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Edge Degree Set of Intuitionistic Fuzzy graphs and its Properties UnderIsomorphisms

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Abstract—In this paper edge degree set of intuitionistic fuzzy graph (IFG) is introduced and some of its properties are studied. If two intuitionistic fuzzy graphs are isomorphic or co-weak isomorphic, then they must have same edge degree set. But weak isomorphism need not preserve edge degree set.

Keywords—Degree of a vertex, Degree of an Edge, Degree set, Edge Degree set, Edge regular, Isomorphism, weak isomorphism, co-weak isomorphism.
AMS Subject Classification: 03F55, 05C72, 03E72

1. INTRODUCTION

Rosenfeld introduced the concept of fuzzy graphs in 1975 [10].Bhattacharya [2] gave some remarks on fuzzy graphs. K.R.Bhutani also introduced the concepts of weak isomorphism, co-weak isomorphism and isomorphism between fuzzy graphs [3]. K.Radha and A. Rosemine introduced degree sequence of fuzzy graph[9]. K.T.Atanassov[1] introduced the concept of intuitionisitic fuzzy sets as a generalization of fuzzy sets. Intuitionistic Fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry, economics and washing machine. R.Parvathi and M.G.Karunambigai discussed some concepts in intuitionistic fuzzy graphs[6].R.Parvathi and M.G.Karunambigai and R.Buvaneswari introduced constant intuitionistic fuzzy graphs[5]. In this paper, we introduce edge degree set of intuitionistic fuzzy graphs and discuss about the edge degree set of isomorphic,co-weak isomorphic and weak isomorphic intuitionistic.

Definition 1.1: [5]
G: $(\mu_1, \gamma_1;\ \mu_2, \gamma_2)$ is an intuitionistic fuzzy graph (IFG) on G* = (V, E) where
(i) $V=\{v_1, v_2, v_3, \dots, v_n\}$, $\mu_1 : V \rightarrow [0,1]$ and $\gamma_1 : V \rightarrow [0,1]$ denote the degree of membership value and the degree of non-membership value of the elements $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$, for every $v_i \in V$, i =1, 2, 3, ⋯, n .
(ii) $E \subseteq V \times V$, $\mu_2 : V \times V \rightarrow [0,1]$ and $\gamma_2 : V \times V \rightarrow [0,1]$ are such that
$\mu_2(v_I, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$,
$\gamma_2(v_i v_j) \leq \gamma_1(v_i) \vee \gamma_1(v_j)$
and $0 \leq \mu_2(v_i v_j) + \gamma_2(v_i v_j) \leq 1$

for every $v_i v_j \in E$, i, j = 1, 2, 3, ⋯, n.

Definition 1.2: [6]

Let G = $(\mu_1, \gamma_1; \ \mu_2, \gamma_2)$ be an IFG on G^*= (V, E). Then degree of a vertex $v_i \in G$ is defined by

$d_G(v_i) = (d_G^{\mu}(v_i), d_G^{\gamma}(v_i))$ where

$d_G^{\mu}(v_i) = \Sigma\mu_2(v_i v_j)$ and $d_G^{\gamma}(v_i) = \Sigma\gamma_2(v_i v_j)$,

where the summation runs over all $v_i v_j \in E$,

Definition: 1.3[8]:

Let G = $(\mu_1, \gamma_1; \ \mu_2, \gamma_2)$ be an IFG on G^* = (V, E). Then degree of an Edge uv ∈ E is defined by d(uv) = $(d\mu_2(uv), d\gamma_2(uv))$ where $d\mu_2(uv) = d\mu_1(u) + d\mu_1(v) - 2\mu_2(uv)$ and $d\gamma_2(uv) = d\gamma_1(u) + d\gamma_1(v) - 2\gamma_2(uv)$.

Definition 1.3:

We use the following order relation to compare two ordered pairs of real numbers:

(i) (u, v)= (x, y) if and only if u = x and v = y.

(ii) (u, v)> (x, y) if and only if either u > x or u = x and v > y.

Definition 1.4: [9]

Let G:(μ_1 , γ_1 ; μ_2 , γ_2)be an IFG on G^* = (V, E). A sequence S = $((d_{11}, d_{12}), (d_{21}, d_{22}), \cdots, (d_{n1}, d_{n2}))$ of ordered pairs of real number is said to be an intuitionistic fuzzy graphic sequence of IFG if there exists an IFG G whose vertices have degree (d_{i1}, d_{i2}) and G is called realization of S.

Definition 1.5: [9]

Let G:$(\mu_1, \gamma_1; \ \mu_2, \gamma_2)$be an IFG on G^*= (V, E). Then a degree sequence of ordered pair of real numbers in which no two of its elements are equal is called Perfect degree sequence of IFG.

Definition1.6: [9]

Let G:$(\mu_1, \gamma_1; \ \mu_2, \gamma_2)$be an IFG on G^*= (V, E). A degree sequence of ordered pair of real numbers in which exactly two of its elements are same is called Quasi-Perfect degree sequence of IFG.

Definition1.7: [13]

An intuitionistic fuzzy graph G(IFG) on G* = (V, E) is said to be (k_1,k_2)-Regular if $d_G(v_i)$ = (k_1,k_2) for all $v_i \in V$ and also G is said to be a regular intuitionistic fuzzy graph of degree (k_1,k_2).

Definition1.8: [4]

LetGbe an intuitionistic fuzzy graphon G*= (V, E).If each edge in G has the same degree(k_1,k_2),then G is said to be an edge regularintuitionistic fuzzy graph.

Definition 1.9: [9]

Let G:$(\mu_1, \gamma_1; \mu_2, \gamma_2)$ and G′: $(\mu_1', \gamma_1'; \ \mu_2', \gamma_2')$be two IFGs on G^*= (V, E) and G'^*= (V′, E′) respectively. A homomorphism of IFG h: G → G′ is a map h: V →V′ such that $\mu_1(x) \leq \mu_1'(h(x))$ ∀x ∈ V, $\gamma_1(x) \leq \gamma_1'(h(x))$ ∀x ∈ V

and $\mu_2(xy) \leq \mu_2'(h(x)h(y))$ ∀x, y ∈ V, $\gamma_2(xy) \leq \gamma_2'(h(x)h(y))$, ∀x, y ∈ V.

Defintion 1.10: [9]

Let G:$(\mu_1, \gamma_1; \ \mu_2, \gamma_2)$ and G′:$(\mu_1', \gamma_1'; \ \mu_2', \gamma_2')$be two IFGs on G^*= (V, E) and G'^*= (V′, E′) respectively. A weak isomorphism of IFGs h:G→G′ is map h:V→V′which is a bijective homomorphism that satisfies

$\mu_1(x) = \mu_1'(h(x))$, ∀x ∈ V, $\gamma_1(x) = \gamma_1'(h(x))$, ∀x ∈ V

and $\mu_2(xy) \leq \mu_2'(h(x)h(y))$∀x, y ∈ S, $\gamma_2(xy) \leq \gamma_2'(h(x)h(y))$∀x, y ∈ S.

Definiton 1.11: [9]

Let G: $(\mu_1, \gamma_1; \mu_2, \gamma_2)$ and G′:$(\mu_1', \gamma_1'; \mu_2', \gamma_2')$be two IFGs on G^*= (V, E) and G'^*= (V′, E′) respectively. A co-weak isomorphism of IFGs h:G→G′ is map h:V→V′which is a bijective homomorphism that satisfies

$\mu_1(x) \leq \mu_1'(h(x))\ \forall x \in V,\ \gamma_1(x) \leq \gamma_1'(h(x))\ \forall x \in V$

and $\mu_2(xy) = \mu_2'(h(x)h(y))\ \forall x, y \in S,\ \gamma_2(xy) = \gamma_2'(h(x)h(y)\ \forall x, y \in S$.

Definition 1.12: [9]

Let G:$(\mu_1, \gamma_1; \mu_2, \gamma_2)$ and G′:$(\mu_1', \gamma_1'; \mu_2', \gamma_2')$be two IFGs on G^*= (V, E) and G'^*= (V′, E′) respectively. An isomorphism of IFGs h : G→G′ is map h : V→V′whish is a bijective homomorphism that satisfies

$\mu_1(x) = \mu_1'(h(x))\ \forall x \in V,\ \gamma_1(x) = \gamma_1'(h(x)) \forall x \in V$

and $\mu_2(xy) = \mu_2'(h(x)h(y))\ \forall x, y \in V$,

$\gamma_2(xy) = \gamma_2'(h(x)h(y),\ \forall x, y \in V$.

Definition 1.13[12]: Let G: $(\mu_1, \gamma_1; \mu_2, \gamma_2)$be an IFG on G^*= (V, E). A sequence of ordered pairs of positive real numbers $(d_1, d_2, d_3, \cdots, d_n)$ with$d_1 \geq d_2 \geq d_3 \geq, \cdots, \geq d_n$,whered_i = $d(e_i) = (d\mu_2(e_i), d\gamma_2(e_i))$, is the edge degree sequence of the IFG G.

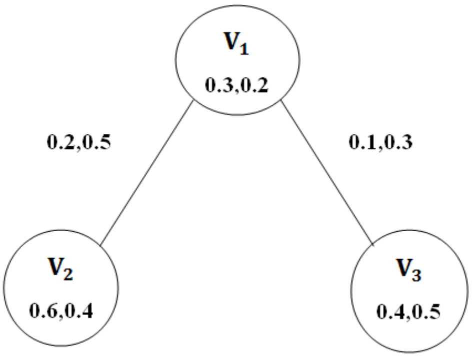


Figure 1

Here $d(v_1v_2)$ = (0.1, 0.3) and $d(v_1v_3)$ = (0.2, 0.5). Hence the edge degree sequence of G is ((0.2, 0.5), (0.1, 0.3)).

2. EDGE DEGREE SET OF INTUITIONISTIC FUZZY GRAPHS

Definition 2.1: The set of distinct ordered pair of positive real numbers occurring in an edge degree sequence of an IFG is called its edge degree set.

Definition 2.2: The set of distinct ordered pair of positive real numbers is called an edge degree set if it is the edge degree set of some IFG.The IFGs is said to realize the edge degree set.

Example 2.3:

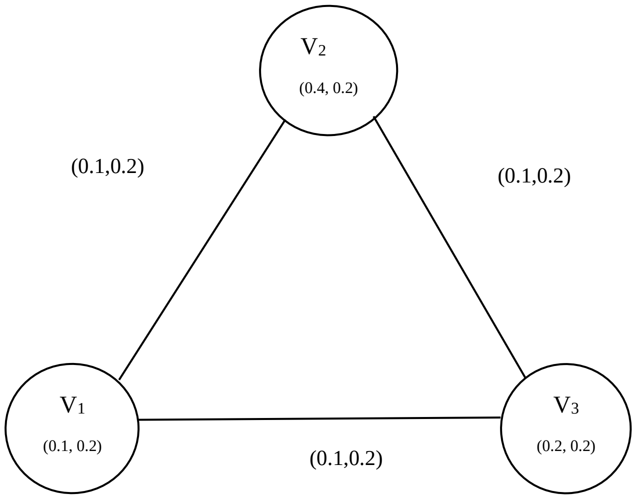


Figure 2

The edge degree sequence of the intuitionistic fuzzy graph in figure 2 is {(0.2, 0.4), (0.2, 0.4),(0.2, 0.4)}. Therefore its edge degree set is {(0.2, 0.4)}. The edge degree set of the intuitionistic fuzzy graphin figure 1 is {(0.2, 0.5), (0.1, 0.3)}.

Theorem 2.4: G is an edge regular intuitionistic fuzzy graph if and only if its edge degree set is a singleton set.

Proof: G is an (k_1, k_2)-edge regular intuitionistic fuzzy graph if and only if each element of the edge degree sequence is (k_1, k_2) if and only if the edge degree set of G is {(k_1, k_2)} which is a singleton set.

Theorem 2.5: The number of elements in the edge degree set is the number of edges of G if and only if the edge degree of the edges of G are all distinct.

Proof: The number of elements in the edge degree set is the number of edges of G if and only if the elements of the edge degree sequence are all distinct which happens if and only if the edge degree of the edges are all distinct.

Theorem 2.6: Let G:(μ_1, γ_1;μ_2, γ_2) be an intuitionistic fuzzy graph on an m-edge regular graph G*. If $\mu_2$and $\gamma_2$are constant functions, then the edge degree set is a singleton set.

Proof: Let G*be an m-edge regular graph.

Let $\mu_2$and $\gamma_2$be constant functions of constant values $c_1$and c_2 respectively.

Then the edge degree of each edge is (mc_1, mc_2).

Therefore the edge degree sequence is ((mc_1, mc_2), (mc_1, mc_2), ..., (mc_1, mc_2)).

Hence the edge degree set is {(mc_1, mc_2)}.

Remark2.7: The converse of Theorem 2.6 need not be true. That is, if the edge degree set is a singleton set, then $\mu_2$and γ_2 need not be constant functions. For example, consider the fuzzy graph G_1 in Figure 3 with $\mu_2(v_1v_2)$ = 0.3 = $\mu_2(v_2v_3)$ and $\mu_2(v_3v_4)$ = 0.4= $\mu_2(v_4v_1)$; $\gamma_2(v_1v_2)$ = 0.5 = $\gamma_2(v_2v_3)$ and $\gamma_2(v_3v_4)$ = 0.3 = $\gamma_2(v_4v_1)$. Then the degree of each edge is (0.7, 0.8). Hence the edge degree set is a singleton set {(0.7, 0.8)} but $\mu_2$and γ_2 are not constant functions.

2. ISOMORPHIC PROPERTIES OF EDGE DEGREE SET OF IFG

Theorem 3.1: If G and G′are isomorphic IFGs, then the edge degree set of G and G′ are identical.

Proof: Let G:(μ_1,γ_1;μ_2,γ_2) and G′: (μ_1',γ_1'; μ_2',γ_2')be two isomorphic IFGs on G^*= (V,E) and G'^*= (V′,E′) respectively.Then there exists an isomorphism h:G→G′is map h : V→V′ whish is a bijective homomorphism that satisfies

$\mu_1(x) = \mu_1'(h(x))\ \forall x \in V$, $\gamma_1(x) = \gamma_1'(h(x)) \forall x \in V$

and $\mu_2(xy) = \mu_2'(h(x)h(y))\ \forall x, y \in V$,

$\gamma_2(xy) = \gamma_2'(h(x)h(y), \forall x, y \in V$.

Hence $d_G^{\mu}(u) = \sum_{u \neq v} \mu_2(uv) = \sum_{h(u) \neq h(v)} \mu_2'((h(u)h(v)) = d_{G'}^{\mu}(h(u))$

and$d_G^{\gamma}(u) = \sum_{u \neq v} \gamma_2(uv) = \sum_{h(u) \neq h(v)} \gamma_2'((h(u)h(v)) = d_{G'}^{\gamma}(h(u))$

Since u ∈ V is arbitrary, $d_G(u) = d_{G'}(h(u))$, ∀u ∈ V.

Therefore

$d_G^{\mu}(uv) = d_G^{\mu}(u) + d_G^{\mu}(v) - 2\ \mu_2(uv)$

$\quad = d_{G'}^{\mu}(h(u)) + d_{G'}^{\mu}(h(v)) - 2\ \mu_2'(h(u)h(v))$

$\quad = d_{G'}^{\mu}(h(u)h(v)) \forall uv \in E$.

Similarly

$d_G^{\gamma}(uv) = d_G^{\gamma}(u) + d_G^{\gamma}(v) - 2\ \gamma_2(uv)$

$\quad = d_{G'}^{\gamma}(h(u)) + d_{G'}^{\gamma}(h(v)) - 2\ \gamma_2'(h(u)h(v))$

$\quad = d_{G'}^{\gamma}(h(u)h(v)), \quad \forall uv \in E$.

Hence $d_G(uv) = \left(d_G^{\mu}(uv),\ d_G^{\gamma}(uv)\right)$

$\quad = \left(d_{G'}^{\mu}(h(u)h(v)),\ d_{G'}^{\gamma}(h(u)h(v))\right)$

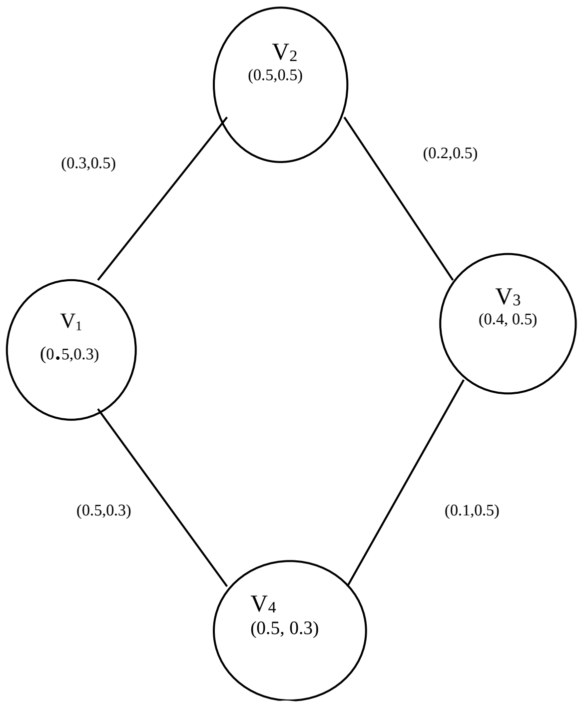
$= d_{G'}(uv)$, ∀ uv ∈ E

Thus the edge degree sequences of G and G′ are same and therefore their corresponding edge degree sets are identical.

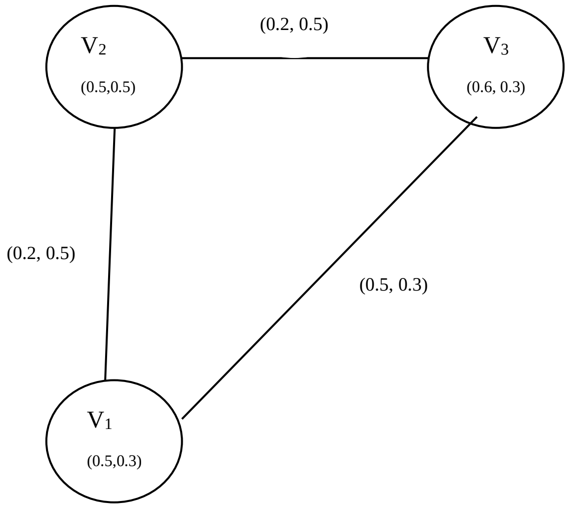
Remark 3.2:

The converse part of theorem 3.1 need not be true. That is, two IFGs with same edge degree set need not be isomorphic. It can be verified by the following example.

The edge degree sequence of intuitionistic fuzzy graph G_1 is {(0.7,0.8), (0.7, 0.8), (0.4,1.0), (0.4,1.0)}.



G_1



G_2

Figure 3

The edge degree sequence of intuitionistic fuzzy graph G_2 is {(0.7,0.8), (0.7, 0.8), (0.4,1.0), (0.4,1.0)}.Therefore the egde degree set of both G_1 and G_2 is {(0.7,0.8),(0.4,1.0)}. But G_1 and G_2can not be isomorphic.

Theorem 3.3: Co-weak isomorphism of IFGs preserve edge degree set.

Proof: Let G:(μ_1, γ_1; μ_2, γ_2) and G′: (μ_1', γ_1'; μ_2', γ_2')be two isomorphic IFGs on G^*= (V, E) and G'^*= (V′, E′) respectively. Then there exists a bijectivemap h:V→V′which satisfies

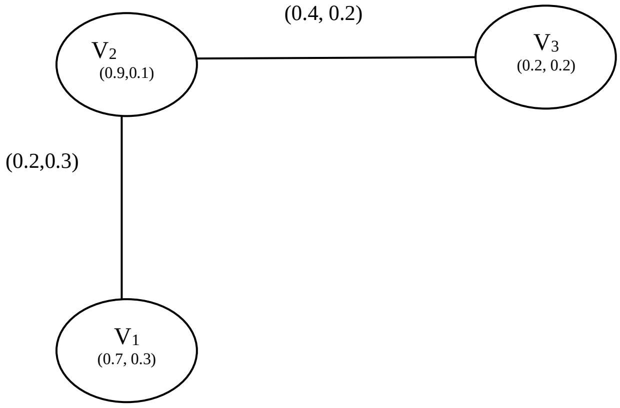
$\mu_1(x) \leq \mu_1'(h(x))\ \forall x \in V$, $\gamma_1(x) \leq \gamma_1'(h(x))\ \forall x \in V$

and $\mu_2(xy) = \mu_2'(h(x)h(y))\ \forall x, y \in V$, $\gamma_2(xy) = \gamma_2'(h(x)\, h(y)\ \forall x, y \in V$.

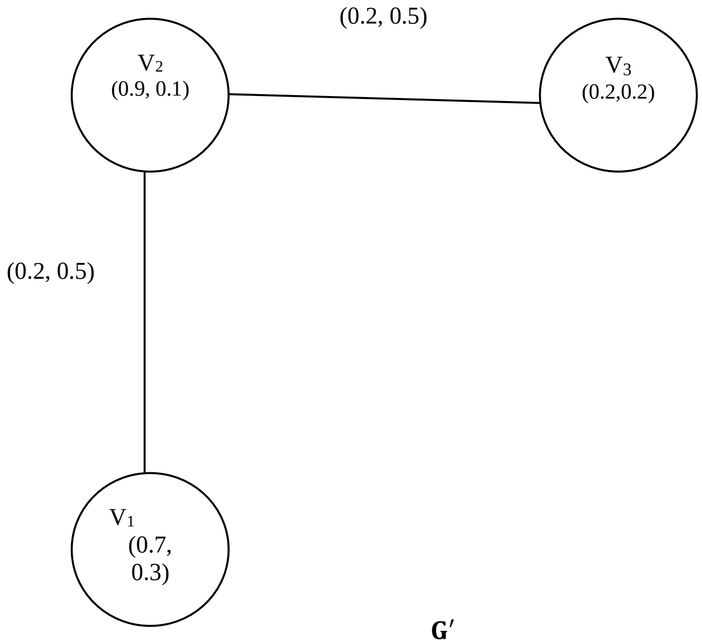
Then proceeding as in the proof of theorem 3.1, $d_G(u) = d_{G'}(h(u))$, $\forall\ u \in V$. Hence G and G′ have identical edge degree sequences so their edge degree sets are same.

Remark 3.4: Two IFGs with same edge degree set need not be co-weak isomorphic. For example consider figure 3, the edge degree set of both G and G′is {(0.7,0.8),(0.4,1.0)}. But G and G′ cannot be co-weak isomorphic.

Remark 3.5:G and G′ are weak isomorphic IFGs, then they need not have the edge degree set.For example the IFGs in figure 4 are weak isomorphic to each other but their edge degree sets are not same. The edge degree set of G is {(0.4,0.2),(0.2,0.3)} and the edge degree set of G′ is {(0.2,0.5)} which are not identical.

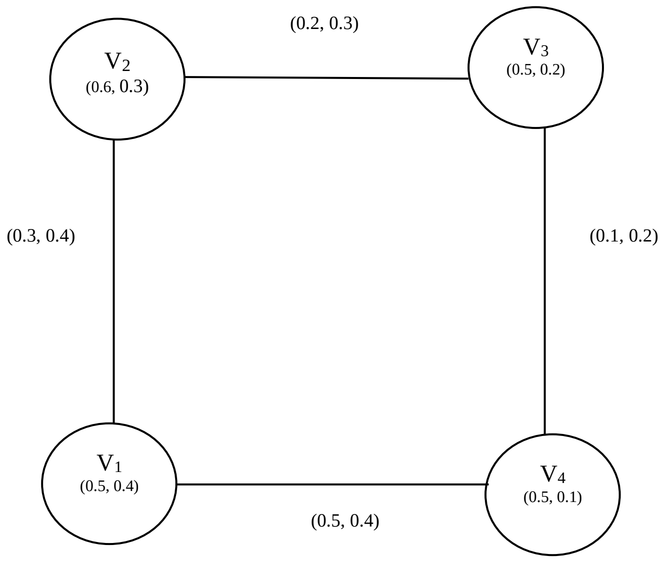


G

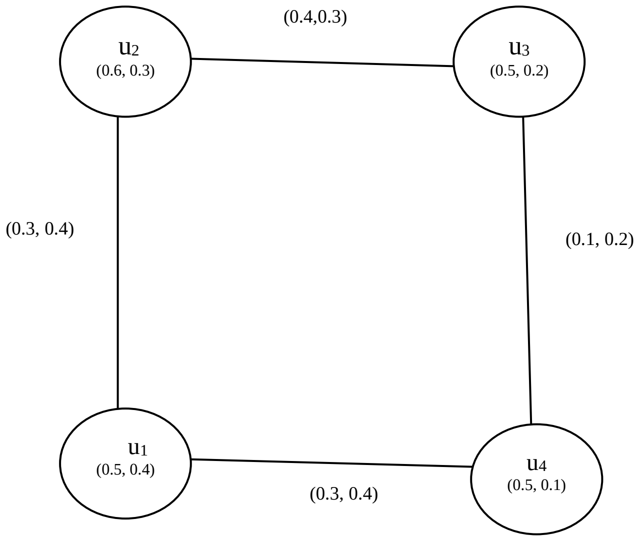


G′
Figure 4

Also two intuitionistic fuzzy graphs with same edge degree set need not be weak isomorphic. The intuitionistic fuzzy graphs G and G' in following Fig 5 are of same edge degree set but they are not weak isomorphic.



G



G′

Figure 5

The edge degree set of both G and G′is {(0.7,0.7),(0.4,0.6)}. But G and G′ cannot be weak isomorphic.

3. CONCLUSION

In this paper, edge degree set of an IFG is introduced and some of its properties are studied. Edge degree set in isomorphic, weak isomorphic and co-weak isomorphic intuitionistic fuzzy graphs are discussed.

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