

A Proposed Method to Solve Transportation Problem by Generalized Pentagonal and Hexagonal Fuzzy Numbers

R. Srinivasan¹, N. Karthikeyan²

¹Department of Ancient Science, Tamil University, Tanjavur, India

²Department of Mathematics, Kongunadu College of Engineering and Technology, Trichy, India

¹srinivasanmaths@gmail.com

²karthi.lect@gmai.com

Abstract: Fuzzy numbers figuring essential parts in issues in the dynamic, examination of information, and economics course of action. Tracking down the Positioning of any Fuzzy numbers is an unavoidable advance in numerous numerical models. A significant number of the strategies proposed, which created the best answer for the transportation issues. This paper presents a proposed positioning technique applying a similar we change the generalized Fuzzy transportation issue to a dazzling esteemed one, hence into another proposed interaction to uncover the Fuzzy reasonable arrangement. The mathematical representation exhibits that the new projected technique delicate marvellous methods for dealing with the transportation issues on Fuzzy calculations

Keywords: Fuzzy set, Ranking of Pentagonal Fuzzy Numbers, Ranking of Hexagonal Fuzzy Numbers, Fuzzy Transportation Problem, Centroid Method.

AMS Mathematics Subject Classification (2010): 90C08, 90C90, 49K, 90B, 94D

1. INTRODUCTION

The transportation issue figures out how to decrease transportation of products by planning less transportation course for a solitary item from the given number of sources to the given number of objections. The Transportation issue is an investigation of ideal transportation costs. The standard assertion of the transportation issue is a lattice with the lines addressing sources and segments addressing objections. This composition may be one of the novel techniques that cut down the ideal arrangement esteem. Here we contrasted and existing strategies named NWCM, LCM, and VAM strategy.

There are various techniques which can take care of transportation issue in the Fuzzy climate however all are addressed by normal Fuzzy numbers. Consider \tilde{a}_i as quantity of items that available at source \tilde{i} and \tilde{b}_j as quantity of items that necessary at destination \tilde{j} . Consider $\tilde{\alpha}_{ij}$ as the price of transferring one item from source \tilde{i} to end terminal \tilde{j} and \tilde{X}_{ij} as the amount of item carried from source \tilde{i} to terminal end \tilde{j} . A fuzzy transportation problem is a progressive method in that we can get the expenditure of the transportation, Demand and supply facts are fuzzy quantities. The first introduced fuzzy set concept by Zadeh [1]. Chen et al. [2] has examined that there is no impulse for enrollment work for normal Fuzzy numbers. He proposed the idea of generalized Fuzzy numbers.

Amarpreet et al. [3] proposed a generalized Fuzzy transportation issue with trapezoidal Fuzzy numbers and addressed mathematical models with positioning capacities. The positioning of normal Fuzzy numbers was first, presented by Jain [4]. A few specialists utilized the positioning technique in various habits. Phanibhushan et al. [5] examine the distance strategy by utilizing a positioning of Fuzzy numbers with the Centroid. Rajarajeswari et al. [6] proposed requesting generalized Fuzzy numbers dependent on region, mode, and difference, spread, and addressed with generalized hexagonal Fuzzy numbers. Malini et al. [7] presented a positioning capacity with octagonal Fuzzy numbers.

Chu et al. [8] used the region between the Centroid dependent on the distance strategy to rank Fuzzy numbers. Wang et al. [9] proposed the reconsidered technique for positioning Fuzzy numbers with a space between the Centroid and unique point. Annie et al. [10] proposed the best up-and-comer technique and take care of the transportation issue. They utilized hexagonal Fuzzy numbers by Centroid positioning strategy. Roseline et al. [11] proposed a positioning procedure for generalized trapezoidal Fuzzy numbers. He additionally utilized a Fuzzy Hungarian strategy to track down an underlying arrangement. Singh Pushpinder [12] proposed a novel strategy to rank generalized Fuzzy numbers. Ghadle and Pathade [13] settled octagonal Fuzzy numbers by the positioning technique. They utilized balanced and unbalanced Fuzzy transportation issues. R. Srinivasan et al. [14] have investigated a two-stage cost-limiting Fuzzy transportation issue where inventory and requirement are Fuzzy numbers utilizing a way to deal with arriving at a completely fuzzy arrangement. Srinivasan R et al [15] have additionally utilized a proposed technique to resolve transportation problems by trapezoidal Fuzzy numbers. The proposed Positioning calculation is to unwind a solid arrangement by utilizing Fuzzy transportation issues assessing inventory, requirement, and thing transportation cost as generalized pentagonal Fuzzy numbers and generalized Hexagonal Fuzzy numbers.

In this manuscript, an unblemished way is suggested for the Positioning of generalized pentagonal Fuzzy numbers and generalized Hexagonal Fuzzy numbers in a worked-on manner. To show this proposed technique and the case is given. As the proposed cycle is straight and easy to comprehend and applying it is undemanding to make out the Fuzzy most select practical result of Fuzzy transportation inconveniences happen in the verifiable conditions.

This manuscript is figured out as follows: In division 2 it is focused on the essential depiction of Fuzzy figures. In division 3, a getting Ranking practice is started and show on a novel calculation to determine the transportation issue by generalized Fuzzy numbers. In division 4, is to uncover the projected strategy a mathematical plan is settled. In division 5, a conclusion component is likewise included.

2. PRELIMINARIES

2.1 Definition: Fuzzy Set

\tilde{A} is a fuzzy set on R is defined as a set of ordered pairs

$$\tilde{A} = \{x_0, \mu_{\tilde{A}}(x_0) / x_0 \in \tilde{A}, \mu_{\tilde{A}}(x_0) \rightarrow [0, 1]\}$$

where $\mu_{\tilde{A}}(x_0)$ is said to be the membership function.

2.2 Definition: Fuzzy Number

\tilde{A} is a fuzzy set on R , likely bounded to the stated conditions given beneath

- i. $\mu_{\tilde{A}}(x_0)$ is part by part continuous

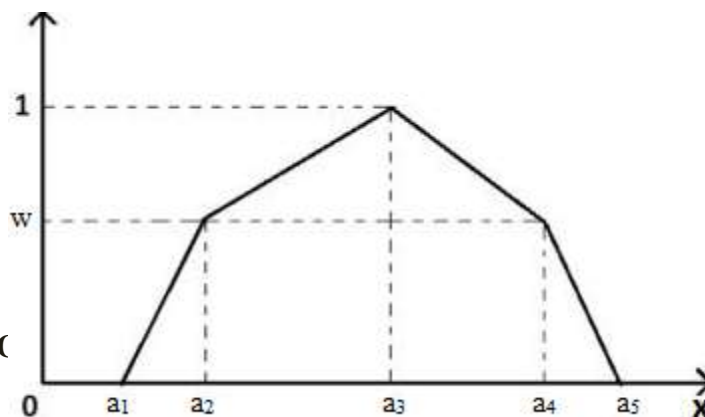
- ii. There exist at least one $x_0 \in \mathfrak{R}$ with $\mu_{\tilde{A}}(x_0) = 1$
- iii. \tilde{A} is a regular and convex

2.3 Definition: Generalized Pentagonal Fuzzy Number (GPFNs)

A fuzzy number \tilde{A} on R is said to be the Generalized pentagonal fuzzy number (GPFNs) or linear fuzzy number which is named as $(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5; \tilde{w})$ if its membership function $\mu_{\tilde{A}}(x)$ has the following characteristic

$$\mu_A(x) = \begin{cases} 0, & x < \tilde{a}_1 \\ \tilde{w} \frac{x - \tilde{a}_1}{\tilde{a}_2 - \tilde{a}_1}, & \tilde{a}_1 \leq x \leq \tilde{a}_2 \\ (1 - (1 - \tilde{w})) \frac{x - \tilde{a}_2}{\tilde{a}_3 - \tilde{a}_2}, & \tilde{a}_2 \leq x \leq \tilde{a}_3 \\ \tilde{w}, & x = \tilde{a}_3 \\ (1 - (1 - \tilde{w})) \frac{\tilde{a}_4 - x}{\tilde{a}_4 - \tilde{a}_3}, & \tilde{a}_3 \leq x \leq \tilde{a}_4 \\ \tilde{w} \frac{\tilde{a}_5 - x}{\tilde{a}_5 - \tilde{a}_4}, & \tilde{a}_4 \leq x \leq \tilde{a}_5 \\ 0, & x > \tilde{a}_5 \end{cases}$$

Here the midpoint \tilde{a}_3 has the grade of membership \tilde{w}



2.4 Definition: (

A fuzzy number \tilde{A} on R is said to be the Generalized hexagonal fuzzy number (HPFNs) or linear fuzzy number which is named as $(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6; \tilde{w})$ if its membership function $\mu_{\tilde{A}}(x)$ has the following characteristic

$$\mu_A(x) = \begin{cases} 0, & x < \tilde{a}_1 \\ \frac{\tilde{w}}{2} \frac{x - \tilde{a}_1}{\tilde{a}_2 - \tilde{a}_1}, & \tilde{a}_1 \leq x \leq \tilde{a}_2 \\ \frac{\tilde{w}}{2} + \frac{\tilde{w}}{2} \left(\frac{x - \tilde{a}_2}{\tilde{a}_3 - \tilde{a}_2} \right), & \tilde{a}_2 \leq x \leq \tilde{a}_3 \\ \tilde{w}, & \tilde{a}_3 \leq x \leq \tilde{a}_4 \\ \tilde{w} - \frac{\tilde{w}}{2} \left(\frac{x - \tilde{a}_4}{\tilde{a}_5 - \tilde{a}_4} \right), & \tilde{a}_4 \leq x \leq \tilde{a}_5 \\ \frac{\tilde{w}}{2} \frac{\tilde{a}_6 - x}{\tilde{a}_6 - \tilde{a}_5}, & \tilde{a}_5 \leq x \leq \tilde{a}_6 \\ 0, & x > \tilde{a}_6 \end{cases}$$

2.5 Arithmetic Operations

Let $\tilde{\alpha}_A = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \dots, \tilde{a}_n; \tilde{w}_1)$ and $\tilde{\alpha}_B = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4, \tilde{b}_5, \dots, \tilde{b}_n; \tilde{w}_2)$ are two fuzzy numbers where $\tilde{a}_1 \leq \tilde{a}_2 \leq \tilde{a}_3 \leq \tilde{a}_4 \leq \tilde{a}_5 \leq \dots \leq \tilde{a}_n$ and $\tilde{b}_1 \leq \tilde{b}_2 \leq \tilde{b}_3 \leq \tilde{b}_4 \leq \tilde{b}_5 \leq \dots \leq \tilde{b}_n$. Then the arithmetic operations are defined as follows

- i. $\tilde{\alpha}_A + \tilde{\alpha}_B = (\tilde{a}_1 + \tilde{b}_1, \tilde{a}_2 + \tilde{b}_2, \tilde{a}_3 + \tilde{b}_3, \tilde{a}_4 + \tilde{b}_4, \tilde{a}_5 + \tilde{b}_5, \dots, \tilde{a}_n + \tilde{b}_n; \min(\tilde{w}_1, \tilde{w}_2))$
- ii. $\tilde{\alpha}_A - \tilde{\alpha}_B = (\tilde{a}_1 - \tilde{b}_n, \tilde{a}_2 - \tilde{b}_{n-1}, \tilde{a}_3 - \tilde{b}_{n-2}, \dots, \tilde{a}_n - \tilde{b}_1; \min(\tilde{w}_1, \tilde{w}_2))$
- iii. $\tilde{\alpha}_A * \tilde{\alpha}_B = (\tilde{a}_1 * \tilde{b}_1, \tilde{a}_2 * \tilde{b}_2, \tilde{a}_3 * \tilde{b}_3, \tilde{a}_4 * \tilde{b}_4, \tilde{a}_5 * \tilde{b}_5, \dots, \tilde{a}_n * \tilde{b}_n; \min(\tilde{w}_1, \tilde{w}_2))$
- iv. $k\tilde{\alpha}_A = \begin{cases} k\tilde{a}_1, k\tilde{a}_2, k\tilde{a}_3, k\tilde{a}_4, k\tilde{a}_5, \dots, k\tilde{a}_n; \tilde{w}_1 & \text{if } k > 0 \\ k\tilde{a}_n, k\tilde{a}_{n-1}, k\tilde{a}_{n-2}, \dots, k\tilde{a}_2, k\tilde{a}_1; \tilde{w}_1 & \text{if } k < 0 \end{cases}$

2.6 Fuzzy transportation problem by means of Mathematical formulation

The mathematical formulation of any fuzzy numbers under the case that the total inventory is equivalent to the total requirement is given as follows

$$\text{Min } Z = \sum_{i=1}^s \sum_{j=1}^t \tilde{\alpha}_{ij} \tilde{x}_{ij}$$

Subject to the constraints

$$\sum_{j=1}^t \tilde{x}_{ij} = \tilde{a}_i \quad j=1, 2, \dots, t$$

$$\sum_{i=1}^s \tilde{x}_{ij} = \tilde{b}_j \quad i=1, 2, \dots, s$$

$$\sum_{i=1}^s \tilde{a}_i = \sum_{j=1}^t \tilde{b}_j; \quad i=1, 2, \dots, s; \quad j=1, 2, \dots, t \quad \text{and}$$

$$\tilde{x}_{ij} \geq 0, \quad i=1, 2, \dots, s, \quad j=1, 2, \dots, t$$

The fuzzy transportation problem is explicitly represented by the fuzzy transportation table:

	1	...	t	Inventory
1	$\tilde{\alpha}_{11}$...	$\tilde{\alpha}_{1t}$	\tilde{a}_1
\vdots	\vdots	...	\vdots	\vdots

S	$\tilde{\alpha}_{s1}$...	$\tilde{\alpha}_{st}$	$\tilde{\alpha}_s$
Requirement	\tilde{b}_1	...	\tilde{b}_t	

2.7 Ranking Function

In this manuscript, we proposed a Centroid ranking technique for generalized Pentagonal Fuzzy numbers and generalized Hexagonal Fuzzy numbers as follows.

- i. Let $\tilde{\alpha}_A = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5; \tilde{w})$ be generalized Pentagonal fuzzy numbers by using Centroid Ranking technique [16] for GPFNs here the ranking was introduced.

$$R(\tilde{\alpha}_A) = \left(\frac{2\tilde{a}_1 + 9\tilde{a}_2 + 2\tilde{a}_3 + 9\tilde{a}_4 + 2\tilde{a}_5}{24} \right) \left(\frac{11\tilde{w} + 2}{24} \right)$$

- ii. Let $\tilde{\alpha}_A = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6; \tilde{w})$ be generalized Hexagonal fuzzy numbers by using Centroid Ranking technique for GHFNs here the ranking was introduced.

$$R(\tilde{\alpha}_A) = \left(\frac{2\tilde{a}_1 + 3\tilde{a}_2 + 4\tilde{a}_3 + 4\tilde{a}_4 + 3\tilde{a}_5 + 2\tilde{a}_6}{18} \right) \left(\frac{5\tilde{w}}{18} \right)$$

3. RECOMMENDED ALGORITHM

Step – 1: Verify given problem is stabled or not.

$$(i.e) \sum_{i=1}^s \tilde{a}_i = \sum_{j=1}^t \tilde{b}_j.$$

If unstable, change into a stabled one by introducing a model source or model destination utilizing zero fuzzy item transportation expenses.

Step – 2: The Ranking value is imparted to transform both inventory and Requirement.

Step – 3: Row-wise, the greatest value of each row and it's divided by a number of the columns of the cost matrix.

Step – 4: Column-wise, the greatest value of each column and it's divided by a number of the rows of the cost matrix.

Step – 5: We find the maximum of the resultant value and do the allocation of that particular cell of the given matrix, suppose we have more than one maximum consequent value. We can select anyone.

Step – 6: Find the smallest amount of Inventory and Requirement and allocate it

Step – 7: Follow the Third, fourth, fifth, and sixth steps, till $(s+t-1)$ groups are allocated. Suppose allocated cell not achieved apply Modi method to find optimality.

4. RESULT AND DISCUSSION

4.1 Numerical example

Using generalized pentagonal Fuzzy figures, we affirm a solution to the fuzzy transportation problem, which includes transportation costs, customer requirements and desires, and the availability of products. Take a look at Sapan Kumar Das's transportation problem [17].

Table 1

	R_a	R_b	R_c	Inventory
I_a	(5,10,13,14,15;1)	(1,2,3,4,5;1)	(2,6,8,10,14;1)	(2,11,,23,34,45;1)
I_b	(3,4,5,6,7;1)	(1,5,6,7,11;1)	(1,4,5,9,16;1)	(10,47,52,65,76;1)
I_c	(3,6,9,12,15;1)	(2,5,7,8,8;1)	(1,1,1,1,1;1)	(3,18,56,76,81;1)

Requirement	(11,16,51,67,75;1)	(20,40,60,80,100;1)	(15,30,45,75,110;1)	
-------------	--------------------	---------------------	---------------------	--

4.1.1 Solution

Table 2

By using the Centroid Ranking Technique, we have to convert generalized pentagonal fuzzy numbers into a crisp value

	R _a	R _b	R _c	Inventory
I _a	6.3946	1.6250	4.3333	10.7207
I _b	2.7083	3.2500	3.6337	28.9792
I _c	4.8750	3.4080	0.586	25.4132
Requirement	19.9514	32.5000	29.0017	

The given problem is unbalanced, and we are adding 0 rows to balance the given problem.

Table 3

	R _a	R _b	R _c	Inventory
I _a	6.3946	1.6250	4.3333	10.7207
I _b	2.7083	3.2500	3.6337	28.9792
I _c	4.8750	3.4080	0.586	25.4132
I _d	0	0	0	16.3400
Requirement	19.9514	32.5000	29.0017	

Then choose the highest of the penalty values, discover the associated minimal cost value, and allocate the provided problem's specific cost cell. We can choose any of the maximum resulting values if we have more than one.

Table 4

	R _a	R _b	R _c	Inventory	$\frac{\text{max}}{\text{column}}$
I _a	6.3946	10.7207 1.6250	4.3333	10.7207	2.1315
I _b	2.7083	3.2500	3.6337	28.9792	1.2112
I _c	4.8750	3.4080	0.586	25.4132	1.625
I _d	0	0	0	16.3400	0
Requirement	19.9514	32.5000	29.0017		
$\frac{\text{max}}{\text{row}}$	1.5987	0.8520	1.0833		

Again, we select the highest penalty value and calculate the matching minimal cost value before allocating the provided problem's cost cell. We can choose anyone if we have more than one maximum outcome value.

Table 5

	R _a	R _b	R _c	Inventory	$\frac{\text{min} \times \text{max}}{\text{row} \times \text{column}}$
I _a	6.3946	10.7207 1.6250	4.3333	10.7207	2.1315
I _b	2.7083	3.2500	3.6337	28.9792	1.2112
I _c	4.8750	3.4080	25.4132 0.586	25.4132	1.625
I _d	0	0	0	16.3400	0
Requirement	19.9514	32.5000	29.0017		
$\frac{\text{min} \times \text{max}}{\text{row} \times \text{column}}$	1.625	1.136	1.2112		

The same approach will be repeated until we arrive at the final allocation. Finally, the best feasible resolutions are as follows when applying the new proposed ranking system.

Table 6

	R _a	R _b	R _c	Inventory
I _a	6.3946	10.7207 1.6250	4.3333	10.7207
I _b	19.9514 2.7083	9.0278 3.2500	3.6337	28.9792
I _c	4.8750	3.4080	25.4132 0.586	25.4132
I _d	0	12.7515 0	3.5885 0	16.3400
Requirement	19.9514	32.5000	29.0017	

4.1.2 Result

A total of $(4+3-1) = 6$ cells are allotted in this case. Then, using the proposed Ranking algorithm, we may obtain the best solution.

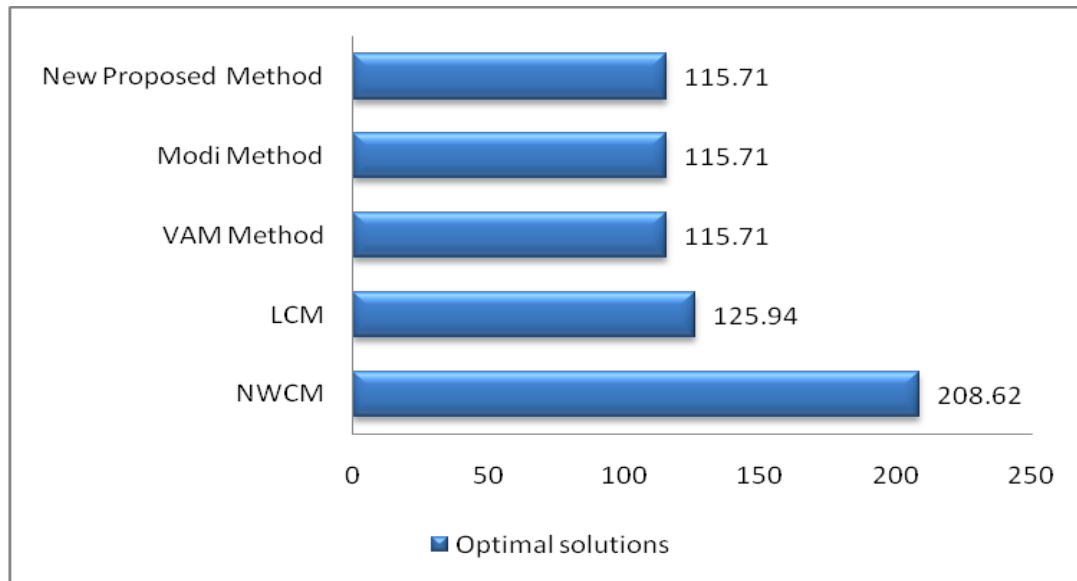
Min

$$Z = 19.9514(2.7083) + 9.0278(3.25) + 12.7515(0) + 3.5885(0) + 25.4132(0.5867) + 10.7207(1.625)$$

$$\text{Min } Z = 115.7057 \approx 115.71$$

4.1.3 Discussion

The new proposed Ranking Technique is compared to the NWCM, the LCM, VAM, and MODI's optimality test in the graphical representation below. It is evident that the new proposed Ranking Technique provides the best results.



4.2 NUMERICAL EXAMPLE

Using generalized hexagonal Fuzzy figures, we affirm a solution to the fuzzy transportation problem, which includes transportation costs, customer requirements and desires, and the availability of products. Take a look at Thamaraiselvi transportation problem [18].

Table 7

	R_a	R_b	R_c	R_d	Inventory
I_a	(3,7,11, 15,19,24;0.5)	(13,18,23, 28,33,40;0.7)	(6,13,20, 28,36,45;0.4)	(15,20,25, 31,38,45;0.8)	16
I_b	(16,19,24, 29,34,39;0.2)	(3,5,7, 9,10,12;0.5)	(5,7,10, 13,17,21;0.6)	(20,23,26, 30,35,40;0.4)	36
I_c	(11,14,17, 21,25,30;0.7)	(7,9,11, 14,18,22;0.6)	(2,3,4, 6,7,9;0.5)	(5,7,8, 11,14,17;0.9)	20
Requirement	24	18	20	10	

4.2.1 Solution

Table 8

By using the Centroid Ranking Technique, we have to convert generalized hexagonal fuzzy numbers into a crisp value

	R_a	R_b	R_c	R_d	Inventory
I_a	0.73	1.45	1.36	1.6	16
I_b	1.48	0.43	0.67	1.6	36
I_c	1.08	0.74	0.28	0.56	20
Requirement	24	18	20	10	

The same approach will be repeated until we arrive at the final allocation. Finally, the best feasible resolutions are as follows when applying the new proposed ranking system.

Table 9

	R _a	R _b	R _c	R _d	Inventory
I _a	16 0.73	1.45	1.36	1.6	16
I _b	1.48	18 0.43	18 0.67	1.6	36
I _c	8 1.08	0.74	2 0.28	10 0.56	20
Requirement	24	18	20	10	

4.2.2 Result

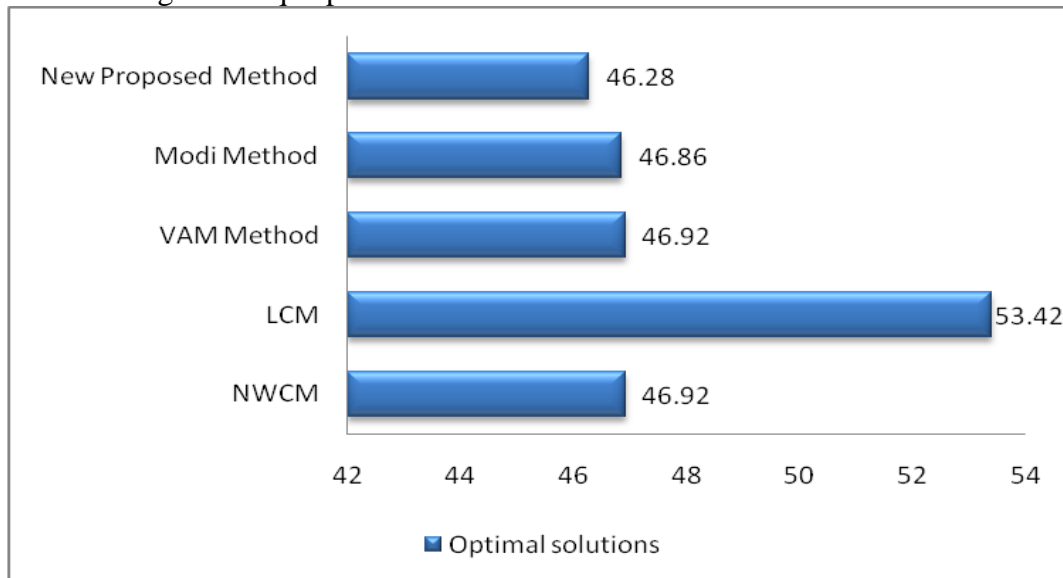
A total of $(4+3-1) = 6$ cells are allotted in this case. Then, using the proposed Ranking algorithm, we may obtain the best solution.

$$\text{Min } Z = 18(0.67) + 2(0.28) + 18(0.43) + 8(1.08) + 10(0.56) + 16(0.73)$$

$$\text{Min } Z = 46.28$$

4.2.3 Discussion

The new proposed Ranking Technique is compared to the NWCM, the LCM, VAM, and MODI's optimality test in the graphical representation below. It is evident that the new proposed Ranking Technique provides the best results.



5. CONCLUSION

The main contribution of this manuscript is to derive the simplest possible viability of a fuzzy transportation problem for generalized Pentagonal fuzzy numbers and generalized hexagonal fuzzy numbers using the newly proposed Ranking algorithm method. This practice can use for every kind of fuzzy transportation problem. The recently proposed Ranking technique may be a regularized practice, simple to relate to, and ready to be operated for the whole styles of transportation problems either to capitalize on or play down an intended

function. This approach can be broadening to resolve transportation problems by way of a further fuzzy algorithm.

REFERENCES

- [1] L.A. Zadeh, Fuzzy sets, *Inf. Control* 8 (1965) 338–353. DOI: [https://dx.doi.org/10.1016/s0019-9958\(65\)90241-x](https://dx.doi.org/10.1016/s0019-9958(65)90241-x)
- [2] Chen, S. “Operations on Fuzzy Numbers with Function Principal,” *Tamkang, Journal of management sciences*, 1985, 6, pp. 13-25
- [3] Amarpreet, K., and Amit, K. “A New Method for Solving Fuzzy Transportation Problems Using Ranking Function,” *Applied Mathematical Modeling*, 2011, 35(12), pp. 5652-5661.
- [4] Jain, R. “Decision Making in the Presence of Fuzzy Variables,” *IEEE Transactions on systems, Man and Cybernetics*, 1976, 6, pp. 698-703.
- [5] Phanibhushan, R., and Ravishankar, N. “Ranking Fuzzy Numbers with Distance Method Using Circumcentre of Centroids and an Index of Modeling, ”*Advances in fuzzy system*, 2011, 10, pp. 1155-1161.
- [6] Rajarajeshwari, P., and Sudha, S. “Ordering Generalized Hexagonal Fuzzy Numbers Using Rank, Mode, Divergence and Spread,” *IOSR Journal Mathematics*, 2014, 10(3), pp. 15-22.
- [7] Malini, P., and Ananthnarayan, M. “Solving Fuzzy Transportation Problem Using Ranking of Octagonal Fuzzy Numbers,” *International journal of pure and applied Mathematics*, 2016, 110(2), pp. 275-282.
- [8] Chu, T. , and Tsao, C. “ Ranking of Fuzzy Numbers with an Area between the Centroid and Original Points,” *Computers and Mathematics with application*, 2002, 43, pp. 111-117,
- [9] Wang, Y., and Lee, S. “The Revised Method of Ranking Fuzzy Numbers with an Area between Cetroid and Original Points,” *Computers and Management with Application*, 2008, 55 (9), pp. 2033-2042.
- [10] Annie, S., and Malini, D. “Solving Transportation Problems with Hexagonal Fuzzy Numbers Using BCM and Different Ranking Techniques,” *International journal of Engineering research and Application*, 2016, 6(2), pp. 76-81.
- [11] Roseline, S., and Amitraj, H. “GFHM to GTFTP with Ranking of GFN,” *International journal of applied mathematics*, 2014, 3(1), pp. 5-12.
- [12] Singh P. “ A Novel Method for Ranking Generalized Fuzzy Numbers, *Journal of Information Sciences and Engineering*, 2015, 3, pp. 1373-1385
- [13] Ghadle, K., and Pathade, P. “Optimal Solution of balanced and Unbalanced Fuzzy Transportation Problem by Using Octagonal Fuzzy Numbers” *IJPAM (Bulgaria)*. 2017, (Accepted)
- [14] Srinivasan, R., Karthikeyan, N., Renganathan, K., & Vijayan, D. V. (2020). Method for solving fully fuzzy transportation problem to transform the materials. *Materials Today: Proceedings*. doi:10.1016/j.matpr.2020.05.423
- [15] Srinivasan R, Karthikeyan N, Jayaraja A (2021) A Proposed Technique to Resolve Transportation Problem by Trapezoidal Fuzzy Numbers. *Indian Journal of Science and Technology* 14(20): 1642-1646. <https://doi.org/10.17485/IJST/v14i20.645>
- [16] Avishek Chakraborty, Sankar Prasad Mondal, The Pentagonal Fuzzy Number: Its Different Representations, Properties, Ranking, Defuzzification and Application in Game Problems, *Symmetry* 2019, 11, 248, 1-31, DOI: doi:10.3390/sym11020248

- [17] Sapan Kumar Das., “ Application of Transportation Problem under Pentagonal Neutrosophic Environment” Journal of Fuzzy Extension & Applications, 2020, 1(1), pp 27-41
- [18] Thamaraiselvi, A., Santhi, R., “Solving Fuzzy Transportation Problem with Generalized Hexagonal Fuzzy Numbers”, IOSR Journal Mathematics, 2015, 11(5), pp. 08-13.