International Journal of Aquatic Science
ISSN: 2008-8019
Vol 12, Issue 02, 2021

Fuzzified M/M/1 Traffic Model

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Abstract: The evaluation of undisturbed traffic flow is basically on experiential methods. We intend some logical queueing models based on traffic counts and we develop the function of traffic flow of the most applicable determining factors. This paper used a method to build the membership functions using DSW algorithm of the traffic flow model in queueing systems where the inter-arrival and service time are to be fuzzified. We plan to model in M/M/1 queue model whose arrivals and service rate are fuzzy numbers. Our research objectives are congestion control, traffic management and environmental impact of road traffic. We illustrate our results for determining maximum traffic flow based on trapezoidal fuzzy numbers.

Keywords: Queueing theory, Performance measure, Congestion management, Traffic-flow modelling, DSW algorithm.

1. INTRODUCTION:

The application of queueing models is in the field of Probability theory, Operation research, Management science and Industrial engineering. In real life queueing models cover the vast area of traffic flow of vehicles, communication system of people, scheduling patients in hospitals, facility design in back sector & post office.

This probability queueing models have poison input and exponential output service times. In our traffic flow model, traffic flows are function of number of vehicles, speed of the vehicles and traffic density. The DSW algorithm is used to obtain diagrams of waiting time of the system, speed of the flow, relative speed and maximum traffic flow.

Srinivasan discussed DSW approach for M/M/1 model. In this paper, $\alpha -$ cut method is used to decompose a fuzzy queue into family of crisp queues. To evaluate the proposed method the fuzzy queue model M/M/1 is used where M and M represents poison arrival process and exponential service process respectively. The solutions from DSW algorithm gives the membership functions of the crisp queues which we obtained from $\alpha -$ cut method.

2. MODEL FORMULATION:

Consider a general queueing system in which arrival rate λ and service rate μ. Also λ is fuzzy number in nature and μ is exponentially considered. Traffic flow theory is the inter-linkage of traffic flow q, traffic density E and speed s.

$$q = E * s \qquad \dots(1)$$

If we know the two variables there, then the third variable can easily be obtained. The following table 1 gives the important parameters used in this paper.

Table 1:

Parameter	**Description**

E	Traffic density (Veh/km)
s	Speed of the flow (km/h)
C	Maximum traffic flow density (Veh/km)
r	Relative speed (km/h)
SN	Nominal speed (km/h)
q_{max}	Maximum traffic flow (Veh/h)
λ	Arrival rate (Veh/h)
μ	Service rate (Veh/h)
ρ	Intensity of traffic flow $\left(\lambda/_{\mu}\right)$
W_s	Waiting time in the system (h)

In this model, C is defined as the maximum traffic density which is depends on the type of road i.e., number of lanes, etc… Consider roads are divided into small segments of equal length $1/_{C}$. This length is the minimal length needed for one vehicle on that particular road. Each segment is considered as a service station, in which each vehicle arrived at rate λ and served at rate μ.

W_s is defined as the total time required to the system, which is equal to the sum of waiting time for all vehicles in the traffic flow. When the traffic intensity is higher, then the time in the system also higher.

The calculation of effective speed of the vehicle is calculated by the following formula:

$$s = \frac{1/_{C}}{W_s} \qquad \ldots(2)$$

The relative speed r is defined by,

$$r = \frac{s}{SN} = \frac{1/_{C}}{W * SN} \qquad \ldots(3)$$

Queueing models are referred to using the Kendall notation, consisting several symbols. We develop traffic model with M/M/1, the first symbol is for the distribution of inter-arrival times, the second for the distribution of service times and last one denotes the number of servers in the system.

Fuzzified M/M/1 traffic model:

The arrival rate λ is the product of the traffic density E and nominal speed SN and inter-arrival times are distributed exponentially. The service rate μ is also distributed exponentially in which the time needed for a vehicle to pass one segment to another road segment. Service rate μ is the product of nominal speed SN and traffic density C.

With these representations of λ and μ, W_s is obtained as

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{SN \times C - SN \times E}$$

$$W_s = \frac{1}{SN(C-E)} \qquad \dots(4)$$

Using this expression for W_s, we obtained effective speed and relative speed as

$$s = \frac{SN(C-E)}{C}$$
$$= SN\left(1-\frac{E}{C}\right) = SN(1-\rho)$$
$$s = SN\left(1-\frac{\lambda}{\mu}\right) \qquad \dots(5)$$
$$r = \frac{s}{SN} = \frac{SN\left(1-\frac{\lambda}{\mu}\right)}{SN}$$
$$r = 1-\frac{\lambda}{\mu} \qquad \dots(6)$$

With ρ the traffic density, we define

$$\rho = \frac{\lambda}{\mu} = \frac{E}{C} \qquad \dots(7)$$

Substitute the value of $E = \frac{q}{s}$ in (5), the following equation is obtained

$$s = \frac{SN\left(C-\frac{q}{s}\right)}{C}$$
$$s.C = \frac{SN(s.C-q)}{s}$$
$$s^2.C = SN.s.C - q.SN$$
$$s^2.C - s.C.SN + SN.q = 0$$

This expression is taken as the function of s and q as,

$$f(s,q) = s^2.C - s.C.SN + SN.q = 0 \qquad \dots(8)$$
$$f'(s,q) = 2s.C - C.SN = 0$$
$$\Rightarrow\ s = \frac{SN}{2}$$

To maximize $f(s,q)$ for s and substitute this value in (8), q_{max} can be written as

$$SN.q_{max} + \frac{SN^2}{4} \times C - \frac{SN.SN.C}{2} = 0$$
$$SN.q_{max} = SN^2 \times C\left(\frac{1}{2}-\frac{1}{4}\right)$$
$$SN.q_{max} = \frac{SN^2 \times C}{4}$$
$$\Rightarrow\ q_{max} = \frac{SN \times C}{4}$$

From the values of W_s, s, r and q_{max},we obtain corresponding graphs using our numerical example.

Interval analysis:

Given $a, b \in \mathbb{R}$ such that $a \le b$, the classical interval $[a,b]$ is defined as $[a,b] = \{x \in \mathbb{R} / a \le x \le b\}$ where $\mathbb{R}$ is the set of all real numbers. The set of classical intervals is represented by I_1 and I_2.

$$I_1 = [a,b], \quad a < b\,; \quad I_2 = [c,d], \quad c < d$$

Generally, arithmetic property with the symbol * is defined as follows, where $* = [+, -, \times, \div]$ symbolically,

$$I_1 * I_2 = [a, b] * [c, d]$$

Which represents another new interval. The calculation in the intervals depends on the magnitudes and sign of the elements a, b, c, d.

$$I_1 + I_2 = [a + c, b + d]$$
$$I_1 - I_2 = [a - d, b - c]$$
$$I_1 \times I_2 = [min(ac, ad, bc, bd), max(ac, ad, bc, bd)]$$
$$I_1 \div I_2 = [a, b] * \left[\frac{1}{d}, \frac{1}{c}\right]$$

Provided $d, c \neq 0$.

$$\alpha[a, b] = \begin{cases} [\alpha a, \alpha b], & \alpha > 0 \\ [\alpha b, \alpha a], & \alpha < 0 \end{cases}$$

DSW algorithm:

DSW (Dong, Shah and Wong) algorithm is one of the approximate methods to make use of intervals at various α −cut levels in defining membership functions. It contains all α −cut intervals from a standard interval analysis.

Importantly the DSW algorithm simplifies manipulation of the extension principle for continuous valued fuzzy variables, such a fuzzy number defined on the real line. It prevents the irregularity in the output membership function due to application of the distinction reaching on the fuzzy variable domain, it can avoid the expanding of the resulting functional expression by conventional interval analysis methods.

Any membership function which is continuous can be represented by a continuous sweep α −cut which is in term from $\alpha = 0$ to $\alpha = 1$. Suppose we have single input functioning given by $y = f(x)$ that is to be extended for fuzzy sets $\tilde{B} = f(\tilde{A})$ and we want to do decomposition of $\tilde{A}$ into the set of α −cut intervals, say I_1 and I_2. It intends the use of full α −cut intervals in a standard interval analysis. The DSW algorithm consists of the following steps:

1. In the interval [0, 1], select the value of α −cut.
2. Corresponding to this α, find the intervals in the input membership functions.
3. For each selected α −cut level, calculate the interval for the output membership function by using standard binary interval operations.
4. Repeat the steps 1 to 3 for various values of α to finish a α −cut representation of two solutions.

Numerical example:

Consider an integrated a system in which the service system consists of two phases both the arrival and service rate are trapezoidal fuzzy numbers and denoted by $\lambda = [3\ \ 4\ \ 5\ \ 6]$ and $\mu = [14\ \ 15\ \ 16\ \ 17]$ per minute respectively.

Let's evaluate the performance measures. The Confidence interval at α are $[3 + \alpha\ \ 6 - \alpha]$ and $[14 + \alpha\ \ 17 - \alpha]$.

Consider $x = [3 + \alpha\ \ 6 - \alpha]$ and $y = [14 + \alpha\ \ 17 - \alpha]$. Our important measures in traffic flow model becomes,

$$W_s = \frac{1}{\mu - \lambda}$$
$$s = SN\left(1 - \frac{\lambda}{\mu}\right)$$

International Journal of Aquatic Science
ISSN: 2008-8019
Vol 12, Issue 02, 2021

$$r = 1 - \frac{\lambda}{\mu}$$

$$q_{max} = \frac{SN \times C}{4}$$

Assume that $SN = [90\ \ 100\ \ 110\ \ 120]$ and $C = [70\ \ 80\ \ 90\ \ 100]$

These values become $[90 + \alpha\ \ 120 - \alpha]$ and $[70 + \alpha\ \ 100 - \alpha]$ by confidence interval at α.

Steps to calculate W_s when $\alpha = 0$:

Given that $\lambda = [3\ \ 6], \mu = [14\ \ 17], SN = [90\ \ 120], C = [70\ \ 100]$

$$W_s = \frac{1}{\mu - \lambda}$$

$$W_s = \frac{1}{[14\ \ 17] - [3\ \ 6]}$$

Using interval analysis method,

$$W_s = \left[min\left(\frac{1}{14-3}, \frac{1}{14-6}, \frac{1}{17-3}, \frac{1}{17-6}\right), max\left(\frac{1}{14-3}, \frac{1}{14-6}, \frac{1}{17-3}, \frac{1}{17-6}\right)\right]$$

$$W_s = [\min(0.0909, 0.125, 0.0714, 0.0909), \max(0.0909, 0.125, 0.0714, 0.0909)]$$

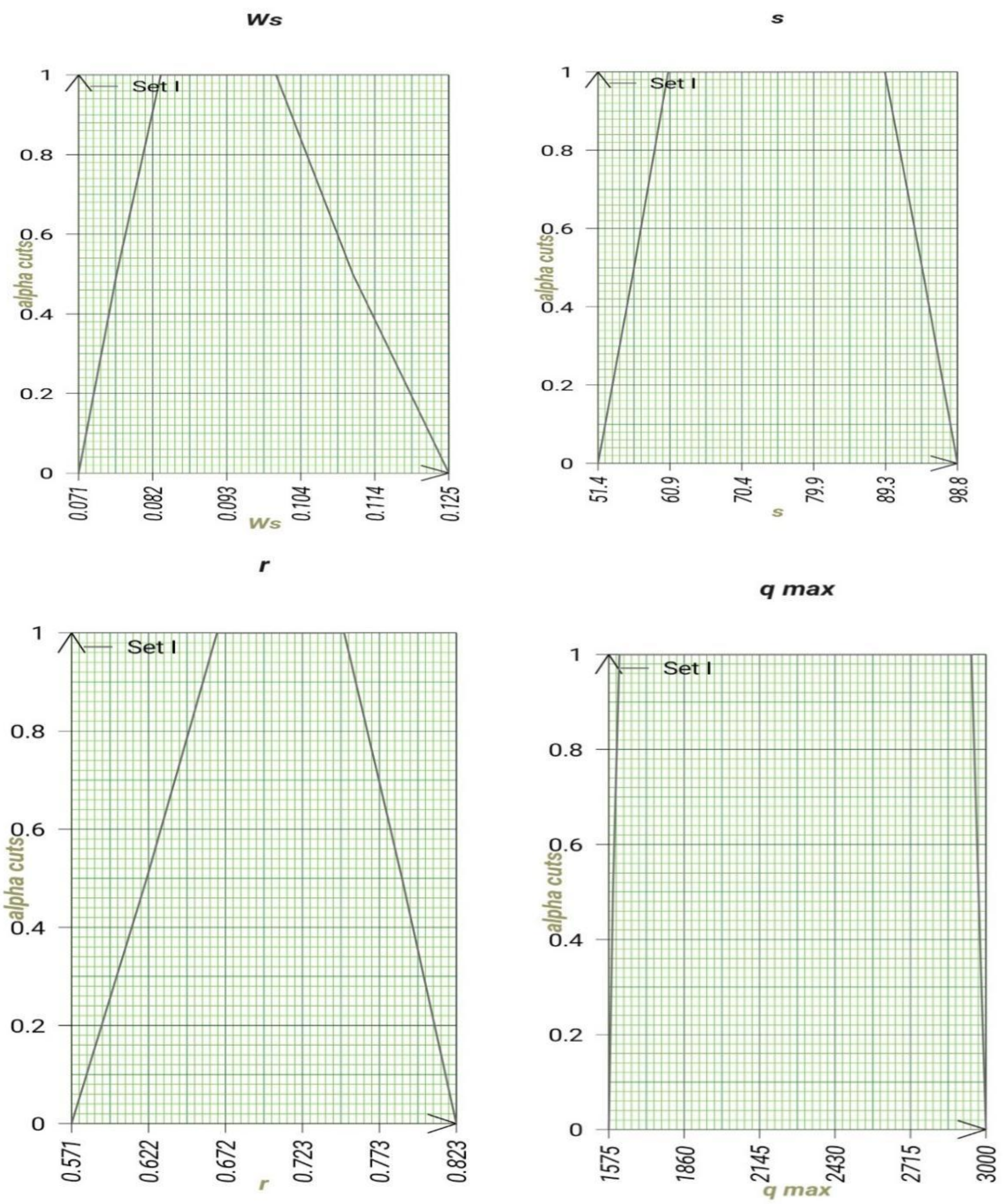
$$W_s = [0.0714, 0.125]$$

Similarly, continuing this procedure to obtain values for $\alpha = 0, 0.1, \ldots, 1$ to W_s, s, r, q_{max}.

Table 2: The α −cut of W_s, s, r, q_{max} at α values.

α	W_s	s	r	q_{max}
0	[0.0714, 0.125]	[51.426, 98.82]	[0.5714, 0.8235]	[1575, 3000]
0.1	[0.0725, 0.1220]	[52.4025, 97.9134]	[0.5816, 0.8166]	[1579, 2994.5]
0.2	[0.0735, 0.1190]	[53.3533, 96.9781]	[0.5915, 0.8095]	[1583.01, 2989.01]
0.3	[0.0746, 0.1163]	[54.3064, 96.0473]	[0.6014, 0.8024]	[1587, 2983.52]
0.4	[0.0758, 0.1136]	[55.2434, 95.1059]	[0.6111, 0.7952]	[1591.04, 2978.04]
0.5	[0.0769, 0.1111]	[56.1734, 94.1541]	[0.6207, 0.7879]	[1595.06, 2972.56]
0.6	[0.0781, 0.1087]	[57.0871, 93.1917]	[0.6301, 0.7805]	[1599.09, 2967.09]
0.7	[0.0794, 0.1064]	[58.0027, 92.2189]	[0.6395, 0.7730]	[1603.12, 2961.62]
0.8	[0.0806, 0.1042]	[58.8929, 91.2357]	[0.6486, 0.7654]	[1607.16, 2956.16]
0.9	[0.0820, 0.1020]	[59.7866, 90.2540]	[0.6577, 0.7578]	[1611.20, 2950.70]
1	[0.0833, 0.1]	[60.6697, 8925]	[0.6667, 0.75]	[1615.25, 2945.25]

In this method, the maximum traffic flow observed in the DSW algorithm is 2945 vehicles per hour. This is our required performance measure in this model.



3. CONCLUSION:

We constructed the diagrams of waiting time, speed and relative speed of the flow and maximum traffic flow from the observations in the numerical example. In this paper, we developed the inter-arrival and service time are fuzzified, according to DSW algorithm, the performance measures of traffic flow model will be fuzzy value. Our model can be effectively used to the environmental impact of road traffic in the case of finding speeds which is influenced on vehicle emissions. Our numerical illustration shows the efficiency of the DSW algorithm in the traffic flow model.

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