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Novel Strategy to Solve Transportation Problems with Interval - Valued Trapezoidal Fuzzy Numbers

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***Abstract** - The special type of linear programming problem where special mathematical structure of restrictions is used is the transportation problem. The central concept of transportation problem is to determine the minimum total transportation cost of a commodity for satisfying the demand at destination using the available supply at the origins. In real–life situations, the parameters of transportation problems may not be known precisely because of uncontrollable factors. Herein, a strategy is proposed for solving fuzzy transportation problems in which the transportation costs , supply and demand are represented as interval-valued trapezoidal fuzzy numbers. Finally, a numerical example is provided to illustrate the strategy and the result show that the proposed method is simpler and computationally more efficient.*

***Keywords** – Transportation problem, Fuzzy Transportation problem, Trapezoidal fuzzy numbers, Interval-valued trapezoidal fuzzy numbers, Signed distance ranking.*

1. INTRODUCTION

Transportation problem is a structured linear programming where the objective is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins. In classical approach, the values of transportation cost, availability and demand of the product are precise. However, in real life situations, the information available is imprecise in nature and there is uncertainty present in the problem under consideration. In order to tackle this uncertainty the concept of Fuzzy Sets can be used as an important decision making tool.

In 1978, Zimmermann [11] developed fuzzy linear programming for solving the transportation problems. Chanas & Kuchta[1] discussed the type of transportation problems with fuzzy cost coefficients and converted the problem into a bicriterial transportation problem with crisp objective function. Their method only gives crisp solutions based on efficient solutions of the converted problems. Kumar & Kaur [9] proposed a new method based on ranking function for solving fuzzy transportation problem by assuming that transportation cost, supply and demand of the commodity as generalized trapezoidal fuzzy numbers. Ebrahimnejad [4] proved that once the ranking function is selected the fuzzy transportation problem considered by Kaur & Kumar [10] is converted into a deterministic one, which is easily solved by the standard transportation algorithms. However, there are only few papers dealing with

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the problems involving interval-valued fuzzy numbers. Chiang [3] used interval-valued fuzzy numbers instead of normal fuzzy numbers to represent the availability and demand and proposed a method for obtaining the optimal solution of single-objective transportation problems by representing the availability and demand as interval-valued triangular fuzzy numbers. Ebrahimnejad [5] proposed a two-step method for solving fuzzy transportation problems where all of the parameters were represented by non- negative triangular fuzzy numbers. Moreover, Ebrahimnejad [6] proposed an fuzzy linear programming approach for solving transportation problem involving interval-valued trapezoidal fuzzy numbers by comparing the interval-valued fuzzy numbers using the signed distance ranking.

In this paper, transportation problems with costs, availability and demand as interval - valued trapezoidal fuzzy numbers is introduced. Herein, a strategy to solve interval-valued trapezoidal fuzzy transportation problem using is proposed. Also, a numerical example is provided to illustrate the strategy and finally the solution is compared with the existing method. This paper is organized as follows: In Section 2, some basic definitions and arithmetic operations are reviewed. In Section 3, a formulation of transportation problem with (w^L, w^U) –interval-valued trapezoidal fuzzy numbers is introduced. In Section 4, a method to solve interval-valued trapezoidal fuzzy transportation problem is proposed. In Section 5, an application example is provided to illustrate the effectiveness of the method. Finally, conclusion is given in Section 6.

II PRELIMINARIES

In this section, some basic definitions, the arithmetic operations and the comparison of the level (w^L, w^U) –interval-valued trapezoidal fuzzy numbers are presented([2] ,[10] [7], [8]) .

Definition 1:

A fuzzy set $\tilde{A}$, defined on the universal set of real numbers $\mathbb{R}$, is said to be a generalized fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}} : \mathbb{R} \to [0,w]$ is continuous.

(ii) $\mu_{\tilde{A}}(\tilde{x}) = 0$ for all $x \in (-\infty, a_1] \cup [a_4, \infty)$.

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a_1, a_2]$ and strictly decreasing on $[a_3, a_4]$.

(iv) $\mu_{\tilde{A}}(x) = w$ for all $x \in [a_2, a_3]$, where $0 < w \leq 1$.

Definition 2:

A level w- trapezoidal fuzzy number $\tilde{A}$ or a generalized trapezoidal fuzzy number $\tilde{A}$, denoted by

$\tilde{A} = (a_1, a_2, a_3, a_4; w)$, $0 < w \leq 1$ is a fuzzy number with membership function as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} w\dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2, \\ w, & a_2 \le x \le a_3, \\ w\dfrac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4, \\ 0, & \text{otherwise.} \end{cases}$$

Let $F_{TN}(w)$ be the family of all level w- trapezoidal fuzzy numbers, that is,

$F_{TN}(w) = \{\tilde{A} = (a_1, a_2, a_3, a_4; w), a_1 \le a_2 \le a_3 \le a_4\}, 0 < w \le 1.$

Definition 3:

If $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ are two Generalized Trapezoidal Fuzzy numbers then the arithmetic operations of $\tilde{A}$ and $\tilde{B}$ are defined as follows
(i) Generalized Fuzzy numbers addition ($\oplus$) :
$\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \text{minimum}(w_1, w_2))$
where $a_1, b_1, c_1, d_1, a_2, b_2, c_2\ d_2$ are any real numbers.
(ii) Generalized Fuzzy numbers subtraction ($\ominus$) :
$\tilde{A} \ominus \tilde{B} = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; \text{minimum}(w_1, w_2))$
where $a_1, b_1, c_1, d_1, a_2, b_2, c_2\ d_2$ are any real numbers.
(iii) Generalized Fuzzy numbers multiplication ($\otimes$) :
$\tilde{A} \otimes \tilde{B} = (a', b', c', d'; \text{minimum}(w_1, w_2))$
where $a' = \text{minimum}(a_1a_2, a_1d_2, a_2d_1, d_1d_2)$ $b' = \text{minimum}(b_1b_2, b_1c_2, c_1b_2, c_1c_2)$
$c' = \text{maximum}\ (b_1b_2, b_1c_2, c_1b_2, c_1c_2)$ $d' = \text{maximum}\ (a_1a_2, a_1d_2, a_2d_1, d_1d_2)$
where $a_1, b_1, c_1, d_1, a_2, b_2, c_2\ d_2$ are any real numbers.
(iv) Generalized Fuzzy numbers scalar multiplication:

$$\lambda\tilde{A} = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; w_1), \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; w_1), \lambda < 0 \end{cases}$$

Definition 4:

Let $\tilde{A} = (a, b, c, d; w)$ be a generalized trapezoidal fuzzy number then
(i) Rank $R(\tilde{A}) = \frac{w(a+b+c+d)}{4}$, (ii) mode $(\tilde{A}) = \frac{w(b+c)}{2}$, (iii) divergence $(\tilde{A}) = w(d-a)$
(iv) Left spread $(\tilde{A}) = w(b-a)$, (v) Right spread $(\tilde{A}) = w(d-c)$

Two generalized trapezoidal fuzzy numbers $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ can be compared using the ranking functions given in [8]

Definition 5:

Let $\tilde{A}^L \in F_{TN}(w^L)$ and $\tilde{A}^U \in F_{TN}(w^U)$. A level (w^L, w^U) – interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$ denoted by
$\tilde{\tilde{A}} = [\tilde{A}^L, \tilde{A}^U] = \langle(a_1^L, a_2^L, a_3^L, a_4^L; w^L), (a_1^U, a_2^U, a_3^U, a_4^U; w^U)\rangle$ is an interval- valued fuzzy set on $\mathbb{R}$ with the lower trapezoidal fuzzy number $\tilde{A}^L$ expressing by

$$\mu_{\tilde{A}^L}(x) = \begin{cases} w^L \dfrac{x - a_1^L}{a_2^L - a_1^L}, & a_1^L \le x \le a_2^L, \\ w^L, & a_2^L \le x \le a_3^L, \\ w^L \dfrac{a_4^L - x}{a_4^L - a_3^L}, & a_3^L \le x \le a_4^L, \\ 0, & \text{otherwise,} \end{cases}$$

and the upper trapezoidal fuzzy number $\tilde{A}^U$ expressing by

$$\mu_{\tilde{A}^U}(x) = \begin{cases} w^U \dfrac{x - a_1^U}{a_2^U - a_1^U}, & a_1^U \le x \le a_2^U, \\ w^U, & a_2^U \le x \le a_3^U, \\ w^U \dfrac{a_4^U - x}{a_4^U - a_3^U}, & a_3^U \le x \le a_4^U, \\ 0, & \text{otherwise,} \end{cases}$$

where $a_1^L \le a_2^L \le a_3^L \le a_4^L$, $a_1^U \le a_2^U \le a_3^U \le a_4^U$,
$0 < w^L \le w^U \le 1$, $a_1^U \le a_1^L$ and $a_4^L \le a_4^U$.

Moreover, $\mu_{\tilde{A}^L}(x) \le \mu_{\tilde{A}^U}(x)$. This means that the grade of membership x belongs to interval $\tilde{\tilde{A}} = [\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)]$, the latest and greatest grade of membership at x are $\mu_{\tilde{A}^L}(x)$ and $\mu_{\tilde{A}^U}(x)$ respectively.

Let $F_{IVTN}(w^L, w^U)$ be the family of all level (w^L, w^U)- interval- valued trapezoidal fuzzy numbers, that is,

$$F_{IVTN}(w^L, w^U) = \left\{ \begin{matrix} \tilde{\tilde{A}} = [\tilde{A}^L, \tilde{A}^U] = \langle (a_1^L, a_2^L, a_3^L, a_4^L;\ w^L), (a_1^U, a_2^U, a_3^U, a_4^U;\ w^U) \rangle : \\ \tilde{A}^L \in F_{TN}(w^L), \\ \tilde{A}^U \in F_{TN}(w^U),\ a_1^U \le a_1^L, a_4^L \le a_4^U \\ 0 < w^L \le w^U \le 1. \end{matrix} \right\},$$

Definition 6:
Let , 0 $\in \mathbb{R}$. The signed distance from r to 0 is defined as $d(r, 0) = r$.

Theorem : Let $\tilde{\tilde{A}} \in F_{IVTN}(w^L, w^U)$. The signed distance of $\tilde{\tilde{A}}$ from O1 (y-axis) is given as follows :

$$d(\tilde{A}, O_1) = \frac{1}{4}\left[a_1^L + a_2^L + a_3^L + a_4^L\right],\ \tilde{A}^L = \tilde{A}^U = \tilde{A}$$

$$d\left(\tilde{\tilde{A}}, O_1\right) = \frac{1}{8}\left[a_1^L + a_2^L + a_3^L + a_4^L + a_1^U + a_2^U + a_3^U + a_4^U\right], \quad 0 < w^L = w^U \le 1$$

$$d\left(\tilde{\tilde{A}}, O_1\right) = \frac{1}{8}\begin{bmatrix} a_1^L + a_2^L + a_3^L + a_4^L + 4a_1^U + 2a_2^U + 2a_3^U + 4a_4^U + \\ 3(a_2^U + a_3^U - a_1^U - a_4^U)\frac{w^L}{w^U} \end{bmatrix}, \quad 0 < w^L < w^U \le 1$$

Definition 7:

Let $\tilde{\tilde{A}}, \tilde{\tilde{B}} \in F_{IVTN}(w^L, w^U)$. Then the ranking of level (w^L, w^U) – interval - valued trapezoidal fuzzy numbers
in $F_{IVTN}(w^L, w^U)$ is defined on the basis of signed distance $d(.,O_1)$ as follows:
$\tilde{\tilde{A}} \prec \tilde{\tilde{B}}$ iff $d\left(\tilde{\tilde{A}}, O_1\right) < d\left(\tilde{\tilde{B}}, O_1\right)$
$\tilde{\tilde{A}} \succ \tilde{\tilde{B}}$ iff $d\left(\tilde{\tilde{A}}, O_1\right) > d\left(\tilde{\tilde{B}}, O_1\right)$
$\tilde{\tilde{A}} \approx \tilde{\tilde{B}}$ iff $d\left(\tilde{\tilde{A}}, O_1\right) = d\left(\tilde{\tilde{B}}, O_1\right)$
Notice that the signed distance $d(.,O_1)$ provides us a linear ranking function,
i.e. for any $\tilde{\tilde{A}}, \tilde{\tilde{B}} \in F_{IVTN}(w^L, w^U)$ and $k \in \mathbb{R}$ we have
$d\left(k\tilde{\tilde{A}} \oplus \tilde{\tilde{B}}, O_1\right) = k\, d\left(\tilde{\tilde{A}}, O_1\right) + d\left(\tilde{\tilde{B}}, O_1\right)$.
In addition, $(F_{IVTN}(w^L, w^U), \approx, \prec)$ satisfies the law of trichotomy
that is, we have
$\tilde{\tilde{A}} \prec \tilde{\tilde{B}}$ or $\tilde{\tilde{A}} \approx \tilde{\tilde{B}}$ or $\tilde{\tilde{B}} \prec \tilde{\tilde{A}}$

TRANSPORTATION PROBLEM IN INTERVAL- VALUED TRAPEZOIDAL FUZZY $F_{IVTN}(w^L, w^U)$ ENVIRONMENT

In this section, transportation problem in interval valued trapezoidal fuzzy environment is introduced.

Suppose that all the parameters (cost, supply, demand and amount of commodity) are represented by (w^L, w^U)- interval- valued trapezoidal fuzzy numbers. Then transportation problem in $F_{IVTN}(w^L, w^U)$ environment is

	IVFD$_1$	IVFD$_2$		IVFDn	IVFC
IVFO$_1$	$\tilde{x}_{11}$ $\tilde{c}_{11}$	$\tilde{x}_{12}$ $\tilde{c}_{12}$	…	$\tilde{x}_{1n}$ $\tilde{c}_{1n}$	$\tilde{a}_1$
IVFO$_2$	$\tilde{x}_{21}$ $\tilde{c}_{21}$	$\tilde{x}_{22}$ $\tilde{c}_{22}$	…	$\tilde{x}_{2n}$ $\tilde{c}_{2n}$	$\tilde{a}_2$
	⋮	⋮	⋮	⋮	
IVFO$_m$	$\tilde{x}_{m1}$ $\tilde{c}_{m1}$	$\tilde{x}_{m2}$ $\tilde{c}_{m2}$	…	$\tilde{x}_{mn}$ $\tilde{c}_{mn}$	$\tilde{a}_m$
IVFD	$\tilde{b}_1$	$\tilde{b}_2$		$\tilde{b}_n$	

IVFO$_i$ (i=1,2,…m) – Interval - valued Fuzzy Origin,
IVFD$_j$ (j= 1,2,…n) – Interval - valued Fuzzy Destination,
IVFC – Interval - valued Fuzzy Capacity,
IVFD – Interval - valued Fuzzy Demand.

where
$\tilde{a}_i = \langle (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; w^L), (a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; w^U) \rangle \in F_{IVTN}^{+}(w^L, w^U), i = 1, 2, \cdots, m,$

$\tilde{b}_j = \langle (b_{j1}^L, b_{j2}^L, b_{j3}^L, b_{j4}^L; w^L), (b_{j1}^U, b_{j2}^U, b_{j3}^U, b_{j4}^U; w^U) \rangle \in F_{IVTN}^{+}(w^L, w^U), j = 1, 2, \cdots, n,$
$\tilde{c}_{ij} = \langle (c_{ij1}^L, c_{ij2}^L, c_{ij3}^L, c_{ij4}^L; w^L), (c_{ij1}^U, c_{ij2}^U, c_{ij3}^U, c_{ij4}^U; w^U) \rangle \in F_{IVTN}^{+}(w^L, w^U),$
$i = 1, 2, \cdots, m; j = 1, 2, \cdots, n$
And
$\tilde{x}_{ij} = \langle (x_{ij1}^L, x_{ij2}^L, x_{ij3}^L, x_{ij4}^L; w^L), (x_{ij1}^U, x_{ij2}^U, x_{ij3}^U, x_{ij4}^U; w^U) \rangle \in F_{IVTN}^{+}(w^L, w^U),$
$i = 1, 2, \cdots, m; j = 1, 2, \cdots, n.$

METHOD TO SOLVE INTERVAL-VALUED TRAPEZOIDAL FUZZY TRANSPORTATION PROBLEM

In this section , algorithm to solve interval valued trapezoidal fuzzy transportation problem is proposed.

Steps to find the solution of interval valued trapezoidal fuzzy transportation problem are as follows

Step 1:
Find the total interval- valued fuzzy supply $\sum_{i=1}^{m} \tilde{a}_i$ and the total interval- valued fuzzy demand $\sum_{j=1}^{n} \tilde{b}_j$.
Let $\sum_{i=1}^{m} \tilde{a}_i = \langle (m_1^L, m_2^L, m_3^L, m_4^L; w^L), (m_1^U, m_2^U, m_3^U, m_4^U; w^U) \rangle$ and
$\sum_{j=1}^{n} \tilde{b}_j = \langle (n_1^L, n_2^L, n_3^L, n_4^L; w^L), (n_1^U, n_2^U, n_3^U, n_4^U; w^U) \rangle$.
Examine the problem is balanced or not, i.e. $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$
or $\sum_{i=1}^{m} \tilde{a}_i \neq \sum_{j=1}^{n} \tilde{b}_j$.

Case (i) If the problem is balanced, i.e., $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$, then go to Step 2.
Case (ii) If $\sum_{i=1}^{m} \tilde{a}_i \neq \sum_{j=1}^{n} \tilde{b}_j$, then convert the unbalanced problem into balanced problem as follows
Case (a) If $m_1^L \leq n_1^L$, $m_2^L \leq n_2^L$, $m_3^L \leq n_3^L$, $m_4^L \leq n_4^L$, $m_1^U \leq n_1^U$, $m_2^U \leq n_2^U$, $m_3^U \leq n_3^U$ and $m_4^U \leq n_4^U$ then introduce a dummy origin with interval-valued fuzzy supply
$\langle (n_1^L - m_1^L, n_2^L - m_2^L, n_3^L - m_3^L, n_4^L - m_4^L; w^L), (n_1^U - m_1^U, n_2^U - m_2^U, n_3^U - m_3^U, n_4^U - m_4^U; w^U) \rangle$.
Assume the interval- valued fuzzy transportation cost for one unit quantity of the commodity from the introduced dummy origin to all destinations as zero interval- valued trapezoidal fuzzy number, i.e., $\tilde{0}$ $\langle (0, 0, 0, 0; 1), (0, 0, 0, 0; 1) \rangle$, then go to Step 2.

Case (b) If $n_1^L \leq m_1^L$, $n_2^L \leq m_2^L$, $n_3^L \leq m_3^L$, $n_4^L \leq m_4^L$, $n_1^U \leq m_1^U$, $n_2^U \leq m_2^U$, $n_3^U \leq m_3^U$ and $n_4^U \leq m_4^U$ then introduce a dummy destination with interval-valued fuzzy demand
$\langle (m_1^L - n_1^L, m_2^L - n_2^L, m_3^L - n_3^L, m_4^L - n_4^L; w^L), (m_1^U - n_1^U, m_2^U - n_2^U, m_3^U - n_3^U, m_4^U - n_4^U; w^U) \rangle$. Assume the interval- valued fuzzy transportation cost for one unit quantity of the commodity from all origins to the introduced dummy destination as zero interval- valued trapezoidal fuzzy number, i.e., $\tilde{0}$ $\langle (0, 0, 0, 0; 1), (0, 0, 0, 0; 1) \rangle$, then go to Step 2.

Case (c) If neither Case (a) nor Case (b) is satisfied then introduce a dummy origin with interval- valued fuzzy supply $\langle (A_1^L, A_2^L, A_3^L, A_4^L; w^L), (A_1^U, A_2^U, A_3^U, A_4^U; w^U) \rangle$ and dummy

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destination with interval- valued fuzzy demand $\langle(B_1^L,\ B_2^L,\ B_3^L, B_4^L;\ w^L), (B_1^U,\ B_2^U,\ B_3^U, B_4^U;\ w^U)\rangle$
where
$A_1^L = |n_1^U - m_1^U| + max\{0,\ n_1^L - m_1^L\}$,
$A_2^L = A_1^L + max\{0,\ (n_2^L - n_1^L) - (m_2^L - m_1^L)\}$
$A_3^L = A_2^L + max\{0,\ (n_3^L - n_2^L) - (m_3^L - m_2^L)\}$,
$A_4^L = A_3^L + max\{0,\ (n_4^L - n_3^L) - (m_4^L - m_3^L)\}$
$B_1^L = |n_1^U - m_1^U| + max\{0,\ m_1^L - n_1^L\}$,
$B_2^L = B_1^L + max\{0,\ (m_2^L - m_1^L) - (n_2^L - n_1^L)\}$
$B_3^L = B_2^L + max\{0,\ (m_3^L - m_2^L) - (n_3^L - n_2^L)\}$,
$B_4^L = B_3^L + max\{0,\ (m_4^L - m_3^L) - (n_4^L - n_3^L)\}$
$A_1^U = max\{0,\ n_1^U - m_1^U\}$,
$A_2^U = |n_1^U - m_1^U| + max\{0, n_1^U - m_1^U\} + max\{0, (n_2^U - n_1^U) - (m_2^U - m_1^U)\}$
$A_3^U = A_2^U + max\{0, (n_3^U - n_2^U) - (m_3^U - m_2^U)\}$,
$A_4^U = A_3^U + max\{0, (n_4^U - n_3^U) - (m_4^U - m_3^U)\} + min\{0, max\{0, (n_4^U - n_3^U) - (m_4^U - m_3^U)\} - A_4^L\}$
$B_1^U = max\{0, m_1^U - n_1^U\}$,
$B_2^U = B_1^U + |n_1^U - m_1^U| + max\{0, (m_2^U - m_1^U) - (n_2^U - n_1^U)\}$
$B_3^U = B_2^U + max\{0, (m_3^U - m_2^U) - (n_3^U - n_2^U)\}$,
$B_4^U = B_3^U + max\{0, (m_4^U - m_3^U) - (n_4^U - n_3^U)\} + min\{0, B_3^U + max\{0, (m_4^U - m_3^U) - (n_4^U - n_3^U)\} - B_4^L\}$
Assume the interval- valued fuzzy transportation cost for one unit quantity of the commodity from the introduced dummy origin to all destinations and from all origins to the introduced dummy destination and as zero interval- valued trapezoidal fuzzy number, i.e., $\tilde{0}\ \langle(0,0,0,0;1), (0,0,0,0;1)\rangle$, then go to Step 2.

Step 2:

Split the interval valued trapezoidal fuzzy transportation table into lower trapezoidal fuzzy transportation table and upper trapezoidal fuzzy transportation table.

Step 3:
Find the row minimum by using the ranking method and then subtract the row minimum from each row entry of that row of lower trapezoidal fuzzy transportation problem.

Step 4:
Find the column minimum by using the ranking method and then subtract the column minimum of the resulting lower trapezoidal fuzzy transportation problem from each column entry of that column. Each column and row now have at least one trapezoidal fuzzy number with rank zero.

Step 5:
Check whether each column trapezoidal fuzzy demand is lesser than the sum of trapezoidal fuzzy supply whose reduced trapezoidal fuzzy cost in that column are trapezoidal fuzzy number with rank zero in the resulting lower trapezoidal fuzzy transportation problem. Also check whether each row trapezoidal fuzzy supply is lesser than the sum of the column trapezoidal fuzzy demands whose reduced trapezoidal fuzzy costs in that row are trapezoidal

fuzzy number with rank zero in the resulting lower trapezoidal fuzzy transportation problem. If so, go to step 7 otherwise go to step 6.

Step 6:

Draw minimum number of horizontal and vertical lines to cover all trapezoidal fuzzy number with rank zero in the resulting lower trapezoidal fuzzy transportation problem. Find the smallest valued trapezoidal fuzzy cost not covered by any line using the ranking and subtract it from all uncovered trapezoidal fuzzy costs and add the same to all trapezoidal fuzzy costs lying at the intersection of any two lines in the resulting lower trapezoidal fuzzy transportation problem. Repeat this step till trapezoidal fuzzy supply satisfies trapezoidal fuzzy demand for all rows and columns in the resulting lower trapezoidal fuzzy transportation problem.

Step 7:

Allocate the maximum trapezoidal fuzzy quantity to be transported where the trapezoidal fuzzy costs have been trapezoidal fuzzy number with rank zero depending on the trapezoidal fuzzy demand and trapezoidal fuzzy supply, starting from the row/column with single trapezoidal fuzzy number with rank zero in the resulting lower trapezoidal fuzzy transportation problem.

Step 8:

Repeat Step7 till all trapezoidal fuzzy supply and trapezoidal fuzzy demand quantities are exhausted. And the same algorithm is to be applied for the upper trapezoidal fuzzy transportation problem also and then the optimal solution for the interval valued trapezoidal fuzzy transportation problem is obtained.

NUMERICAL EXAMPLE

In this section, an application example is solved by using the above discussed method.

Example 1: A company has two origins O_1 and O_2 and three destinations D_1, D_2 and D_3; the approximate transportation cost for unit commodity from i^{th} source to j^{th} destination, the approximate supply of the commodity at two origins and the approximate demand of the commodity at three destinations are represented by interval- valued trapezoidal fuzzy numbers and shown in table below:

Table 1:

Destinations / Origins	D_1	D_2	D_3	Supply
O_1	$< (10,20,30,40;\frac{2}{3})$ $(5,15,35,45;1) >$	$< (50,60,70,90;\frac{2}{3})$ $(45,55,75,95;1) >$	$< (80,90,110,120;\frac{2}{3})$ $(75,85,115,125;1) >$	$< (70,90,90,100;\frac{2}{3})$ $(65,85,95,105;1) >$
O_2	$< (60,70,80,90;\frac{2}{3})$ $(55,65,85,95;1) >$	$< (70,80,100,120;\frac{2}{3})$ $(65,75,105,125;1) >$	$< (20,30,50,60;\frac{2}{3})$ $(15,25,55,65;1) >$	$< (40,60,70,80;\frac{2}{3})$ $(35,55,75,85;1) >$
Demand	$< (30,40,50,70;\frac{2}{3})$ $(25,35,55,75;1) >$	$< (20,30,40,50;\frac{2}{3})$ $(15,25,45.55;1) >$	$< (40,50,50,80;\frac{2}{3})$ $(35,45,55,85;1) >$	

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The company wants to determine the approximate quantity of the commodity that should be transported from each origin to each destination so that the total approximate transportation cost is minimum.

The total interval-valued fuzzy supply and total interval-valued fuzzy demand are $\langle\left(110,\ 150, 160, 180;\ \frac{2}{3}\right), (100, 140, 170, 190; 1)\rangle$and $\langle\left(90, 120, 140, 200;\ \frac{2}{3}\right), (75, 105, 155, 215; 1)\rangle$, respectively. Since total interval- valued fuzzy supply and total interval- valued fuzzy demand are not equal, so this is an unbalanced interval- valued fuzzy transportation problem. Now, as described in the method (Case (c) of Step1), unbalanced interval- valued fuzzy transportation problem can be converted into a balanced interval- valued fuzzy transportation problem, by introducing a dummy origin O_3 with interval- valued fuzzy supply $\langle\left(25, 25, 35, 75;\ \frac{2}{3}\right)(0, 25, 45, 85; 1)\rangle$ and a dummy destination D_4 with interval-valued fuzzy demand $\langle\left(45, 55, 55, 55;\ \frac{2}{3}\right), (25, 60, 60, 60; 1)\rangle$. Assume the interval- valued fuzzy transportation cost for one unit quantity of the commodity from the introduced dummy origin O_3 to all destinations and from all origins to the introduced dummy destination D_4 are as zero interval- valued trapezoidal fuzzy number i.e., $\tilde{c}_{14} = \tilde{c}_{24} = \tilde{c}_{31} = \tilde{c}_{32} = \tilde{c}_{33} = \tilde{c}_{34} = \tilde{0}$.

Using the step2 of the proposed method, split the interval - valued trapezoidal fuzzy table as follows

Table 2:

$(10,20,30,40;\frac{2}{3})$	$(50,60,70,90;\frac{2}{3})$	$(80,90,110,120;\frac{2}{3})$	$\tilde{0}$	$(70,90,90,100;\frac{2}{3})$
$(60,70,80,90;\frac{2}{3})$	$(70,80,100,120;\frac{2}{3})$	$(20,30,50,60;\frac{2}{3})$	$\tilde{0}$	$(40,60,70,80;\frac{2}{3})$
$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$(25,25,35,75;\frac{2}{3})$
$(30,40,50,70;\frac{2}{3})$	$(20,30,40,50;\frac{2}{3})$	$(40,50,50,80;\frac{2}{3})$	$(45,55,55,55;\frac{2}{3})$	

Table 3 :

(5,15,35,45;1)	(45,55,75,95;1)	(75,85,115,125;1)	$\tilde{0}$	(65,85,95,105;1)
(55,65,85,95;1)	(65,75,105,125;1)	(15,25,55,65;1)	$\tilde{0}$	(35,55,75,85;1)
$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	(0,25,45,85;1)
(25,35,55,75;1)	(15,25,45.55;1)	(35,45,55,85;1)	(25,60,60,60;1)	

Using steps 3 to 8 of the proposed method, the optimal solution for the Interval -valued trapezoidal fuzzy transportation problem is obtained as in the following table

Table 4 :

$<(30,40,50,70;\frac{2}{3})$ $(25,35,55,75;1)>$			$<(0,40,50,70;\frac{2}{3})$ $(-10,30,60,80;1)>$
		$<(-15,45,65,105;\frac{2}{3})$ $(-35,25,75,140;1)>$	$<(-25,5,15,55;\frac{2}{3})$ $(-55,0,30,70;1)>$
	$<(20,30,40,50;\frac{2}{3})$ $(15,25,45.55;1)>$	$<(-25, -15, 5,55;\frac{2}{3})$ $(-55,-20,20,70;1)>$	

And the signed distance of the optimal solution of the interval-valued trapezoidal fuzzy transportation problem is 8,093.75

Comparing the above result with the existing method [4], it is observed that total transportation cost obtained by using the proposed method is minimum, since, the signed distance of the optimal solution of the above interval-valued trapezoidal fuzzy transportation problem using the existing method [4] is 10,075.

The classical LP problem [4] applied for the above interval-valued trapezoidal fuzzy transportation problem has 152 constraints (without considering the non- negative constraints) and 88 variables (without considering slack variables), whereas the proposed method is simpler and computationally more efficient.
Hence, from the computation point of view the proposed method is preferable compared to the existing method [4] for solving the interval- valued trapezoidal fuzzy transportation problem.

2. CONCLUSION

In this paper, Interval-valued trapezoidal fuzzy transportation problem is considered and a strategy to solve fuzzy transportation problems in which the transportation costs, supply and demand are represented as interval-valued trapezoidal fuzzy numbers is proposed. Finally, a numerical example is provided to illustrate the strategy and the result is compared with the existing method. And it is concluded from the computation point of view, the proposed method is preferable compared to the existing method for solving the interval- valued trapezoidal fuzzy transportation problem.

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