eta = 0$

Now, we construct Total Expected Cost function and analyse its solution in 2 cases :

CASE (I):

Analysing the Total Expected Cost of the system when the system is turned on and turned off.

$T_1 = C_s k\theta + C_0 H_{off} + C_n H_{on}$

$= C_s k\theta + \frac{C_0(N-1)(1-\eta)}{2} + C_n\frac{r}{t}$ ----------(2)

Where r = $\eta(N+1) - \eta^2(N-\frac{1}{k})$

t = $2(1-\eta)$

$r' = \frac{-1}{\theta}\left[\eta(N+1) + 2\eta^2(N-\frac{1}{k})\right]$

$r'' = \frac{1}{\theta^2}\left[2\eta(N+1) - 6\eta^2(N-\frac{1}{k})\right]$

$t' = \frac{1}{\theta}(2\eta)$

$t'' = \frac{-1}{\theta^2}(4\eta)$

Equation (2) after differentiating with respect to θ gives that

$T_1' = C_s k + C_0\frac{N-1}{2}\frac{\eta}{\theta} + C_n\frac{tr'}{t^2} - \frac{rt'}{t^2}$

$T_1' = C_s k + C_0\frac{N-1}{2}\frac{\eta}{\theta} + C_n\frac{r'}{t} - \frac{rt'}{t^2}$

$T_1'' = -C_0\,(N-1)\frac{\eta}{\theta^3} + C_n\left[\frac{tr''-r't'}{t^2} - \frac{t^2rt'-(rt')'2tt'}{t^4}\right]$

$= -C_0\,(N-1)\frac{\eta}{\theta^3} + C_n\left[\frac{tr''-r't'}{t^2} - \frac{t^2(rt''+(r't'))-2rt(t'^2)}{t^4}\right]$

$$= -C_0\ (\text{N-1})\ \frac{\eta}{\theta^3} + C_n\left[\frac{tr'' - r't'}{t^2} - \frac{rt'' + r'tt' - 2r(t'^2)}{t^3}\right]$$
$$= -C_0\ (\text{N-1})\ \frac{\eta}{\theta^3} + C_n\left[\frac{t^2r'' - r'tt' - rtt'' - r'tt' + 2r(t'^2)}{t^3}\right]$$
$$= -C_0\ (\text{N-1})\ \frac{\eta}{\theta^3} + C_n\left[\frac{t^2r'' - rtt'' - 2r'tt' + 2r(t'^2)}{t^3}\right]$$

Now for optimum θ we have,

$$T_1' = C_s k + C_0\ \frac{N-1}{2}\frac{\eta}{\theta} + C_n\frac{tr' - rt'}{t^2} = 0$$

It implies that,

$$\theta^4\,\frac{2C_s k}{\phi^3} + \theta^3\left[\frac{-4C_s k}{\phi^2}\right] + \theta^2\left[\frac{2C_s k}{\phi} + \frac{C_0(N-1)}{\phi^2}\right] + \theta\left[\frac{-2C_0(N-1) - 2C_n(N-\frac{1}{k})}{\phi}\right] + C_0(N-1) + 3C_n(N-\frac{1}{k}) = 0$$

CASE II :

Analysing that the Total Expected Cost of the system with utilization of idle time,

$$T_2 = C_s k\eta + C_n H_n + C_u\,\frac{E(I)}{E(C)}$$

Substituting the values of Setup cost, Holding cost, Queue length and ratio $\frac{E(I)}{E(C)}$ turns out to be,

$$T_2 = C_s k\eta + C_h\left[\frac{N-1}{2}\right] + C_h\frac{r}{t} + C_u(1\text{-}\eta) \quad \text{--------} \ (3)$$

Where,

r = η^2(1-k) + 2k η

t = 2k(1-η)

$$r' = \frac{-1}{\theta}\left[2(1-\text{k})\eta^2 + 2k\eta\right]$$
$$r'' = \frac{1}{\theta^2}\left[6(1-\text{k})\eta^2 + 4k\eta\right]$$
$$t' = \frac{1}{\theta}[2k\eta]$$
$$t'' = \frac{-1}{\theta^2}[4k\eta]$$

Now differentiating (3) with respect to θ we get,

$$T_2' = C_s k + C_h\left[\frac{tr' - rt'}{t^2}\right] + C_u\frac{\eta}{\theta}$$
$$T_2' = C_s k + C_h\left[\frac{tr'}{t^2} - \frac{rt'}{t^2}\right] + C_u\frac{\eta}{\theta}$$
$$T_2' = C_s k + C_h\left[\frac{r'}{t} - \frac{rt'}{t^2}\right] + C_u\frac{\eta}{\theta}$$
$$T_2'' = C_h\left[\frac{tr'' - r't'}{t^2} - \frac{t^2(rt')' - 2rt'tt'}{t^4}\right] - C_u\left[\frac{2\eta}{\theta^3}\right]$$
$$T_2'' = C_h\left[\frac{tr'' - r't'}{t^2} - \frac{t^2(rt'' + r't') - 2rt(t')^2}{t^4}\right] - C_u\left[\frac{2\eta}{\theta^3}\right]$$
$$T_2'' = C_h\left[\frac{tr'' - r't'}{t^2} - \frac{(r'tt' + r'tt') - 2r(t')^2}{t^3}\right] - C_u\left[\frac{2\eta}{\theta^3}\right]$$
$$T_2'' = C_h\left[\frac{t^2r'' - r'tt' - rtt'' - r'tt' + 2r(t')^2}{t^3}\right] - C_u\left[\frac{2\eta}{\theta^3}\right]$$
$$T_2'' = C_h\left[\frac{t^2r'' - rtt'' - 2r'tt' + 2r(t')^2}{t^3}\right] - C_u\left[\frac{2\eta}{\theta^3}\right]$$

For optimum θ we have,

$$C_s k + C_h\left[\frac{tr' - rt'}{t^2}\right] + C_u\frac{\eta}{\theta} = 0$$

$\theta^4 \frac{4C_s k^3}{\phi^3} + \theta^3 \left[\frac{8C_s k}{\phi^2}\right] + \theta^2 \left[\frac{4C_s k^3}{\phi} + \frac{4C_u k^2}{\phi^2}\right] + \theta \left[\frac{2C_h k(k-2) - 8C_u(k^2)}{k^2}\right] \quad 2C_h k(1-k) +$
$4C_u(k^2) \quad = 0$

Profit Analysis of the system:

TEC = $C_s k\theta + C_h H_n$

TER = RH_n , where R is the expected Revenue

TEP = TER – TEC

$= \left[\frac{N-1}{2} + \frac{\phi^2(1-k)+2k\phi\theta}{2k\theta(\theta-\phi)}\right]$ (R-C_h) - $C_s k\theta$

$= \left[\frac{N-1}{2} + \frac{\alpha}{\beta}\right]$(R-$C_h$) - $C_s k\theta$

Where $\alpha = \phi^2(1-k) + 2k\phi\theta$

$\beta = 2k\theta(\theta - \phi)$

NUMERICAL EXAMPLE :

(i) To find Total Expected Profit,

TEP = $\frac{\tilde{N}-1}{2} + \left[\frac{(\tilde{\phi}^2(1-k)) + 2k\tilde{\phi}\tilde{\theta}}{2k\tilde{\theta}(\tilde{\theta}-\tilde{\phi})}\right](\tilde{R} - \tilde{C}_h)$- $\tilde{C}_s$k$\tilde{\theta}$

Where,

$\tilde{N}$ =(9,10,11)

$\tilde{\phi}$ = (3,4,5)

$\tilde{\theta}$ = (9,10,11)

$\tilde{R}$= (699,700,701)

$\tilde{C}_h$ = (14, 15, 16)

$\tilde{C}_s$= (19,20,21)

Calculation of $(\tilde{\phi}^2(1-k))$, $2k\tilde{\phi}\tilde{\theta}$, $2k\tilde{\theta}(\tilde{\theta}-\tilde{\phi})$, *(*$\tilde{R}$ *-* $\tilde{C}_h$*)-* $\tilde{C}_s k\tilde{\theta}$

$\tilde{\phi}^2(1-k)$ = $\tilde{\phi}.\tilde{\phi}$(1-3)
= -2(3,4,5)(3,4,5)
= -2(4,1,1) (4,1,1)
= -2 (16,1,1)
= (-32,-2,-2)

$2k\tilde{\phi}\tilde{\theta}$ = 2(3)(3,4,5)(9,10,11)
= 6(4,1,1)(10,1,1)
= (240,6,6)

$2k\tilde{\theta}(\tilde{\theta}-\tilde{\phi})$ = 2(3)(9,10,11)[(9,10,11) − (3,4,5)]
= 6(9,10,11) [(10,1,1) − (4,1,1)]
= 6(9,10,11)(6,1,1)
= 6(10,1,1)(6,1,1)
= (360,6,6)

($\tilde{R}$ – $\tilde{C}_h$)- $\tilde{C}_s$k$\tilde{\mu}$ =[(699,700,701) − (14,15,16)] − [(19,20,21)3(9,10,11)]
=[(700,1,1) − (15,1,1)] −3[(20,1,1)(10,1,1)]
=[(685,1,1) − 3(200,1,1)]
=[(685,1,1) − (600,3,3)]
= (85,3,3)

TEP = $\frac{(9,10,11)-(1,1,1)}{(2,2,2)} + \left[\frac{(-32,-2,-2)+(240,6,6)}{(360,6,6)}\right]$ (85,3,3)

$= \frac{(10,1,1)-(1,1,1)}{(2,0,0)} + \left[\frac{(208,6,6)}{(360,6,6)}\right]$ (85,3,3)

$= \frac{(9,1,1)}{(2,0,0)} + (0.577,6,6)(85,3,3)$
$= (4.5,1,1) + (49.045,6,6)$
TEP = (47.545,53.545,59.545)
(ii)To find Total Expected Revenue,
TER = $\tilde{R}\ \tilde{H}_n$
$= \tilde{R}\left[\frac{\tilde{N}-1}{2} + \frac{\eta^2(1-K)+2K\eta}{2K(1-\eta)}\right]$
Where,
$\tilde{R}$ = (699,700,701)
$\tilde{N}$ = (9,10,11)
η =4,
k = 3
$= (699,700,701)\left[\frac{(9,10,11)-(1,1,1)}{(2,2,2)} + \frac{4(1-3)+2(3)(4)}{2(3)(1-4)}\right]$
$= (699,700,701)\left[\frac{(10,1,1)-(1,1,1)}{(2,2,2)} + \frac{4(-2)+6(4)}{6(1-4)}\right]$
$= (700,1,1)\left[\frac{(9,1,1)}{(2,2,2)} + \frac{(-8)+24}{-18}\right]$
$= (700,1,1)[(4.5,2,2) - (0.88,1,1)]$
$= (700,1,1)(3.62,2,2)$
TER = (2532,2534,2536)
(iii)To find Total Expected Cost :
TEC = $\tilde{C}_s k\tilde{\theta} + \tilde{C}_h\tilde{H}_n$
$= \tilde{C}_s k\tilde{\theta} + \tilde{C}_h\left[\frac{\tilde{N}-1}{2} + \frac{\eta^2(1-k)+2k\eta}{2k(1-\eta)}\right]$
Where,
$\tilde{C}_s$= (19,20,21)
k =3
$\tilde{\theta} = (9,10,11)$
$\tilde{C}_h$ = (14,15,16)
$\tilde{N}$ = (9,10,11) η = 4
$= (19,20,21)(3)(14,15,16)\left[\frac{(9,10,11)-1}{2} + \frac{4^2(1-3)+2(3)4}{2(3)(1-4)}\right]$
$= 3(20,1,1)(10,1,1) + (15,1,1)\left[\frac{(10,1,1)-(1,1,1)}{(2,2,2)} + \frac{16(-2)+24}{-18}\right]$
$= 3(200,1,1) + (15,1,1)\left[\frac{(9,1,1)}{(2,2,2)} + \frac{(8,8,8)}{(18,18,18)}\right]$
$= (600,3,3) + (15,1,1)\left[(4.5,2,2) + \frac{(8,1,1)}{(18,1,1)}\right]$
$= (600,3,3) + (15,1,1)(1.98,2,2)$
$=(600,3,3) + (29.7,2,2)$
$= (629.7,3,3)$
TEC = (626.7, 629.7, 632.7)

The Total Expected Profit is (47.545,53.545,59.545)
The Total Expected Revenue is (2532,2534,2536)
The Total Expected Cost is (626.7, 629.7, 632.7)

It is evident that Total Expected Revenue is greater than the Total Expected Cost and Total Expected Profit.

Thus, if we utilize the idle time,we can improve the Total Expected Revenue.

5. CONCLUSION :

We observe that, as N increases upto a certain number of N, Total Operating Cost increases. When we increase the number of customers required for starting service, Total Expected Revenueof the system also gets increased. Thus Total Expected Revenue is achieved. There are many valuable applications of the theory, most of which have been well documented in the literature of probability, operations research, management science and Industrial Engineering.

6. REFERENCES:

[1] D.GROSS and C.M. HARRIS, Fundamentals of Queueing theory, 2nd edition , Wiely, Newyork, 1985.
[2] K.H. WANG, Optimal control of an M/E_k/1 queueing system with removable service station subject to breakdowns, Journal of the Operational Research Society, 48(1997), 936-942
[3] K.H. WANG and H.M.HUANG, Optimal control of an M/E_k/1 queueing system with removable service station subject to breakdowns, Journal of the Operational Research Society, 46(1995), 1014-1022
[4] M. YADIN and NAOR, Queueing systems with a removable service station, Operational Research Queue (14)(1963)193-405
[5] PEARN and W.L. CHANG, Y.C., Optimal management of the N-policy M/E_k/1 queueing system with removable service station: a sensitivity investigation, computers and operation research, 31(2004) 1001-1015
[6] R.J.LI., E.S.LEE, Analysis of Fuzzy Queues, Proc. NAFIPS (1998) 158- 162
[7] R.J.LI., E.S.LEE, Analysis of Fuzzy Queues, Computers and Mathematics with Applications, 17 (1989), 1143-1147
[8] D.S. NEGI, E.S. LEE, Analysis and Simulation of Fuzzy Queues, Fuzzy sets and Systems, 46(1992) 321-330.
[9] J.P. MUKEBA KANYINDA, R. MABELA MAKENGO MATENDO, B. ULUNGU EKUNDA LUKATA, Computing fuzzy queueing Performance Measures by L-R method Journal of Fuzzy set Valued Analysis (2015) 57-67
[10] S. SHANMUGASUNDARAM, S. THAMOTHARAN, M. RAGAPRIYA, A Study on Single Server Fuzzy Queueing Model, International Journal of Latest Trends in Engineering and Technology., volume 6 issue 1, September 2015, 162-169
[11] H.J. ZIMMERMANN, Fuzzy Set Theory and its Applications, Springer Science + Business Media, New York, fourth edition, (2001)
[12] B.KALPANA AND ANUSHEELA, Analysis of a single server non- preemptive fuzzy queue using L-R method, ARPN Journal of Engineering and Applied Sciences, volume 13 no.23 December 2018,9306-9310
[13] J.B. MUKEBA, Applications of L-R method to single server fuzzy queue with patient customers, Journal of Pure and Applied Mathematics, Advances and Appllications, volume 16, No. 1, 2016, 43-59