International Journal of Aquatic Science
ISSN: 2008-8019
Vol 12, Issue 02, 2021

An Analytical Study Of The M/M/1/N Imprecise Queuing System With Encouraged Arrivals

W. Ritha and N. Rajeswari

Department of Mathematics, Holy Cross College (Autonomous),Affiliated to Bharathidasan University,Tiruchirappalli – 620 002.Tamil Nadu, India

Abstract : We propose a single-server Markovian fuzzy queuing system with encouraged arrivals in this article. Recursive steady-state solutions are used to solve the model. Performance measures are drawn up as needed. By constructing a cost model, an economic study of the model is presented. The model is numerically examined and randomly simulated. The term "encouraged arrivals" was formulated to describe the situation that a system faces when firms make deals and discounts. An encouraged arrival is a modern concept in queuing theory that adds to current customer behaviour.

Keywords: Encouraged arrivals, stochastic models, fuzzy queuing theory, Nonagonal fuzzy number, Ranking function method.

1. INTRODUCTION AND LITERATURE SURVEY

Queuing theory was first proposed by (AK Erlang, 1909) as a field of study. He developed templates to describe the Copenhagen telephone exchange as part of his thesis. He began to understand the applicability of queuing systems in many other fields when designing a solution to the problem, and he began to improve queuing theory further. He is now widely regarded as the inventor of queuing theory. Understanding consumer behaviour is critical in queuing systems. In addition, customers act in the following ways: 1) Balking 2) Reneging 3) Jockeying 4) Collusion. There is a huge amount of information available on consumer behaviour (Haight, 1957, 1959), is a pioneer in this field (Ancker and Gafarian, 1963).
From a company viewpoint, the complex existence of the competitive business environment, as well as the uncertainity of customer behaviour, motivates companies to analyze customer behaviour. This aids businesses in developing policies to attract prospective clients. The margin of error is so poor that even a minor deviation from the plan will result in a business loss.
In 1965, the concepts of fuzzy set theory and the degree of membership introduced by Prof. L.A. Zadeh. The fuzzy number defined as a fuzzy subset of the real line by D. Dubais and H. Prade. A fuzzy number is a quantity whose value as precise, rather than exact, as in the case with single value. To deal imprecise in real-life situations, many researchers used triangular and trapezoidal fuzzy numbers. Most of the researchers have focused on hexagonal, octagonal and decagonal fuzzy numbers. If the vagueness arises in nine different points, we restricted to use other fuzzy numbers.
In this situation, a Nonagonal fuzzy number can be used to clear the uncertainity and get a clear solution.Firms also release different deals and incentives in order to attract new buyers. It doesn't matter whether the retailers are online (or) offline. Customers are often drawn to

visit a certain company because of enticing sales and incentives provided by different organisations. In this article, those drawn customers are referred to as encouraged arrivals.
The term "encouraged arrivals" contributes to the fundamental literature on queuing (Jain et al., 2014) invented the term "customer mobilisation" which they called "reverse balking". They stated that a new client is drawn to a system by looking at a vast customer base. Encouraged arrivals deals with the percentage shift of consumers due to deals and incentives, while reverse balking deals with the possibility of entering and not joining the system. Encouraged arrivals can also be thought of as the precise opposite to discouraged arrivals discussed by (Kumar et al., 2014).
They stated in their work that discouraged arrivals occur as a result of a Poisson process described by parameter $\frac{\lambda}{n+1}$. When customers peer at a big system, they are excluded from joining (Reynolds, 1968) presents multi-server queuing model with discouragement.He discovered an equilibrium distribution of queue length and used it to derive other performance measures. (Natvig, 1975) investigated the birth-death queuing process on a single server in the presence of state dependent parameters, $\lambda_n = \frac{\lambda}{n+1}$, $n \geq 0$ and $\mu_n = \mu$, $n \geq 1$.
Ascending to the study of literature, no one has established a model with encouraged arrivals. In this article, we establish a single-server Markovian fuzzy queuing system with encouraged arrivals to solve the above-mentioned contemporary dynamic nature. Rest of the paper is organized as follows: The mathematical model formulation is discussed in section 2, and the steady-state solution is described in section 3. In section 4, performance measures are derived. Numerical illustration is presented in section 5. Section 6 deals with economic analysis of the model. Special case of no encouragement is covered in section 7. Section 8 concludes with a discussion of the potential spectrum.

A) PRELIMINARIES

a. Fuzzy Set

A fuzzy set A characterized by a membership function mapping element of a domain, space or the universe of discourse X to the unit internal [0,1], i.e., $A = \{(x, \mu_A(x)) / x \in X\}$. Here $\mu_A(x): X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades often represented by real numbers ranging [0,1].

b. Fuzzy Number

A fuzzy set A defined on the universal set of real line R with membership function $\mu_A(x): R \rightarrow [0,1]$ is called a fuzzy number if

(i) A is normal and convexity
(ii) A must be bounded
(iii) $\mu_A(x)$ is piecewise continuous.

c. Nonagonal fuzzy number

A nonagonal fuzzy number $\tilde{A}_N$ denoted as $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$ and the membership function is defined as:

$$\mu_{\tilde{A}_N}(x)=\begin{cases}\frac{1}{4}\frac{(x-a_1)}{(a_2-a_1)} & ;\ a_1\le x\le a_2\\ \frac{1}{4}+\frac{1}{4}\frac{(x-a_2)}{(a_3-a_2)} & ;\ a_2\le x\le a_3\\ \frac{1}{2}+\frac{1}{4}\frac{(x-a_3)}{(a_4-a_3)} & ;\ a_3\le x\le a_4\\ \frac{3}{4}+\frac{1}{4}\frac{(x-a_4)}{(a_5-a_4)} & ;\ a_4\le x\le a_5\\ 1-\frac{1}{4}\frac{(x-a_5)}{(a_6-a_5)} & ;\ a_5\le x\le a_6\\ \frac{3}{4}-\frac{1}{4}\frac{(x-a_6)}{(a_7-a_6)} & ;\ a_6\le x\le a_7\\ \frac{1}{2}-\frac{1}{4}\frac{(x-a_7)}{(a_8-a_7)} & ;\ a_7\le x\le a_8\\ \frac{1}{4}\frac{(a_9-x)}{(a_9-a_8)} & ;\ a_8\le x\le a_9\\ 0 & ;\ \text{otherwise}\end{cases}$$

d. Ranking of Nonagonal fuzzy number

The new ranking method for Nonagonal fuzzy number is given below:

$$R(\tilde{A}_N)=\left(\frac{a_1+3a_2+3a_3+5a_4+3a_5+5a_6+3a_7+3a_8+a_9}{27}\right)$$

2. MATHEMATICAL MODEL FORMULATION

A single-server Markovian fuzzy queuing model is formulated under following assumptions.

(i) Customers arrive one at a time, according to a Poisson process with fuzzy parameter $\tilde{\lambda}(1+\eta)$, where "η" represents the percentage change in the number of customers determined from previous or observed results. For example, if a company formerly gave discounts and the percentage change in the number of customers are monitored +50% (or) 150% then η = 0.5 or η = 1.5 respectively.

(ii) With the fuzzy parameter $\tilde{\mu}$, service times are exponentially distributed.

(iii) Customers are served in the order, in which they arrive, i.e., the queue discipline is first come, first served.

(iv) There is a single server through which the service is provided.

(v) The capacity of the system is finite, say, N.

Differential difference equations determining the model are given by:

$$\frac{d}{dt}P_0(t)=-\tilde{\lambda}(1+\eta)P_0(t)+\tilde{\mu}P_1(t) \qquad ...(1)$$

$$\frac{d}{dt}P_n(t)=\tilde{\lambda}(1+\eta)P_{n-1}(t)-\left\{\tilde{\lambda}(1+\eta)+\tilde{\mu}\right\}\cdot P_n(t)+\tilde{\mu}P_{n+1}(t) \qquad ...(2)$$

$$\frac{d}{dt}P_N(t) = \tilde{\lambda}(1+\eta)P_{N-1}(t) - \tilde{\mu}P_N(t) \qquad \text{... (3)}$$

In steady state, as $t \to \infty$, $P_n(t) = P_n$ and therefore, $\frac{d}{dt}P_n(t) = 0$ as $t \to \infty$ and hence, equations (1)-(3) becomes

$$0 = -\tilde{\lambda}(1+\eta)P_0 + \tilde{\mu}P_1 \qquad \text{... (4)}$$

$$0 = \tilde{\lambda}(1+\eta)P_{n-1} - \left\{\tilde{\lambda}(1+\eta) + \tilde{\mu}\right\}P_n + \tilde{\mu}P_{n+1} \qquad \text{... (5)}$$

$$0 = \tilde{\lambda}(1+\eta)P_{N-1} - \tilde{\mu}P_N \qquad \text{... (6)}$$

3. STEADY-STATE SOLUTION

On solving (4)-(6) iteratively we get

$$P_n = Pr\left\{n \text{ customers in the system}\right\} = \left[\frac{\tilde{\lambda}(1+\eta)}{\tilde{\mu}}\right]^n P_0; 1 \le n \le N-1 \qquad \text{... (7)}$$

And the probability that system is full is given by:

$$P_N = Pr\left\{\text{System is full}\right\} = \left[\frac{\tilde{\lambda}(1+\eta)^N}{\tilde{\mu}}\right] P_0 \qquad \text{... (8)}$$

Using condition of normality, $\sum_{n=0}^{N} P_n = 1$.

$$P_0 = Pr\left\{Systemisempty\right\} = \left\{1 + \sum_{n=1}^{N}\left[\frac{\tilde{\lambda}(1+\eta)}{\tilde{\mu}}\right]^n\right\}^{-1}$$

$$= \frac{1 - \left(\frac{\tilde{\lambda}(1+\eta)}{\tilde{\mu}}\right)}{1 - \left(\frac{\tilde{\lambda}(1+\eta)}{\tilde{\mu}}\right)^{N+1}} \qquad \text{... (9)}$$

4. MEASURES OF PERFORMANCE

1. Expected system size $\left(\tilde{L}_S\right)$

$$\tilde{L}_S = \sum_{n=1}^{N} nP_n$$

$$\tilde{L}_S = \sum_{n=1}^{N} n\left[\frac{\tilde{\lambda}(1+\eta)}{\tilde{\mu}}\right]^n P_0 \qquad \text{... (10)}$$

2. Expected queue length $\left(\tilde{L}_q\right)$

$$\tilde{L}_q = \sum_{n=1}^{N} (n-1)P_n$$

$$\tilde{L}_q = \sum_{n=1}^{N} (n-1)\left[\frac{\tilde{\lambda}(1+\eta)}{\tilde{\mu}}\right]^n P_0 \qquad \text{... (11)}$$

5. NUMERICAL ILLUSTRATION

Let us consider a nonagonal fuzzy numbers are $\tilde{\lambda} = (0,1,2,3,4,5,6,7,8)$ and $\tilde{\mu} = (2,3,4,5,6,7,8,9,10)$. By applying, Ranking technique of Nonagonal fuzzy number, we get

$$R(\tilde{\lambda}) = \left(\frac{0+3(1)+3(2)+5(3)+3(4)+5(5)+3(6)+3(7)+8}{27}\right)$$

$$= \frac{108}{27}$$

$$= 4$$

$\therefore R(\tilde{\lambda}) = 4$, and

$$R(\tilde{\mu}) = \left(\frac{2+3(3)+3(4)+5(5)+3(6)+5(7)+3(8)+3(9)+10}{27}\right)$$

$$= \frac{162}{27}$$

$$= 6$$

$\therefore R(\tilde{\mu}) = 6$

In this section, we present numerical illustration of the above model.

Variation in $\tilde{L}_S$ and $\tilde{L}_q$ with respect to $\tilde{\lambda}$. Here, we take $N = 10$, $\tilde{\mu} = 6$, $\eta = 0.5$.

Table – 1

Average rate of Arrival ($\tilde{\lambda}$)	**Expected System Size ($\tilde{L}_S$)**	**Expected Queue Length ($\tilde{L}_q$)**
4	5	4.090910
4.1	5.246621	4.326726
4.2	5.485556	4.555945
4.3	5.715630	4.777327
4.4	5.935946	4.989909
4.5	6.145880	5.192993
4.6	6.345059	5.386128
4.7	6.533325	5.569082
4.8	6.710709	5.741812
4.9	6.877394	5.904432
5	7.033686	6.057179

International Journal of Aquatic Science
ISSN: 2008-8019
Vol 12, Issue 02, 2021

5.1	7.179976	6.200384
5.2	7.316723	6.334452
5.3	7.444426	6.459830
5.4	7.563606	6.576993
5.5	7.674793	6.686434
5.6	7.778516	6.788639

Source: Simulated data

The above table reveals that as the arrival rate rises, the predicted system size and queue length rise as well. The phenomenon is shown in the graph below.

Variation in $\tilde{L}_s$ and $\tilde{L}_q$ w.r.to $\tilde{\lambda}$.

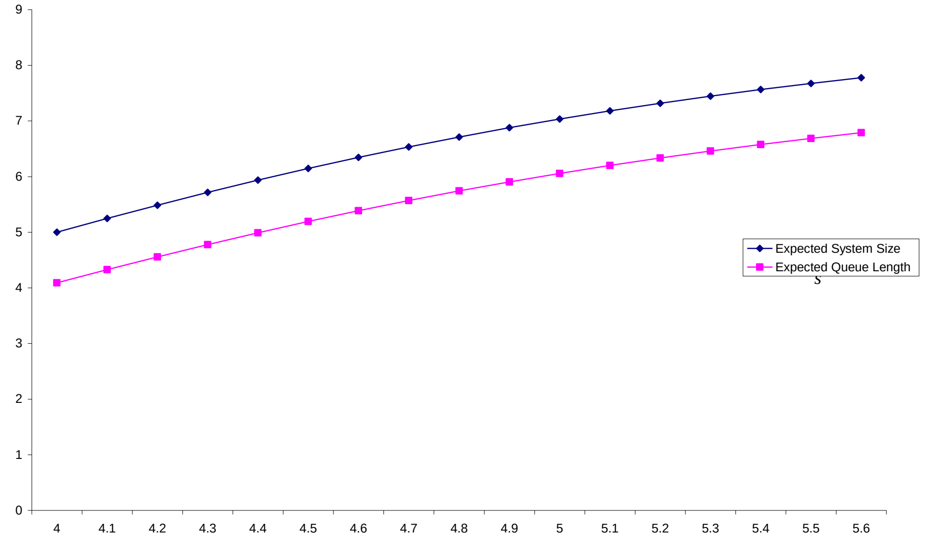

Figure – 1

Similarly, the numerical results are obtained by varying service rate.

Variation in $\tilde{L}_s$ and $\tilde{L}_q$ with respect to $\tilde{\mu}$. Here we take, $N = 10$, $\tilde{\lambda} = 5$, $\eta = 0.5$; Let us consider $\tilde{\lambda} = \{1,2,3,4,5,6,7,8,9\}$, $\therefore R(\tilde{\lambda}) = 5$ and

$\tilde{\mu} = \{1,2,3,4,5,6,7,8,9\}$, $\therefore R(\tilde{\mu}) = 5$.

Table – 2

Average rate of service ($\tilde{\mu}$)	**Expected system size $(\tilde{L}_s)$**	**Expected Queue Length $(\tilde{L}_q)$**
5	8.128658	7.134507
5.1	8.036785	7.043633
5.2	7.937355	6.945383
5.3	7.837372	6.846691
5.4	7.733762	6.744528
5.5	7.626822	6.639200

5.6	7.516923	6.531083
5.7	7.399463	6.415665
5.8	7.279368	6.297809
5.9	7.157224	6.178097
6	7.033686	6.057179
6.1	6.909473	5.935762
6.2	6.778652	5.808063
6.3	6.641052	5.673943
6.4	6.496545	5.533310
6.5	6.493706	5.386128
6.6	6.266693	5.310087

Source: Simulated data

The table above reveals that as the service rate increases, the expected system size decreases, as does the expected length of the queue. The graph below illustrates the phenomenon.
Variation in $\tilde{L}_S$ and $\tilde{L}_q$ w.r.to $\tilde{\mu}$.

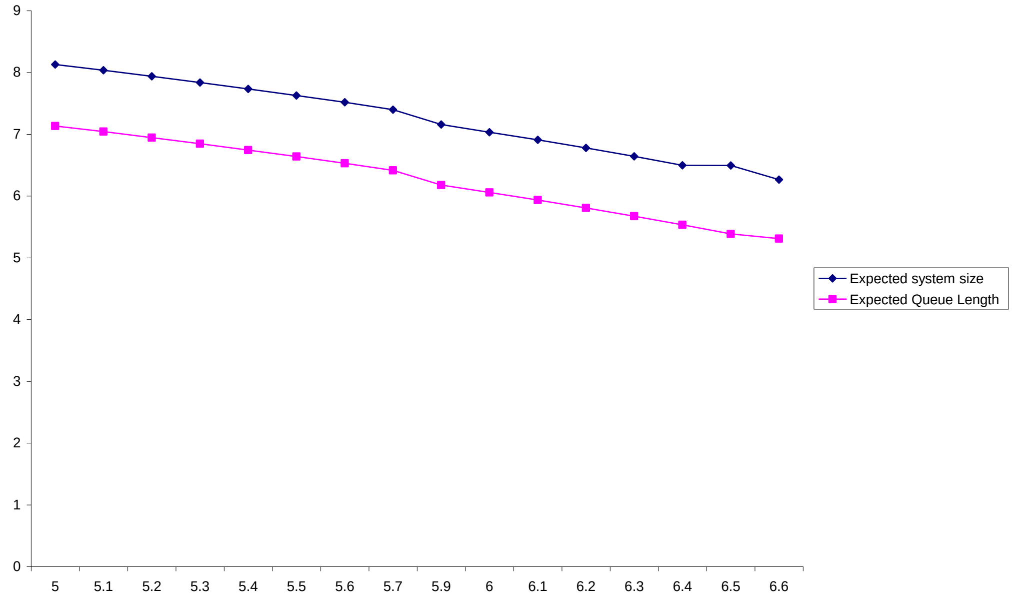

Figure – 2

6. ECONOMIC ANALYSIS

Economic analysis of the model is discussed by developing Total Expected Cost (TEC), Total Expected Revenue (TER) and Total Expected Profit (TEP) functions.

Total Expected Cost of the system (TEC) is given by:

$$TEC = C_S \tilde{\mu} + C_h \sum_{n=1}^{N} n \left[\frac{\tilde{\lambda}(1+\eta)}{\tilde{\mu}} \right]^n P_0 + C_L \tilde{\lambda} \left[\frac{\tilde{\lambda}(1+\eta)}{\tilde{\mu}} \right]^N P_0$$

International Journal of Aquatic Science
ISSN: 2008-8019
Vol 12, Issue 02, 2021

Where $P_0 = \dfrac{1-\left(\dfrac{\lambda(1+\eta)}{\beta}\right)}{1-\left(\dfrac{\lambda(1+\eta)}{\beta}\right)^{N+1}}$

Total Expected Revenue (TER) of the system is given by:

$TER = R \times \beta \times (1-P_0)$

Total expected profit (TEP) of the system is given by:

$TEP = TER - TEC$

Where,

C_S = Cost per service per unit time

C_h = Holding cost per unit per unit time

C_L = Cost associated to each lost unit per unit time

R = Revenue earned per unit per unit time

Sensitivity analysis is carried out on the above-mentioned cost model with varying arrival and service rates.

Table – 3

Variation in TEC, TER and TEP with respect to λ.

We take, $N = 10$, $\beta = 6$, $\eta = 0.5$, $C_S = 10$, $C_L = 15$, $C_h = 2$, $R = 100$.

Average rate of arrival (λ)	**Total Expected Cost (TEC)**	**Total Expected Revenue (TER)**	**Total Expected Profit (TEP)**
4	75.45454	545.4546	470.0001
4.1	76.79960	551.9364	475.1368
4.2	78.19446	557.7667	479.5722
4.3	79.63311	562.9815	483.3484
4.4	81.10968	567.6221	486.5124
4.5	82.61850	571.7326	489.1141
4.6	84.15426	575.3586	491.2044
4.7	85.71210	578.5455	492.8334
4.8	87.28764	581.3377	494.0501
4.9	88.87705	583.7774	494.9004
5	90.47694	585.9043	495.4273
5.1	92.08446	587.7549	495.6704
5.2	93.69718	589.3627	495.6655
5.3	95.31306	590.7579	495.4448
5.4	96.93046	591.9675	495.0371
5.5	98.54803	593.0156	494.4676
5.6	100.16468	593.9234	493.7587

Source: Simulated data

The total expected profit rises as the average rate of arrival increases, reaches a maximum value at a certain stage, and then begins to decline. This is due to the fact that since the service rate is constant, after a certain level of load on the service, the cost rises more than the revenue.

Table – 4
Variation in TEC, TER and TEP with respect to μ.
We take, $N=10$, $\lambda=5$, $\eta=0.5$, $C_S=10$, $C_L=15$, $C_h=2$, $R=100$.

Average rate of Service (μ)	**Total Expected Cost (TEC)**	**Total Expected Revenue (TER)**	**Total Expected Profit (TEP)**
5	91.54972	497.0759	405.5262
5.1	91.43699	506.5076	415.0706
5.2	91.33116	515.8253	424.4941
5.3	91.18015	525.0609	433.8808
5.4	91.20911	534.1860	442.9769
5.5	90.89383	543.1918	452.2980
5.6	90.75079	552.0707	461.3199
5.7	90.70119	560.7644	470.0633
5.8	90.63824	569.3040	478.6658
5.9	90.56279	577.6847	487.1219
6	90.47694	585.9043	495.4273
6.1	90.44632	599.1854	508.7391
6.2	90.35729	601.7654	511.4081
6.3	90.38616	609.2787	518.8926
6.4	90.46119	616.4705	526.0093
6.5	90.87152	623.3052	532.4336
6.6	90.21941	631.3598	541.1404

Source: Simulated data

The table shows that as the rate of service improves, the revenue rises and the profit of the company rises.

7. SPECIAL CASE

When $\eta=0$,

$$P_n = Pr\{n \text{ customers in the system}\}$$

$$= \left(\frac{\lambda}{\mu}\right)^n P_0 = \rho^n P_0; 1 \le n \le N-1$$

$$P_N = Pr\{\text{System is full}\} = \left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^N P_0 = \rho^N P_0$$

$$P_0 = Pr\{\text{System is empty}\} = \left\{1 + \sum_{n=1}^{N}\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^n\right\}^{-1}$$

$$= \frac{1-\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)}{1-\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^{N+1}} = \frac{1-\rho}{1-\rho^{N+1}}$$

Where $\rho = \frac{\tilde{\lambda}}{\tilde{\mu}} < 1$, is the traffic intensity.

The system reduces to classical single-server fuzzy queuing model with finite capacity.

8. CONCLUSIONS AND FUTURE SCOPE

A single server fuzzy queuing model with encouraged arrivals is analyzed in this article. The paper's results would be extremely useful to any company dealing with the phenomena of encouraged consumers and load on service. A successful approach can be developed by adopting and applying this model. Using the proposed ranking method for arrival and service rates as a Nonagonal fuzzy number, the economic analysis of the facility can be calculated and the financial aspect of the business can be observed.

Further service rate and system size optimization can be accomplished, and the system can be analysed in a transient state. It's also interesting to arrive at the framework for heterogeneous service. It is also possible to build a multi-server fuzzy queuing model. The system, on the other hand, can be examined indefinitely.

9. REFERENCES

[1] Ancker, C.J. and A.V. Gafarian, “Some Queuing Problems with Balking and Reneging-I”, Operations Research, vol. 11, no. 1, 1963, pp.88-100.

[2] Ancker, C.J. and A.V. Gafarian, “Some Queuing Problems with Balking and Reneging-II”, Operations Research, vol. 11, no. 6, 1963, pp. 928-937.

[3] Dr. M.S. Annie Christi, Mrs. Kasthuri, B., “Transportation Problem with Pentagonal Intuitionistic Fuzzy Number solved using Ranking Technique and Russell’s method”, International Journal of Engineering Research and Applications, ISSN: 2248-9622, Vol. 6, Issue 2 (part 4) Feb 2016, pp. 82-86.

[4] Bhupendar kumar som, “An M/M/1/N Queuing system with Encouraged Arrivals”, Global Journal of Pure and Applied Mathematics, ISSN:0973-1768,Vol 13,No.7(2017),pp.3443-3453.

[5] I. Deli, Y. Subas, “A ranking method of single-valued Neutrosophic numbers and its applications to multi-attribute decision-making problems”, International Journal of Mechanical Learn & Cyber, 25 January 2016.

[6] Erlang, A.K., “The theory of probabilities and telephone conversations”, “Nyt Tidsskrift for Matematik” B, vol. 20, 1909, p.33-39.

[7] A. Felix, S. Christopher and A. Victor Devadoss, “A Nonagonal Fuzzy Number and it’s the arithmetic operation”, International Journal of Mathematics and its Applications, vol. 3, Issue 2 (2015), 185-195, ISSN: 2347-1557.
[8] Haight, F.A. “Queuing with balking”, Biometrika, vol. 44, no. 3, 1957, pp.360-369.
[9] Haight, Frank, A., “Queuing with reneging”, Metrika, vol. 2, no. 1, 1959, pp.186-197.
[10] C. Jesuraj, A. Rajkumar, “Intuitionistic Nonagonal Fuzzy Number and its Application in FDE”, International Journal of Recent Technology and Engineering (IJRTE), ISSN: 2277-3878, vol. 8, Issue 2, July 2019.
[11] Jain.N.K., Kumar.R., & Som.B.K., “An M/M/1/N Queuing system with reverse balking”. American Journal of Operational Research, vol.4, no.2, 2014, pp.17-20.
[12] Kumar, Rakesh and Kumar Sharma, “A Single-server Markovian queuing system with discouraged arrivals and retention of reneged customers”, Yugoslav Journal of Operations Research, vol. 24, no. 1, 2014, pp. 119-126.
[13] A. Nagoor Gani, V.N. Mohammed, “An algorithm for solving assignment problems with costs as generalized trapezoidal intuititonistic fuzzy numbers”, International Journal of Pure and Applied Mathematics, vol. 104, no. 4, 2015, 561-575.
[14] Natvig, B., “On a Queuing Model where Potential Customers are Discouraged by Queue Length”, Scandinavian Journal of Statistics, vol. 2, 1975, pp. 34-42.
[15] Reynolds, John F., “The Stationary Solution of a Multiserver Queuing Model with Discouragement”, Operations Research, vol. 16, no. 1, 1968, pp. 64-71.
[16] S. Subasri and K. Selvakumari, “Fuzzy Assignment Problem with Ranking of Generalized Nonagonal Fuzzy number using the centroid of centroid”, Journal of Adv. Research in Dynamical and Control Systems, vol. 12, 04-Special issue, 2020, ISSN No: 1943-023X, Accepted 16 March 2020.
[17] L. Sudha, R. Shanmugapriya, B. Rama, “Fuzzy Transportation problem using Nonagonal fuzzy number”, JASC: Journal of Applied Science and Computations, vol. VI, Issue III, March 2019, ISSN No: 1076-5131.
[18] Supriya Kar, Kaila Basu, Sathi Mukherjee, “Application of Neutrosophic set theory in Generalized Assignment Problem”, Neutrosophic Sets and Systems (NSS), vol. 9, 2015.
[19] L.A. Zadeh, Fuzzy sets, Information and Computation, 8(1965), 338-353.