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The Sensitivity Analysis Of Service And Waiting Costs Of A Multi-Server Fuzzy Queuing Model Using Ranking Technique

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Abstract: Queuing principle is associated with waiting lines, which are most often used at service providers. The application of a multi server fuzzy queuing model will generate certain results in terms of the possible number of consumers, the possible waiting period in the system and in the queue, as well as service and waiting costs. By applying Robust ranking algorithm and it is used to define a membership function of the performance measures in multi-server fuzzy queuing model. This research article explores the sensitivity analysis between expected waiting cost of consumer and expected service cost of server, as well as the overall expected cost of the multiserver fuzzy queuing model.

Keywords: FM/FM/S fuzzy queuing model, Trapezoidal fuzzy number, waiting lines, number of servers, expected waiting cost, expected service cost, fuzzy ranking, sensitivity analysis.

1. INTRODUCTION

In classical queuing theory, the inter arrival time and service time would pursue probability distributions. However, in many real-world implementations, observational data can be obtained subjectively; (i.e.,) the arrival and service type are better represented by linguistic terms such as quick, slow (or) moderate rather than by probability distribution.

Fuzzy queuing model was first introduced by R.J. Lie and E.S Lee in 1989, further developed this model by many authors J.J. Buckley [2] in 1990, R.S. Negi and E.S. Lee in 1992, S.P. Chen[3] in 2005, and R. Srinivasan in 2014. Here the parameters fuzzy arrival rate and fuzzy service rate are best described by linguistic terms very high, high, low, very low and moderate.

Our aim of this research article discussed about multi-server fuzzy queuing model and first come first served discipline using trapezoidal fuzzy numbers under α-cut representation. The basic concept is to use the Robust ranking technique to transform the fuzzy inter arrival rate and service rate into crisp values. Then, in classical queuing theory performance measure formulas obtain in the crisp values. This ranking technique is very convenient, easy to use and can be used for a variety of purposes.

If the waiting lines are long, a productive atmosphere among customers is not created; additionally long lines result in a loss for both the customer and the service provider. Consumers may experience high levels of disappointment if a service provider has low expectations and spends little money, and potential market success may suffer as a result.

If the service provider, on the other hand, provides duplicate services of consumers, there is a risk that the consumer will be dissatisfied with the cost of the service. The waiting cost and the service cost are two fundamental costs that are critical to the smooth operation of a system.

2. PRELIMINARIES

Fuzzy set was first introduced by Zadeh [25] in 1965. It is a mathematical way of representing impreciseness or vagueness in real life.

Definition: (Fuzzy Set)

A fuzzy set A in X is characterized by its membership function: A: $X \to [0,1]$. Here X is a non-empty set. (i.e.,) $A=\{(x,\mu_A(x)):x\varepsilon X\}$, here $\mu_A : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set. A and $\mu_A(X)$ is called the membership value of $x \in X$ in the fuzzy set A.

Definition

A fuzzy set A of the universe of discourse X is called a **normal fuzzy set** if there exists atleast one $x \in X$ such that: $\mu_A(x)=1$.

Definition

The fuzzy set A is **convex** if and only if for any x_1, $x_2 \in X$, the membership function of A satisfies the condition:

$$\mu_A\{\lambda x_1+(1-\lambda)x_2\} \ge \min\{\mu_A(x_1),\mu_A(x_2)\}, 0 \le \lambda \le 1.$$

Definition: (α - cut of a fuzzy number)

The α-cut of a fuzzy number $A(X)$ is defined as: $A(\alpha)=\{x:\mu(x)\ge\alpha,\alpha\,\varepsilon[0,1]\}$

Addition of two Trapezoidal fuzzy number can be performed as:

$$(a_1,b_1,c_1,d_1)+(a_2,b_2,c_2,d_2)=(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2)$$

Definition : (Trapezoidal fuzzy number)

A Trapezoidal fuzzy number $\tilde{P}$ is defined by (p_1,p_2,p_3,p_4) where $P_i \in R$ and $p_1 \le p_2 \le p_3 \le p_4$. Its membership function is

$$\mu_{\tilde{P}}(x)=\begin{cases} \dfrac{x-p_1}{p_2-p_1} & for \quad p_1 \le x \le p_2 \\ 1 & for \quad p_2 \le x \le p_3 \\ \dfrac{p_4-x}{p_4-p_3} & for \quad p_3 \le x \le p_4 \\ 0 & otherwise \end{cases}$$

Crisp model

The performance measures of (M/M/S/∞/FCFS) model:

The probable number of consumers in waiting line:

$$N_q=\left[\frac{1}{(S-1)!}\left(\frac{\lambda}{\mu}\right)^s\left(\frac{\lambda\mu}{(S\mu-\lambda)^2}\right)\right]P_0$$

Where $P_0=\left[\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n+\frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s\left(\frac{s\mu}{s\mu-\lambda}\right)\right]^{-1}$

The probable number of consumers in the system $N_S = N_q + \frac{\lambda}{\mu}$

The probable waiting time of a consumer in the waiting line $T_q = \frac{N_q}{\lambda}$

The probable waiting time of a customer in the system $T_s = T_q + \frac{1}{\mu}$

Mathematical expectation of service cost of multi-server model $E(S_C) = C_S * S$
Expected waiting cost in the system $E(W_C) = C_W * N_S$
The total expected cost $E(T_C) = C_W * N_S + C_S * S$

To make the sensitivity analysis between service and waiting costs in a multiserver model one can consider the average rate of arrival $(\lambda = 14)$, the average rate of service $(\mu = 8)$. The fundamental principle for the existence of system is $\frac{\lambda}{S\mu} < 1\,(or) S > \frac{\lambda}{\mu}(i.e) S > 2$.

Robust Ranking Technique – Algorithm

To find the performance measures in terms of crisp values we defuzzify the fuzzy numbers into crisp ones by a fuzzy number ranking method. Robust ranking technique which satisfies compensation, linearity, and additive properties and provides results which are consistent with human intuition. Give a convex fuzzy number $\tilde{a}$, the Robust ranking index is defined by,

$$R(\tilde{a}) = \int_0^1 0.5\left(a_\alpha^L + a_\alpha^U\right) d\alpha$$

where $\left(a_\alpha^L, a_\alpha^U\right)$ is the α-level cut of the fuzzy number $\tilde{a}$. In this paper we use this method for ranking the fuzzy numbers. The Robust ranking index $R(\tilde{a})$ gives the representative value of the fuzzy number $\tilde{a}$. It satisfies the linearity and additive property.

Symbols and Notations

The fundamental indices of queuing systems

$\tilde{\lambda}_A =$ Average rate of arrival

$\mu_S =$ Average rate of service

$\tilde{L}_{N_q} =$ Expected number of consumers in the queue

$\tilde{L}_{N_S} =$ Expected number of consumers in the system

$\tilde{W}_{T_q} =$ Possible waiting period of a consumer in the waiting line.

$\tilde{W}_{T_s} =$ Possible waiting time of a consumer in the system.

Fuzzy model:

The performance measures of $(FM / FM / S / \infty / FCFS)$ *Model:*

The Expected number of consumers in the queue.

$$\tilde{L}_{N_q} = \left[\frac{1}{(S-1)!} \left(\frac{\tilde{\lambda} A}{\tilde{\mu}_S} \right)^S \left(\frac{\tilde{\lambda}_A \tilde{\mu}_S}{\left(S\tilde{\mu}_S - \tilde{\lambda}_A \right)^2} \right) \right] P_0$$

Where $P_0 = \left[\sum_{n=0}^{S-1} \frac{1}{n!} \left(\frac{\tilde{\lambda}_A}{\tilde{\mu}_S} \right)^n + \frac{1}{S!} \left(\frac{\tilde{\lambda}_A}{\tilde{\mu}_S} \right)^S \left(\frac{S\tilde{\mu}_S}{S\tilde{\mu}_S - \tilde{\lambda}_A} \right) \right]^{-1}$

Expected number of customers in the system

$$\tilde{L}_{N_s} = \tilde{L}_{N_q} + \frac{\tilde{\lambda}_A}{\tilde{\mu}_S}$$

The possible waiting period of a consumer in the waiting line

$$\tilde{W}_{T_q} = \frac{\tilde{L}_{N_q}}{\tilde{\lambda}_A}$$

The possible waiting time of a consumer in the system:

$$\tilde{W}_{T_s} = \tilde{W}_{T_q} + \frac{1}{\tilde{\mu}_S}$$

Mathematical expectation of service cost of multi-server model $E(S_C) = C_S * S$

Expected waiting cost in the system $E(W_C) = C_W * \tilde{L}_{N_s}$

The total expected cost $E(T_C) = C_W * \tilde{L}_{N_s} + C_S * S.$

S.No	**Description**	**Symbol**
1.	Number of server	S
2.	Each server's service cost	C_S
3.	Each consumer waiting cost	C_W

Numerical Example: (For Trapezoidal fuzzy number)

Here the arrival rate and service rate are Trapezoidal fuzzy numbers represented by $\tilde{\lambda} = (11,13,15,17)$ and $\tilde{\mu} = (5,7,9,11)$ per minute whose intervals of confidence are (11+2α, 17-2α) and (5+2α, 11-2α) respectively. Now we evaluate R(11,13,15,17) by applying Robust ranking method. The membership function of the Trapezoidal fuzzy number (11,13,15,17) is:

$$\lambda(X) = \begin{cases} \frac{x-11}{13-11}, & 11 \le x \le 13 \\ 1, & 13 \le x \le 15 \\ \frac{17-x}{17-15}, & 15 \le x \le 17 \\ 0, & otherwise \end{cases}$$

The α-cut of the fuzzy number (11,13,15,17) is $\left(a_\alpha^L, a_\alpha^U\right) = (11+2\alpha, 17-2\alpha)$ for which

$$R(\tilde{\lambda}) = \int_0^1 (0.5)\left(a_\alpha^L + a_\alpha^U\right) d_\alpha$$

$$= \int_0^1 (0.5)(11+2\alpha+17-2\alpha)\, d\alpha$$

$= \int_0^1 0.5(28)\,d\alpha = (0.5)(28)\int_0^1 d\alpha$

= 14

Therefore $R(\tilde{\lambda}) = 14$, proceeding similarly, the Robust ranking indices for fuzzy number $\tilde{\mu}$ are calculated as follows:

$R(\tilde{\mu}) = \int_0^1 0.5(5+2\alpha+11-2\alpha)\,d\alpha$

$R(5,7,9,11) = \int_0^1 (0.5)(16)\,d\alpha$

= 8

$R(\tilde{\mu}) = 8$.

3. RESULTS AND DISCUSSION

To make the sensitivity analysis between service and waiting costs in a multi-server fuzzy queuing model. Let us consider trapezoidal fuzzy numbers $\tilde{\lambda}_A = (11,13,15,17)$ as a fuzzy arrival rate and $\tilde{\mu}_S = (5,7,9,11)$ as a fuzzy service rate. By applying Robust ranking method for $\tilde{\lambda}_A$ and $\tilde{\mu}_S$ are calculated as follows:

$R(\tilde{\gamma}_A) = 14$, $R(\tilde{\mu}_S) = 8$ and number of servers S = 3.

The fundamental principle for the existence of system is $\frac{\tilde{\lambda}_A}{S\tilde{\mu}_S} < 1 (or) S > \frac{\tilde{\lambda}_A}{\tilde{\mu}_S} (i.e.,) S > 2$

Table: 1 Performance measure $\tilde{L}N_S$ of multi server fuzzy queuing model.

S.No	S	$\tilde{L}_{N_S}$
1	3	2.2171
2	4	1.8421
3	5	1.7696
4	6	1.7540
5	7	1.7508

Sensitivity analysis of service and waiting costs in multi-server fuzzy queuing model:
Case – I

In this case, each consumer's waiting cost has been assigned a fixed value, while each server's service cost has been assigned a range of values in ascending order. In each case of a multiserver fuzzy queuing model, the estimated consumer waiting cost and the expected system service cost are calculated. For the graphical representation, here we consider $\tilde{L}N_S$ as **N**S.

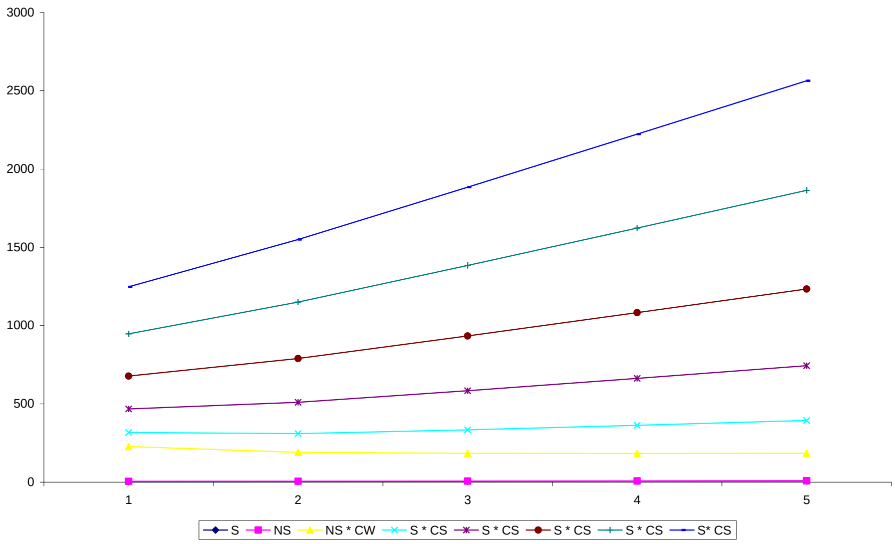
Case I:

Table 2: Fixed waiting cost of each consumer and variant service cost of each server**.**

S	$\tilde{L}_{N_S}$	$\tilde{L}_{N_S}$ * Cw	S * CS	S * Cs	S * Cs	S * Cs	S* Cs

3	2.2171	222	90	150	210	270	300
4	1.8421	184	120	200	280	360	400
5	1.7696	177	150	250	350	450	500
6	1.7540	175	180	300	420	540	600
7	1.7508	175	210	350	490	630	700

Fixed Cw Vs variant Cs



Graph: 1 Fixed waiting cost of each consumer and variant service cost of each server

The graph shows the number of servers and estimated cost, and it is obvious from the graph that as the service cost of each server decreases, the system's expected service cost decreases as well. When each server's service cost rises, So does the estimated service cost. When the average service cost is lower, the expected consumer waiting time is lower as well, but more servers are needed. As a result, in order to simply the system, the service provider would need to have a larger number of servers.

When the actual operating cost of the system rises, so does the expected waiting cost, indicating that if the service provider delivers high-quality service (expensive), customers are inclined to settle even if the expected waiting cost is high. This tendency suggests that customers have a strong desire to stay in the scheme, as shown by the following two examples:

i) If the service provider's service is of high quality.
ii) If the service provider has a larger number of servers in the system.

Case II

In this example, each server's service cost is a nominal cost, and each consumer's waiting cost is a variable amount that increases in order. In each case of the multi-server fuzzy queuing model, the average consumer waiting cost and the expected system service cost are calculated.

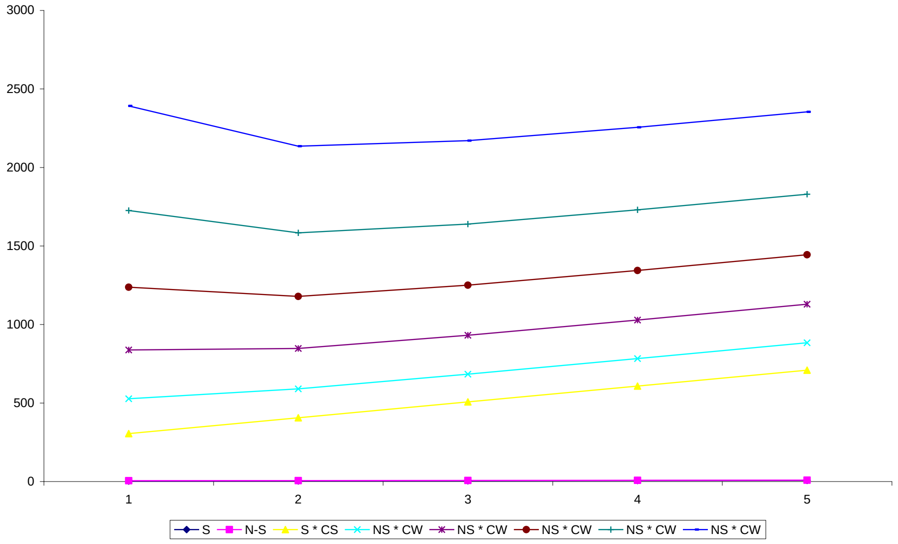
Table 3: Fixed service cost of each server and variant waiting cost of each consumer

S	N_S	$S * C_S$	$N_S * C_W$	$N_S * C_W$	$N_S * C_W$	$N_S * CW$	$N_S * CW$

3	2.2171	300	222	310.80	399.6	488.4	666
4	1.8421	400	184	257.6	331.2	404.8	552
5	1.7696	500	177	247.8	318.6	389.4	531
6	1.7540	600	175	245.6	315.7	385.9	526

Fixed Cs Vs variant Cw

7	1.7508	700	175	245.1	315.2	385.2	525



Graph: 2 Fixed service cost of each server and variant waiting cost of consumer

The graph shows the number of servers and expected cost, and it is obvious from the graph that since the waiting cost of each consumer is low, the system's expected waiting cost is low as well. If each consumer's waiting cost rises, the estimated waiting cost rises with it. When the average waiting time is minimal, the system's expected service cost is low as well, but the number of servers is small. As a result, the service provider must have fewer servers in order to optimize the system when the system's estimated waiting cost rises, So does the demand cost, indicating that when a service provider has a high quality (expensive) service, customers are able to wait even though the expected waiting time is long.

This tendency indicates that the service provider is interested in providing the service, as shown by the following two examples:

i) If the customer pays a high waiting cost to the service provider, the service provider would deliver good quality service.

ii) The service provider will increase the number of services available. If customers are willing to stick with the system.

4. CONCLUSION

The minimum number of needed servers is determined using a multi-server fuzzy queuing model in which waiting costs and service costs are measured and subjected to sensitivity analysis. Consumers are more likely to stay with the system if the service provider, provides a larger number of servers. If the customer pays a high waiting rate to the service provider, the service provider can have quality service. Furthermore, it has been identified in the preceding discussions that the service provider can offer a greater number of services if the customers are able to stay in the scheme. In this research article, we discussed about the performance measures of multi-server fuzzy queuing model in trapezoidal fuzzy numbers using ranking technique.

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