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Gη-Closed, Continuity And Contra Continuity In Topological Ordered Spaces

K. Sumathi[1],T. Arunachalam[2],D. Subbulakshmi[3],K. Indirani[4]

[1]Associate Professor, Department of Mathematics, PSGR Krishnammal College for Women, Coimbatore, Tamilnadu, India
[2] Professor, Department of Mathematics, Kumaraguru College of Technology, Coimbatore, Tamilnadu, India
[3] Assistant Professor, Department of Mathematics, RathnavelSubramaniam College of Arts and Science, Coimbatore, Tamilnadu, India
[4]Associate Professor, Department of Mathematics, Nirmala College for Women, Coimbatore, Tamilnadu, India

Email : [1]ksumathi@psgrkcw.ac.in, [2]tarun_chalam@yahoo.com.
[3] subbulakshmi169@gmail.com, [4] indirani009@ymail.com.

Abstract: The aim of this paper is to introduce a new class of closed sets in topological ordered spaces called xgη-closed sets and obtain some of its characteristics. The concept of continuous and contra continuous functions are called xgη-continuous,xcontragη-continuous is defined and obtained some of its properties.

Keywords: xgη-closed set,xgη-continuous, xcontragη-continuousfunctions.

1. INTRODUCTION

In 1965, Nachbin [16] initiated the study of topological ordered spaces.In 2001, Veerakumar [27] introduced the study of i-closed, d-closed and b-closed sets.A new class of gη-closed sets, gη-continuity functions and contra gη-continuity functions has been introduced Subbulakshmi et al [22, 23, 24]. In 2017, Amarendrababu [1] introduced g*-closed sets in topological ordered spaces. In 2019, Dhanapakyam [7] introduced βg*-closed sets in topological ordered spaces. In this paper a new class of xgη-closed set, xgη-continuous, xcontragη-continuous in topological ordered spaces are defined and some of their properties are analyzed. [Throughout this paper x = i, d, b].

2. PRELIMINARIES

Definition : 2.1 A subset A of a topological space (X, τ) is called
(i) α-open set [2] if A ⊆int(cl(int(A))), α-closed set if cl (int (cl(A))) ⊆ A.
(ii) semi-open set [12] if A ⊆cl(int (A)), semi-closed set if int (cl(A) ⊆ A.
(iii) regular-open set [17] if A = int (cl(A)), regular-closed set if A = cl (int (A))).
(iv) η-open set [21] if A ⊆int (cl(int(A))) ∪ cl (int (A)), η-closed set if cl (int (cl (A))) ∩ int(cl(A)) ⊆ A.

Definition : 2.2 A subset A of a space (X, τ) is called
(i) g-closed set [13] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
(ii) g*-closed set [26] if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).

(iii) sg-closed set [4] if scl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).
(iv) gη-closed set [22] if ηcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

Definition : 2.3 A function f : (X , τ) → (Y, σ) is called
(i) continuous [3] if f^{-1} (V) is a closed in (X, τ) for every closed set V of (Y, σ).
(ii) semi-continuous [12] if f^{-1} (V) is a semi-closed in (X, τ) for every closed set V of (Y, σ).
(iii)α-continuous [6] if f^{-1} (V) is a α-closed in (X, τ) for every closed set V of (Y, σ).
(iv)r-continuous [14] if f^{-1} (V) is a r-closed in (X, τ) for every closed set V of (Y, σ).
(v) sg-continuous [25] if f^{-1} (V) is a sg-closed in (X, τ) for every closed set V of (Y, σ).
(vi)η-continuous [23] if f^{-1} (V) is a η-closed in (X, τ) for every closed set V of (Y, σ).
(vii) gη-continuous [23] if f^{-1} (V) is a gη-closed in (X, τ) for every closed set V of (Y, σ).

Definition : 2.4 A function f :(X, τ) ⟶ (Y, σ) is called
(i) contra continuous [8] if f^{-1} (V) is a closed in (X, τ) for every open set V of (Y, σ).
(ii) contra semi-continuous [9] if f^{-1} (V) is a semi-closed in (X, τ) for every open set V of (Y, σ).
(iii) contra α-continuous [10] if f^{-1} (V) is an α-closed in (X, τ) for every open set V of (Y, σ).
(iv) contra r-continuous [14] if f^{-1} (V) is a r-closed in (X, τ) for every open set V of (Y, σ).
(v) contra g-continuous [5] if f^{-1} (V) is a g-closed in (X, τ) for every open set V of (Y, σ).
(vi) contra g*-continuous [15, 26] if f^{-1} (V) is a g*-closed in (X, τ) for every open set V of (Y,σ).
(vii) contrasg-continuous [19] if f^{-1} (V) is a sg-closed in (X, τ) for every open set V of (Y, σ).
(viii) contraη-continuous [24] if f^{-1} (V) is a η-closed in (X, τ) for every open set V of (Y, σ).
(ix) contragη-continuous [24] if f^{-1} (V) is a gη-closed in (X, τ) for every open set V of (Y, σ).

Definition 2.5: [27] A topological ordered spaces is a triple (X, τ,≤) where 'τ' is a topology on X and '≤' is a partial order on X.
Let A be a subset of topological ordered space (X, τ, ≤).
For any x ∈ X,
(i) $[x, \rightarrow] = \{ y \in X / x \leq y\}$and
(ii) $[\leftarrow, x]$= $\{y \in X/y \leq x\}$.
The subset A is said to be
(i) increasing if A = i(A), where i(A) = $\bigcup_{a \in A}[a, \rightarrow]$ and
(ii) decreasing if A = d (A), where d(A) = $\bigcup_{a \in A}[\leftarrow, a]$
(iii) balanced if it is both increasing and decreasing.
The complement of an increasing set is a decreasing set and the complement of a decreasing set is an increasing set.

Definition: 2.6A subset A of a topological ordered space (X, τ, ≤) is called
(i)x-closed set [7]ifitis bothincreasing (resp. decreasing, increasing and decreasing) set and closed set.
(ii) xα-closed set [11] ifit isbothincreasing(resp. decreasing,increasing and decreasing) set and α-closed set.
(iii) xsemi-closed set [11]ifit is both increasing(resp. decreasing, increasing and decreasing) set and semi-closed set.

(iv) xr-closed set [7] ifit is both increasing(resp. decreasing, increasing and decreasing) set and r-closed set.
(v) xg-closed set [20]if it is both increasing(resp. decreasing, increasing and decreasing) set and g-closed set.
(vi) xg*-closed set [1] if it is both increasing(resp. decreasing, increasing and decreasing) set and g*-closed set.
(vii) xsg-closed set [18] if it is both increasing(resp. decreasing, increasing and decreasing) set and sg-closed set.

Definition :2.7A function f: (X, □□ ≤) → (Y, σ, ≤) is called
(i) x-continuous[7] if $f^{-1}(V)$ is x-closed subset of (X, □□ ≤) for every closed subset of (Y, σ, ≤).
(ii) xα-continuous[11] if $f^{-1}(V)$ is xα-closed subset of (X, □□ ≤) for every closed subset of (Y, σ, ≤).
(ii) xsemi-continuous [11] if $f^{-1}(V)$ is xsemi-closed subset of (X, □□ ≤) for every closed subset of (Y, σ, ≤).
(ii) xr-continuous[7] if $f^{-1}(V)$ isxr-closed subset of (X, □□ ≤) for every closed subset of (Y, σ, ≤).
(ii) xsg-continuous [18] if $f^{-1}(V)$ isxsg-closed subset of (X, □□ ≤) for every closed subset of (Y, σ, ≤).
[Throughout this paper x = i, d, b]

3. On igη-closed set :

Definition : 3.1 A subset A of a topological ordered space (X, τ, ≤) is called an iη-closed set if it is both increasing and η-closed set.
Definition : 3.2A subset A of a topological ordered space (X, τ, ≤) is called an igη-closed set if it is both increasing and gη-closed set.

Theorem : 3.3 Every i-closed, isemi-closed, iα-closed, ir-closed, ig*-closed, iη-closed sets are igη-closed set, but not conversely.
Proof: Every closed, semi-closed, α-closed, r-closed, g*-closed, η-closed sets are gη-closed set [22]. Then every i-closed, isemi-closed, iα-closed, ir-closed, ig*-closed, iη-closed sets areigη-closed set.
EXAMPLE : 3.4 Let X = {a, b, c}, □□= { φ, X, {a}, {b, c}} and ≤= {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, □□□≤) is a topological ordered space. igη-closed sets are {φ, X, {c}, {b, c}}. i-closed, isemi-closed, iα-closed, ir-closed, ig*-closed, iη-closed sets are {φ, X, {b, c}}. Let A = {c}. Clearly A is an igη-closed set but not an i-closed, isemi-closed, iα-closed, ir-closed, ig*-closed, iη-closed set in X.

Theorem : 3.5 Every ig-closed set is an igη-closed set, but not conversely.
Proof: Every g-closed set is a gη-closed set [22]. Then every ig-closed set is an igη-closed set.
EXAMPLE : 3.6 Let X = {a, b, c}, □□= { φ, X, {a}, {b}, {a, b}} and ≤= {(a, a), (b, b), (c, c), (a, b), (a, c)}. Clearly (X, □□□≤) is a topological ordered space. igη-closed sets are {φ, X, {b}, {c}, {b, c}}. ig-closed sets are {φ, X, {c}, {b, c}}. Let A = {b}. Clearly A is an igη-closed set but not an ig-closed set in X.

Theorem :3.7 Every isg-closed set is an igη-closed set, but not conversely.
Proof: Every sg-closed set is a gη-closed set [22]. Then every isg-closed set is an igη-closed set.
EXAMPLE :3.8 Let X = {a, b, c}, □□= { φ, X, {a}} and ≤= {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, □□□≤) is a topological ordered space. igη-closed sets are {φ, X, {b}, {a, b}, {b, c}}. isg-closed sets are {φ, X, {b}, {b, c}}. Let A = {a, b}. Clearly A is an igη-closed set but not an isg-closed set in X.

4. On dgη-closed set :

Definition : 4.1 A subset A of a topological ordered space (X, τ, ≤) is called a dη-closed set if it is both decreasing anddη-closed set.
Definition : 4.2A subset A of a topological ordered space (X, τ, ≤) is called a dgη-closed set if it is both decreasing anddgη-closed set.

Theorem : 4.3 Every d-closed set, dα-closed, dg-closed, dg*-closedsets are dgη-closed set, but not conversely.
Proof: Every closed set, α-closed, g-closed, g*-closed sets are gη-closed set [22]. Then every d-closed, dα-closed, dg-closed, dg*-closed sets are dgη-closed set.
EXAMPLE : 4.4 Let X = {a, b, c}, □□= { φ, X, {a}, {b}, {a, b}} and ≤= {(a, a), (b, b), (c, c), (a, c)}. Clearly (X, □□□≤) is a topological ordered space. dgη-closed sets are {φ, X, {a}, {a, c}}.
d-closed, dα-closed, dg-closed, dg*-closed sets are {φ, X, {a, c}}. Let A = {a}. Clearly A is a dgη-closed set but not a d-closed, dα-closed, dg-closed, dg*-closed set in X.

Theorem : 4.5 Every dsemi-closed, dsg-closed, dη-closed sets are dgη-closed set, but not conversely.
Proof: Every semi-closed, sg-closed, η-closed sets are gη-closed set [22]. Then every dsemi-closed, dsg-closed, dη-closed sets are dgη-closed set.
EXAMPLE : 4.6 Let X = {a, b, c}, □□= { φ, X, {a}} and ≤= {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, □□□≤) is a topological ordered space. dgη-closed sets are {φ, X, {c}, {a, c}}. dsemi-closed, dsg-closed, dη-closed sets are {φ, X, {c}}. Let A = {a, c}. Clearly A is a dgη-closed set but not a dsemi-closed, dsg-closed, dη-closed set in X.

Theorem :4.7 Every dr-closed set is a dgη-closed set, but not conversely.
Proof: Every r-closed set is a gη-closed set [22]. Then every dr-closed set is a dgη-closed set.
EXAMPLE :4.8 Let X = {a, b, c}, □□= { φ, X, {a}, {b, c}} and ≤= {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, □□□≤) is a topological ordered space. dgη-closed sets are { φ, X, {a}, {a, b}}. dr-closed sets are {φ, X, {a}}. Let A = {a, b}. Clearly A is a dgη-closed set but not a dr-closed set in X.

5. On bgη-closed set :

Definition : 5.1 A subset A of a topological ordered space (X, τ, ≤) is called a bη-closed set if it is both increasing and decreasing η-closed set.
Definition : 5.2A subset A of a topological ordered space (X, τ, ≤) is called a bgη-closed set if it is both increasing and decreasing gη-closed set.

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Theorem : 5.3 Every b-closed, bα-closed, bg-closed, bg*-closed sets are bgη-closed set, but not conversely.
Proof: Every closed, α-closed, g-closed, g*-closed sets are gη-closed set [22]. Then every b-closed, bα-closed, bg-closed, bg*-closed sets are bgη-closed set.
EXAMPLE : 5.4 Let X = {a, b, c}, □□= { φ, X, {a}, {b}, {a, b}} and ≤= {(a, a), (b, b), (c, c), (a, c)}. Clearly (X, □□□≤) is a topological ordered space. bgη-closed sets are { φ, X, {b}, {a, c}}.
b-closed, bα-closed, bg-closed, bg*-closed sets are {φ, X, {a, c}}. Let A = {b}. Clearly A is a bgη-closed set but not a b-closed, bα-closed, bg-closed, bg*-closed set in X.

Theorem :5.5 Every br-closed set is a bgη-closed set, but not conversely.
Proof: Every r-closed set is a gη-closed set [22]. Then every br-closed set is a bgη-closed set.
EXAMPLE : 5.6 Let X = {a, b, c}, □□= { φ, X, {a}, {b}, {a, b}} and ≤= {(a, a), (b, b), (c, c), (a, c)}. Clearly (X, □□□≤) is a topological ordered space. bgη-closed sets are { φ, X, {b}, {a, c}}. br-closed sets are {φ, X, {a, c}}. Let A = {b}. Clearly A is a bgη-closed set but not a br-closed set in X.

Theorem :5.7 Every bsemi-closed, bsg-closed, bη-closed sets are bgη-closed set, but not conversely.
Proof: Every semi-closed, sg-closed, η-closed sets are gη-closed set [22]. Then every bsemi-closed, bsg-closed, bη-closed sets are bgη-closed set.
EXAMPLE : 5.8 Let X = {a, b, c}, □□= { φ, X, {a}} and ≤= {(a, a), (b, b), (c, c), (a, c)}. Clearly (X, □□□≤) is a topological ordered space. bgη-closed sets are { φ, X, {b}, {a, c}}. bsemi-closed, bsg-closed, bη-closed sets are {φ, X, {b}}. Let A = {a, c}. Clearly A is a bgη-closed set but not a bsemi-closed, bsg-closed, bη-closed set in X.

6. Onigη-continuity

Definition : 6.1 A function f: (X, □□ ≤) → (Y, σ, ≤) is called iη-continuous if $f^{-1}(V)$ is iη-closed subset of (X, □□ ≤) for every closed subset of (Y, σ, ≤).
Definition : 6.2 A function f: (X, □□ ≤) → (Y, σ, ≤) is called igη-continuous if $f^{-1}(V)$ is igη-closed subset of (X, □□ ≤) for every closed subset of (Y, σ, ≤).

Theorem : 6.3 Every i-continuous functions isigη-continuous, but not conversely.
Proof: The proof follows from the fact that every i-closedsets are igη-closed set.
Example : 6.4 Let X = Y = {a, b, c},τ={X,φ□□{a}} and σ ={Y, φ, {a}, {b}, {a, b}} . ≤ = {(a, a),
(b, b), (c, c), (a, c), (b, c)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = a, f (b) = b, f (c) = c. This map is igη-continuous, but not i-continuous, since for the closed set V={c} in (Y, σ, ≤), $f^{-1}(V)$ ={c} is not i-closedin (X, □□ ≤).

Theorem: 6.5 Everyisemi-continuous, iα-continuous, isg-continuous, iη-continuous functions are igη-continuous, but not conversely.
Proof: The proof follows from the fact that every isemi-closed, iα-closed, isg-closed, iη-closed sets areigη-closed set.

Example : 6.6 Let X = Y = {a, b, c},τ={X,φ□□{a}} and σ ={Y, φ, {a}, {b}, {a, b}} . ≤ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = a, f (b) = c, f (c) = b. This map is igη-continuous, but notisemi-continuous, iα-continuous, isg-continuous, iη-continuous, since for the closed set V = {a, c} in (Y, σ, ≤), $f^{-1}(V)$ = {a, b} is not isemi-closed, iα-closed, isg-closed, iη-closed in (X, □□ ≤).

Theorem: 6.7 Everyir-continuous function is igη-continuous, but not conversely.
Proof: The proof follows from the fact that every ir-closed set is an igη-closed set.
Example: 6.8 Let X = Y = {a, b, c},τ={X,φ□□{a}, {b}, {a, b}} and σ ={Y, φ, {a}, {b, c}} . ≤ = {(a, a), (b, b), (c, c), (a, b), (a, c), (b, c)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = a, f (b) = b, f (c) = c. This map is igη-continuous, but not ir-continuous, since for the closed set V = {c} in (Y, σ, ≤), $f^{-1}(V)$ = {c} is not ir-closed in (X, □□ ≤).

7. Ondgη-continuity

Definition : 7.1 A function f: (X, □□ ≤) → (Y, σ, ≤) is called dη-continuous if $f^{-1}(V)$ is dη-closed subset of (X, □□ ≤) for every closed subset of (Y, σ, ≤).
Definition : 7.2A function f: (X, □□ ≤) → (Y, σ, ≤) is called dgη-continuous if $f^{-1}(V)$ is dgη-closed subset of (X, □□ ≤) for every closed subset of (Y, σ, ≤).

Theorem:7.3 Every d-continuous, dsemi-continuous, dα-continuous, dr-continuous, dη-continuous functions are dgη-continuous, but not conversely.
Proof: The proof follows from the fact that every d-closed, dsemi-closed, dα-closed, dr-closed, dη-closed sets are dgη-closed set.
Example : 7.4 Let X = Y = {a, b, c},τ={X,φ□□{a}, {b, c}} and σ ={Y, φ, {a}} . ≤ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = c, f (b) = b, f (c) = a. This map is d gη-continuous, but not d-continuous, dsemi-continuous, dα-continuous, dr-continuous, dη-continuous, since for the closed set V={b, c} in (Y, σ, ≤), $f^{-1}(V)$ ={a, b} is not d-closed, dsemi-closed, dα-closed, dr-closed, dη-closed set in (X, □□ ≤).

Theorem: 7.5 Everydsg-continuous function is dgη-continuous, but not conversely.
Proof: The proof follows from the fact that every dsg-closed set is a dgη-closed set.
Example:7.6 Let X = Y = {a, b, c},τ={X,φ□□{a}} and σ ={Y, φ, {a}, {b, c}} . ≤ = {(a, a), (b, b), (c, c), (a, c)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = b, f (b) = a, f (c) = c. This map is dgη-continuous, but not dsg-continuous, since for the closed set V = {a} in (Y, σ, ≤), $f^{-1}(V)$ = {a, c} is not dsg-closed in (X, □□ ≤).

8. Onbgη-continuity

Definition: 8.1 A function f: (X, □□ ≤) → (Y, σ, ≤) is called bη-continuous if $f^{-1}(V)$ is bη-closed subset of (X, □□ ≤) for every closed subset of (Y, σ, ≤).
Definition: 8.2A function f: (X, □□ ≤) → (Y, σ, ≤) is called bgη-continuous if $f^{-1}(V)$ is bgη-closed subset of (X, □□ ≤) for every closed subset of (Y, σ, ≤).

Theorem:8.3 Every b-continuous, bsemi-continuous, bα-continuous, br-continuous functions are bgη-continuous, but not conversely.

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Proof: The proof follows from the fact that every b-closed, bsemi-closed, bα-closed, br-closedsets are bgη-closed set.

Example:8.4 Let X = Y = {a, b, c},τ={X,φ□□{a}, {b}, {a, b}} and σ ={Y, φ, {a}, {b, c}} . ≤ = {(a, a), (b, b), (c, c), (a, c)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = b, f (b) = a, f (c) = c. This map is b gη-continuous, but not b-continuous, bsemi-continuous, bα-continuous, br-continuous, since for the closed set V={b, c} in (Y, σ, ≤), $f^{-1}(V)$ = {a, c} is not b-closed, bsemi-closed, bα-closed, br-closed set in (X, □□ ≤).

Theorem: 8.5 Everybη-continuous, bsg-continuous functions are bgη-continuous, but not conversely.

Proof: The proof follows from the fact that every bη-closed, bsg-closed sets are bgη-closed set.

Example : 8.6 Let X = Y = {a, b, c},τ={X,φ□□{a}} and σ ={Y, φ, {a}, {b, c}} . ≤ = {(a, a), (b, b), (c, c), (a, c)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = b, f (b) = a, f (c) = c. This map is bgη-continuous, but not bη-continuous, bsg-continuous, since for the closed set V= {a} in (Y, σ, ≤), $f^{-1}(V)$ = {b} is not bη-closed, bsg-closed sets in (X, □□ ≤).

9. On Contra xgη-continuity

Definition:9.1A function f: (X,□□□□ ≤) → (Y, σ, ≤) is called

(i) xcontra-continuous if $f^{-1}(V)$ is x-closed in (X, □□□≤) for every open set V in (Y, σ, ≤).

(ii) xcontraα-continuous if $f^{-1}(V)$ is xα-closed in (X, □□□≤) for every open set V in (Y, σ, ≤).

(iii) xcontrasemi-continuous if $f^{-1}(V)$ is xsemi-closed in (X, □□□≤) for every open set V in (Y, σ, ≤).

(iv) xcontrar-continuous if $f^{-1}(V)$ is xr-closed in (X, □□□≤) for every open set V in (Y, σ, ≤).

(v) xcontra g-continuous if $f^{-1}(V)$ is xg-closed in (X, □□□≤) for every open set V in (Y, σ, ≤).

(vi) xcontrag*-continuous if $f^{-1}(V)$ is xg*-closed in (X, □□□≤) for every open set V in (Y, σ, ≤).

(vii) xcontrasg-continuous if $f^{-1}(V)$ is xsg-closed in (X, □□□≤) for every open set V in (Y, σ, ≤).

(viii) xcontraη-continuous if $f^{-1}(V)$ isxη-closed in (X, □□□≤) for every open set V in (Y, σ, ≤).

(ix) xcontragη-continuous if f^{-1} (V) isxgη-closed in (X, □□ ≤) for every open set V in (Y, σ, ≤).

Theorem: 9.2 Every icontra-continuous, icontrasemi-continuous, icontraα-continuous, icontra r-continuous, icontraη-continuous functions are icontragη-continuous, but not conversely.

Proof: Every contra continuous, contra semi-continuous, contra α-continuous, contra r-continuous, contra η-continuous functions are contra gη-continuous [24]. Then every icontra-continuous, icontrasemi-continuous, icontraα-continuous, icontra r-continuous, icontraη-continuous functions are icontragη-continuous.

Example: 9.3 Let X = Y = {a, b, c},τ={X,φ□□{a}, {b, c}} and σ ={Y, φ, {a}, {b}, {a, b}} . ≤ = {(a, a), (b, b), (c, c), (a, b), (a, c)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = c, f (b) = a, f (c) = b. This map is icontragη-continuous, but not icontra-continuous, icontrasemi-continuous, icontraα-continuous, icontrar-continuous, icontraη-continuous, since for the open set V = {b} in (Y, σ, ≤), f^{-1}(V) = {c} is not i-closed, isemi-closed, iα-closed, ir-closed, iη-closed in (X, □□ ≤).

Theorem: 9.4 Everyicontrag-continuous, icontrag*-continuous functions are icontragη-continuous, but not conversely.
Proof: Every contra g-continuous, contra g*-continuous functions are contra gη-continuous [24]. Then everyicontrag-continuous, icontrag*-continuousfunctions are icontragη-continuous map.
Example: 9.5 Let X = Y = {a, b, c},τ={X,φ□□{a}, {b}, {a, b}} and σ ={Y, φ, {a}, {b, c}} . ≤ = {(a, a), (b, b), (c, c), (a, c)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = b, f (b) = a, f (c) = c. This map is icontragη-continuous, but not icontrag-continuous, icontra g*-continuous, since for the open set V = {b, c} in (Y, σ, ≤), f^{-1}(V) = {a, c} is not ig-closed, ig*-closed in (X, □□ ≤).

Theorem: 9.6 Everyicontrasg-continuous function is icontragη-continuous, but not conversely.
Proof: Every contra sg-continuous function is contra gη-continuous [24]. Then every icontrasg-continuous function is icontragη-continuous.
Example : 9.7 Let X = Y = {a, b, c},τ={X,φ□□{a}{b}, {a, b}} and σ ={Y, φ, {a}, {b, c}} . ≤ = {(a, a), (b, b), (c, c), (a, c)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = b, f (b) = a, f (c) = c. This map is icontragη-continuous, but not icontrasg-continuous, since for the open set V = {a} in (Y, σ, ≤), f^{-1}(V) = {b} is not isg-closed in (X, □□ ≤).

Theorem: 9.8 Everydcontrasemi-continuous, dcontraα-continuous, dcontrar-continuous, dcontrag*-continuous, dcontraη-continuous functions are dcontragη-continuous, but not conversely.
Proof: Every contra semi-continuous, contra α-continuous, contra r-continuous, contra g*-continuous, contra η-continuous functions are contra gη-continuous [24]. Then every dcontrasemi-continuous, dcontraα-continuous, dcontrar-continuous, dcontrag*-continuous, dcontraη-continuous functions are dcontragη-continuous.
Example: 9.9 Let X = Y = {a, b, c},τ={X,φ□□{a}, {b, c}} and σ ={Y, φ, {b, c}} . ≤ = {(a, a), (b, b), (c, c), (a, b), (a, c)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = b, f (b) = a, f (c) = c. This map is dcontragη-continuous, but not dcontrasemi-continuous, dcontra α-continuous, dcontrar-continuous, dcontrag*-continuous, dcontraη-continuous, since for the open set V = {b, c} in (Y, σ, ≤), f^{-1}(V) = {a, c} is not dsemi-closed, dα-closed, dr-closed, dg*-closed, dη-closed in (X, □□ ≤).

Theorem: 9.10 Everydcontra-continuous, dcontrag-continuousfunctions are dcontragη-continuous, but not conversely.
Proof: Every contra continuous, contra g-continuous functions are contra gη-continuous [24]. Then every dcontracontinuous, dcontrag-continuous functions are dcontragη-continuous.
Example: 9.11 Let X = Y = {a, b, c},τ={X,φ□□{a}, {b}, {a, b}} and σ ={Y, φ, {a}} . ≤ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = a, f (b) = c, f (c) = b. This map is dcontragη-continuous, but not dcontra-continuous, dcontrag-continuous, since for the open set V = {a} in (Y, σ, ≤), f^{-1}(V) = {a} is not dclosed, dg-closed in (X, □□ ≤).

Theorem: 9.12 Everydcontrasg-continuous function is dcontragη-continuous, but not conversely.

Proof: Every contra sg-continuous function is contra gη-continuous [24]. Then every dcontrasg-continuous function is dcontragη-continuous.

Example:9.13 Let X = Y = {a, b, c},τ={X,φ□□{a}{b}, {a, b}} and σ ={Y, φ, {a}, {b, c}} . ≤ = {(a, a), (b, b), (c, c), (a, c)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = b, f (b) = a, f (c) = c. This map is dcontragη-continuous, but not dcontrasg-continuous, since for the open set V = {a} in (Y, σ, ≤), $f^{-1}(V)$ = {b} is not dsg-closed in (X, □□ ≤).

Theorem:9.14 Every bcontra-continuous, bcontrag-continuous, bcontraα-continuous, bcontrar-continuous, bcontrag*-continuous functions are bcontragη-continuous, but not conversely.

Proof: Every contra continuous, contra g-continuous, contra α-continuous, contra r-continuous, contra g*-continuous functions are contra gη-continuous [24]. Then every bcontra-continuous, bcontrag-continuous, bcontraα-continuous, bcontrar-continuous, bcontrag*-continuous functions are bcontragη-continuous.

Example:9.15 Let X = Y = {a, b, c},τ={X,φ□□{a}, {b}, {a, b}} and σ ={Y, φ, {a}} . ≤ = {(a, a), (b, b), (c, c), (a, c)}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = b, f (b) = a, f (c) = c. This map is bcontragη-continuous, but not bcontra-continuous, bcontrag-continuous, bcontraα-continuous, bcontra r-continuous, bcontrag*-continuous, since for the open set V = {a} in (Y, σ, ≤), $f^{-1}(V)$ = {b} is not b-closed, bg closed, bα-closed, br-closed, bg*-closed in (X, □□ ≤).

Theorem :9.16 Every bcontrasemi-continuous, bcontrasg-continuous, bcontraη-continuous functions are bcontragη-continuous, but not conversely.

Proof: Every contra semi-continuous, contra sg-continuous, contra η-continuous functions are contra gη-continuous [24]. Then every bcontrasemi-continuous, bcontrasg-continuous, bcontraη-continuous functions are bcontragη-continuous.

Example : 9.17 Let X = Y = {a, b, c},τ={X,φ□□{a}} and σ ={Y, φ, {a, b}} . ≤ = {(a, a), (b, b), (c, c), (a, c))}. Define a map f: (X, □□ ≤) → (Y, σ, ≤) by f (a) = a, f (b) = c, f (c) = b. This map is bcontragη-continuous, but not bcontrasemi-continuous, bcontrasg-continuous, bcontraη-continuous, since for the open set V = {a, b} in (Y, σ, ≤), $f^{-1}(V)$ = {a, c} is not bsemi-closed, bsg-closed, b η-closed in (X, □□ ≤).

10. REFERENCES:

[1] Amarendrababu. V., Aswini. J., g*-closed sets in topological ordered spaces, International journal of Advanced in Management, Technology and Engineering Sciences, 7 (12) (2017), 113-125.

[2] Andrijevic D. "Some properties of the topology of α-sets", Mat. Vesnik 36(1984).

[3] Balachandran K, Sundaram P & Maki H, On generalized continuous maps in topological spaces, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math, 12(1991), 5-13.

[4] Bhattacharya P, and Lahiri B. K, Semi–generalized closed set in topology, Indian J. Math. 29 (3) (1987), 375 – 382.

[5] Caldas M, Jafari S, Noiri T, Simeos. M, A new generalization of contra-continuity via Levine s. g-closed sets, chaos solitons Fractals 42(2007), 1595-1603.

[6] Devi R &Balachandran K (2001), Some generalizations of α-homeomorphisms in topological spaces , Indian J.PureAppl.Math, 32(4) : 551-563.

[7] Dhanapakyam C., Indirani K.,Onβg* Closed Sets in Topological Ordered Spaces, International Journal of Mathematics Trends and Technology, 65(1), (2019). 7-8.
[8] Dontchev J. Contra-continuous functions and strongly S-closed spaces, Internat. J. Math. Sci. 19 (2) (1996), 303–10.
[9] Dontchev J. and Noiri T., Contra-semi continuous functions, Math. Pannonica 10 (1999), 159–168.
[10] Jafari S. and Noiri T, Contra α-continuous functions between topological spaces, Iran. Int. J. Sci., 2 (2001), pp. 153-167.
[11] Krishna rao K., Some Concepts In Topological Ordered Spaces using semi-open sets, pre-open sets, α-open sets and β-open sets, AcharyaNagarjuna University Thesis, 2014.
[12] Levine N., Semi open sets and semi continuity in Topological spaces, Amer. Math. Monthly,70(1963), 36-41.
[13] Levine. N, Generalized closed sets in topology, Rend. Circ. Mat. Ser. III, 10, (1975), 347 – 350.
[14] Mahmood. S. I. (2012), on generalized regular continuous functions in topological spaces, Kyungpook Math. J., 14:131 -143.
[15] Murugalingam. M, Somasundaram. S and Palaniammal. S, A generalized star sets. Bulltin of Pure and Applied Science, 24(2):233:238, 2005.
[16] Nachbin. L, Topology and order, Van Nostrand. D, Inc., Princeton, New Jersey [1965].
[17] Pious Missier. S. and Annalakshmi.M., Between Regular Open Sets and Open Sets, Internat. J. Math. Archive, 7(5) (2016), 128-133.
[18] Ramachandram V. V. S., SankaraRao B., Sg-continuity in Topological Ordered Spaces, Asian Research Journal of Mathematics, 11(1), 1-5, 2018.
[19] Ravi O, LellisThivagar M, and Latha R, Properties of contra sg continuous maps, Gen. Math. Notes, vol.4, no.1, (2011), pp. 70 - 84.
[20] Srinivasarao G., Srinivasarao. N., Ramprasad C. H., Subbarao M. V., g-closed type sets AND g*-closed type sets in topological ordered spaces, International Journal of Scientific & Engineering Research, 5(6), 1276- 1285, 2014
[21] Subbulakshmi. D, Sumathi. K, Indirani. K., η-open set in topological spaces, International Journal of Innovative Technology and Exploring Engineering, 8(10s) (2019), 276-282.
Subbulakshmi. D, Sumathi. K, Indirani. K., gη-closed set in topological spaces, International Journal of Recent Technology and Engineering, 8(3) (2019), 8863-8866.
[23] Subbulakshmi. D, Sumathi. K, Indirani. K., gη-continuous in topological spaces, Advances in Mathematics: Scientific Journal 8 (2019), no.3, 677-682.
[24] Subbulakshmi. D, Sumathi. K, Indirani. K., contra gη-continuity in topological spaces, International Journal of Advanced Science and Technology ,Vol. 28, No. 16, (2019), pp. 1253-1262.
[25] Sundaram. P, Maki. H and Balachandran. K, Semi-generalized continuous maps and semi-
[26] $T_{1/2}$ spaces, Bull. Fukuoka Univ. Ed. Part-III, 40:33-40, 1991.
[27] Veerakumar. M. K. R. S, Between g* closed sets and g closed sets Antartica J. Math, Reprint.
[28] Veera Kumar. M.K.R.S., Homeomorphisms in topological ordered spaces, ActaCienciaIndica, XXVIII(M), No.1.(2002), 67-76.