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$\mathrm{H_k}$ − Cordial labeling of Some Graph and its Corona Graph

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Abstract: - A graph $G = (V, E)$ is called $H_k -$ cordial if for each edge e and each vertex v of G have the label $1 \le |f(e)| \le k$, $1 \le |f(v)| \le k$ and $|v_f(i) - v_f(-i)| \le 1$, $|e_f(i) - e_f(-i)| \le 1$ for each i with $1 \le i \le k$. In this paper we investigate $H_k -$ cordial labeling of H-graph , $K_{3,m}$ graph , $T_n \odot K_1$,$L_{n,1} = P_n \times P_2$ graph, $H \odot K_1$, $K_{3,m} \odot K_1$, $L_{n,1} \odot K_1$.

Keywords: - $H -$cordial labeling, $H_k -$ cordial labeling, H- graph, Kite graph, Ladder graph, Comb graph, Crown, $T_n \odot K_1$, $H \odot K_1$, $K_{3,m} \odot K_1$, $L_{n,1} \odot K_1$.

1. INTRODUCTION

In this paper we consider only finite, simple and undirected graph $G = (V, E)$ where E is a set of edges of G and V is a set of vertices of G. We represent edge as $e = uv$,where $u, v \in V$. Most graph labeling methods trace their origin to one introduced by Rosa [1], or one given by Graham and Sloane [11]. Several types of graph labeling have been investigated both from a purely combinatorial perspective as well as from an application point of view. A detailed survey of various graph labeling is explained in Gallian[5]. The concept of cordial labeling and $H -$ cordial labeling was introduced by I. Cahit [4].D.Parmar and J.Joshi [3] prove that a triangular snake graph T_n is $H -$ cordial if n is even and $H_3 -$ cordial if n is odd.

Definition 1.1 Let $G = (V, E)$ be a graph. A mapping $f: E \to \{1, -1\}$ is called H-cordial, if there exists a positive constant k, such that for each vertex v, $|f(v)| = k$ with vertex labeling $f(v) = \sum_{e \in I(v)} f(e)$, where $I(v)$ is the set of all edges incident to vertex v and the following two conditions are satisfied $|e_f(1) - e_f(-1)| \le 1$ and $|v_f(k) - v_f(-k)| \le 1$. A graph admits $H -$ cordial labeling is called $H -$ cordial graph. Following lemma gives important relation between vertex labeling and edge labeling. [9]

Lemma 1.2 If f is assignment of integer numbers to the vertices and edges of graph G such that for each vertex v, labeling $f(v) = \sum_{e \in I(v)} f(e)$, where $I(v)$ is the set of all edges incident to vertex v then $\sum_{v \in V(G)} f(v) = 2 \sum_{e \in E(G)} f(e)$.[9]

Definition 1.3 An assignment f of integer labels to the edges of a graph is called $H_k -$ cordial labeling, if for each edge e and each vertex v of graph we have $1 \le |f(e)| \le k$ and

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$1 \leq |f(v)| \leq k$ with vertex labeling $f(v) = \sum_{e \in I(v)} f(e)$, where $I(v)$ is the set of all edges incident to vertex v and for each i with $1 \leq i \leq k$ we have $\left|e_f(\mathrm{i}) - e_f(-\mathrm{i})\right| \leq 1$ and $\left|v_f(i) - v_f(-i)\right| \leq 1$. A graph is called H_k – cordial if it admits a H_k – cordial labeling.[9]

It is clear from definition that if graph admits $H -$ cordial labeling then it is $H_k -$ cordial labeling graph. Also if graph is $H_k -$ cordial then it is $H_{k+1} -$ cordial labeling, but converse is not true. [7]

Definition 1.4 A Triangular Snake Graph T_n is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n.$ that is every edge of a path is replaced by a triangle. [7][3]

Definition 1.5 Let G and H be two graphs with $|V(G)| = n, |V(H)| = m$, corona product of G and H is the graph obtained by taking n copies of H and attaching each such copy of H to every vertex of G . It is denoted by $G \odot H$.[2]

Definition 1.6 The H-graph of path P_n is the graph obtained from two copies of P_n with vertices $u_1, u_2, \dots \dots . . u_n$ and $v_1, v_2, \dots \dots . . v_n$ by joining the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ by an edge if n is odd and the vertices $u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ if n is even. [14]

Definition 1.7 A kite graph is obtained by attaching a path of length m with cycle of length n and it is denoted by $K_{n,m}$. It is also known as Dragon graph OR Canoe paddle graph. [2]

Definition 1.8 The ladder graph is obtained by $P_n \times P_2$ .It is denoted by $L_{n,1}$. [10]

Definition 1.9 A circular ladder graph is defined as the Cartesian product $C_n \times K_2$ where K_2 is the complete graph on two vertices and C_n is the cycle graph with n vertices.[8]

2. MAIN RESULT

Theorem 2.1 The graph $T_n \odot K_1$ is $H -$ cordial if $n \geq 4$ is even.

Proof: Let P_n be the path $u_1, u_2, \dots, u_n$. We can obtain triangular snake graph from path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i < n$. The graph $T_n \odot K_1$ is obtained by adding edge to each vertex .Hence, we have new vertex v'_i for $1 \leq i < n$ and u'_i for $1 \leq i \leq n$ and edges $v_i v'_i$, $u_i u'_i$.Let $V = \{u_i, u'_i, v_j, v'_j : 1 \leq i \leq n, 1 \leq j < n-1\}$ and $E = \{u_i u_{i+1}, u_i u'_i, u_i v_i, v_i u_{i+1}, v_i v'_i : 1 \leq i \leq n-1\}$ be a vertex and edge set of graph $T_n \odot K_1$.

Consider a function $f: E \to \{-1,1\}$ defined as

$$f(u_i, v_i) = f(u_{i+1}, v_i) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n}{2} \\ -1 & ; \frac{n}{2} + 1 \leq i \leq n-1 \end{cases}$$

$$f(u_i u'_i) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n}{2} \\ -1 & ; \frac{n}{2} + 1 \leq i \leq n \end{cases}$$

$$f(u_i u_{i+1}) = f(v_i v'_i) = \begin{cases} -1 & ; 1 \leq i \leq \frac{n}{2} \\ 1 & ; \frac{n}{2} + 1 \leq i \leq n-1 \end{cases}$$

$n \geq 4$	Edge Condition	Vertex Condition

n is even	$e_f(1) = \dfrac{5n-4}{2} = e_f(-1)$	$v_f(1) = 2n - 1 = v_f(-1)$

In each case, the graph satisfies the condition $|e_f(\mathrm{i}) - e_f(-\mathrm{i})| \leq 1$ and $|v_f(i) - v_f(-i)| \leq 1$.

Hence, $T_n \odot K_1$ is $H-$ cordial if n is even.

Example 2.2 $T_6 \odot K_1$ is H – cordial shown in Figure 1.

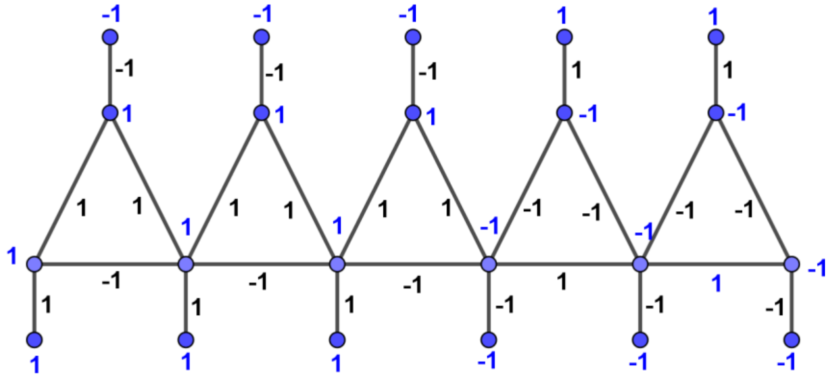

Figure 1: $T_6 \odot K_1$

Theorem 2.3 The graph $T_n \odot K_1$ is H_3- cordial.

Proof: Let P_n be the path $u_1, u_2, \dots, u_n$. We can obtain triangular snake graph from path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i < n$. Let $V = \{u_i, u'_i, v_j, v'_j : 1 \leq i \leq n, 1 \leq j < n-1\}$ and $E = \{u_i u_{i+1}, u_i u'_i, u_i v_i, v_i u_{i+1}, v_i v'_i : 1 \leq i \leq n-1\}$ be a vertex and edge set of graph $T_n \odot K_1$.

Case 1 : If n is even then by Theorem 2.1 $T_n \odot K_1$ is $H-$cordial. Therefore it is H_2- cordial. Hence it is H_3- cordial.

Case 2 : If n is odd, then

Consider a function $f: E \rightarrow \{-1,1\}$ defined as

$$f(u_i u_{i+1}) = f(u_i, v_i) = f(u_{i+1}, v_i) = (-1)^{i+1}\ ; 1 \leq i \leq n-1$$

$$f(u_i u'_i) = f(v_i v'_i) = (-1)^i$$

$n \geq 3$	Edge Condition	Vertex Condition
n is odd	$e_f(1) = \dfrac{5n-5}{2}, e_f(-1) \dfrac{5n-3}{2}$	$v_f(1) = 2n-1, v_f(-1) = 2n-2$ $v_f(3) = 0, v_f(-3) = 1$

Hence, $T_n \odot K_1$ is H_3- cordial

Example 2.4 $T_5 \odot K_1$ is H_3- cordial shown in Figure 2.

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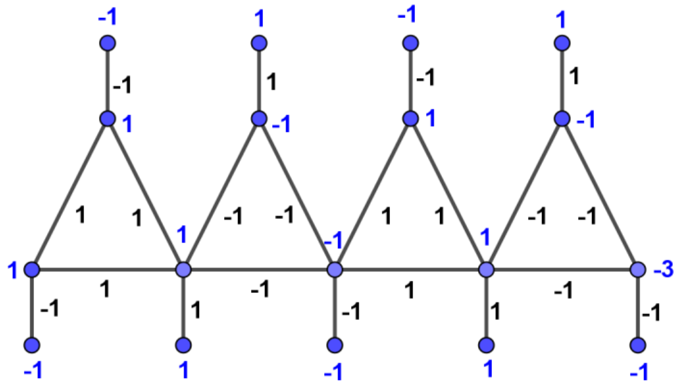

Figure 2: $T_5 \odot K_1$

Theorem 2.5 The H-graph of path P_n is H_3 -cordial.

Proof: Let H graph of path P_n with vertices $u_1, u_2, \ldots \ldots .. u_n$ and $v_1, v_2, \ldots \ldots .. v_n$ by joining the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ by an edge if n is odd and the vertices $u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ if n is even.

Let $E = \left\{u_i u_{i+1}, v_i\, v_{i+1}, u_{\left\lfloor\frac{n+1}{2}\right\rfloor}, v_{\left\lceil\frac{n+1}{2}\right\rceil} : 1 \le i \le n-1\right\}$ be an edge set of $H-$graph.

Consider a function $f: E \to \{-1,1\}$ defined as

$f(u_i u_{i+1}) = 1\ ; 1 \le i \le n-1,$

$f(v_i\, v_{i+1}) = -1\,; 1 \le i \le n-1,$

$f\left(u_{\left\lfloor\frac{n+1}{2}\right\rfloor}\, v_{\left\lceil\frac{n+1}{2}\right\rceil}\right) = 1.$

n	Edge Condition	Vertex Condition
$n \ge 3$	$e_f(1) = n$ $e_f(-1) = n-1$	$v_f(1) = 2, v_f(-1) = 3$ $v_f(2) = n-3 = v_f(-2)$ $v_f(3) = 1, v_f(-3) = 0$

In each case, the graph satisfies the condition $|e_f(\mathrm{i}) - e_f(-\mathrm{i})| \le 1$ and $|v_f(i) - v_f(-i)| \le 1$.

Hence, H-graph of path P_n is $H_3 -$ cordial

Example 2.6 H-graph of path P_6 is $H_3 -$ cordial shown in Figure 3.

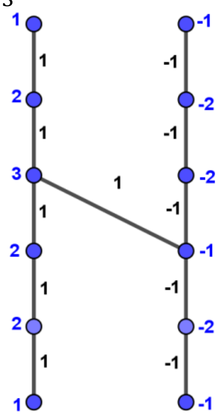

Figure 3: H-graph of path P_6

Theorem 2.7 The $H \odot K_1$ graph of path P_n is H_3 -cordial.

Proof: Let H-graph of path P_n with vertices $u_1, u_2, \dots \dots . . u_n$ and $v_1, v_2, \dots \dots . . v_n$ by joining the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ by an edge if n is odd and the vertices $u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ If n is even. $u_1, u_2, \dots \dots . . u_n$, $v_1, v_2, \dots \dots . . v_n$ are join by edge to the vertices $u'_1, u'_2, \dots \dots . . u'_n$, $v'_1, v'_2, \dots \dots . . v'_n$ respectively. Let $E = \left\{u_i u_{i+1}, v_i\, v_{i+1}, u_i u'_i, v_i\, v'_i, u_{\left\lfloor\frac{n+1}{2}\right\rfloor}\ \ v_{\left\lceil\frac{n+1}{2}\right\rceil} : 1 \le i \le n-1\right\}$ be an edge set of $H \odot K_1$ graph.

Consider a function $f: E \to \{-2, -1, 1, 2\}$ defined as

$$f(u_i u_{i+1}) = 1\,; 1 \le i \le n-1$$

$$f(v_i\, v_{i+1}) = -1\,; 1 \le i \le n-1$$

$$f(u_i\,, u'_i) = \begin{cases} 1 & ; if\ i = 1, n \\ -1 & ; Otherwise \end{cases}$$

$$f(v_i\,, v'_i) = \begin{cases} -1 & ; if\ i = 1, n \\ 1 & ; Otherwise \end{cases}$$

$$f\left(u_{\left\lfloor\frac{n+1}{2}\right\rfloor}\ \ v_{\left\lceil\frac{n+1}{2}\right\rceil}\right) = 2\,.$$

n	Edge Condition	Vertex Condition
$n \ge 3$	$e_f(1) = 2n-1 = e_f(-1)$ $e_f(2) = 1, e_f(-2) = 0$	$v_f(1) = 2n-2, v_f(-1) = 2n-3$ $v_f(2) = 2 = v_f(-2)$ $v_f(3) = 1, v_f(-3) = 0$

Hence, $H \odot K_1$ graph of path P_n is $H_3 -$ cordial.

Example 2.8 $H \odot K_1$ graph of path P_5 is $H_3 -$ cordial shown in Figure 4.

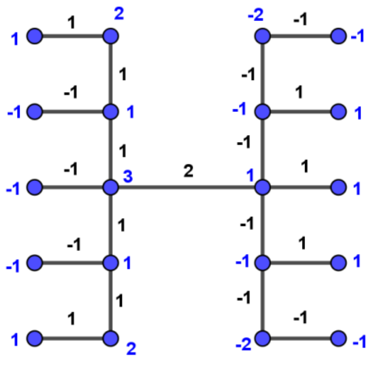

Figure 4: $H \odot K_1$ graph of path P_5

Theorem 2.9 Kite graph $K_{3,m}$ is H_3 -cordial.

Proof: Kite graph $K_{3,m}$ is obtained by attaching a path of length m with C_3 . Let $u_1, u_2, \dots \dots u_{m+3}$ are vertices of graph and u_1, u_2, u_3 form a cycle C_3 .Let u_3 be a common vertex of a cycle C_3 and path of length m . Let $E = \{u_1 u_3, u_i u_{i+1} : 1 \le i \le m+3\}$ be an edge set of kite graph $K_{3,m}$.

Consider a function $f: E \to \{-2, -1, 1, 2\}$ defined as

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Case 1 If $m = 1$, then

$f(u_1u_2) = 2, f(u_1u_3) = -1, f(u_2u_3) = 1,$

$f(u_3u_4) = -1.$

Case 2 If $m = 2$, then

$f(u_1u_2) = 2, f(u_1u_3) = -1, f(u_2u_3) = 1,$

$f(u_3u_4) = -2 , f(u_4u_5) = -1.$

Case 3 If $m \geq 3$, then

$f(u_1u_2) = 2, f(u_1u_3) = -1,$

$$f(u_iu_{i+1}) = \begin{cases} 1 ; 2 \leq i \leq \left\lfloor \frac{m+3}{2} \right\rfloor \\ -2 ; i = \left\lfloor \frac{m+3}{2} \right\rfloor + 1 \\ -1 ; \left\lfloor \frac{m+3}{2} \right\rfloor + 2 \leq i \leq m+2 \end{cases}$$

m	Edge Condition	Vertex Condition
$m = 1$	$e_f(1) = 1, e_f(-1) = 2$ $e_f(2) = 1, e_f(-2) = 0$	$v_f(1) = 1, v_f(-1) = 2$ $v_f(3) = 1, v_f(-3) = 0$
$m = 2$	$e_f(1) = 1, e_f(-1) = 2$ $e_f(2) = 1, e_f(-2) = 1$	$v_f(1) = 1, v_f(-1) = 1$ $v_f(2) = 0, v_f(-2) = 1$ $v_f(3) = 1, v_f(-3) = 1$
m is odd	$e_f(1) = \frac{m+1}{2} = e_f(-1)$ $e_f(2) = 1, e_f(-2) = 1$	$v_f(1) = 2 = v_f(-1)$ $v_f(2) = \frac{m-3}{2} = v_f(-2)$ $v_f(3) = 1 = v_f(-3)$
m is even	$e_f(1) = \frac{m}{2}, e_f(-1) = \frac{m+2}{2}$ $e_f(2) = 1, e_f(-2) = 1$	$v_f(1) = 2 = v_f(-1)$ $v_f(2) = \frac{m-4}{2}, v_f(-2) = \frac{m-2}{2}$ $v_f(3) = 1 = v_f(-3)$

Hence, $K_{3,m}$ Graph is $H_3 -$ cordial.

Example 2.10 $K_{3,5}$ Graph is $H_3 -$ cordial shown in Figure 5.

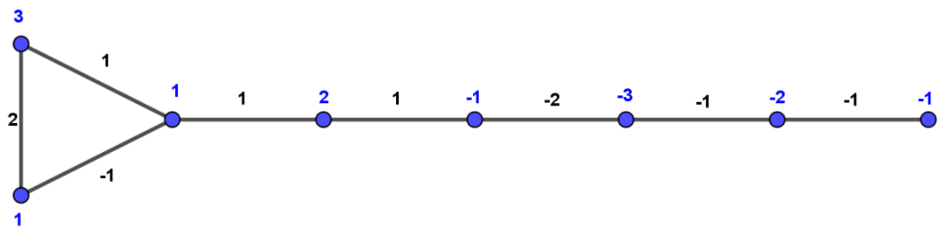

Figure 5: $K_{3,5}$

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Theorem 2.11 The graph $K_{3,m} \odot K_1$ is H_2 -cordial.
Proof: A graph $K_{3,m} \odot K_1$ is obtained by attaching an edge to each vertex of graph $K_{3,m}$. Let $u_1, u_2, \dots \dots u_{m+3}$ are vertices of graph $K_{3,m}$ .Hence new vertices are $u'_1, u'_2, \dots, u'_{m+3}$ and edges $u_i u'_i$,$1 \le i \le m+3$ .let u_3 be a common vertex of a cycle C_3 and path of length m . Let $E = \{u_1u_3, u_iu_{i+1}, u_iu'_i : 1 \le i \le m+3\}$ be an edge set of kite graph $K_{3,m} \odot K_1$.

Consider a function $f: E \to \{-1,1\}$ defined as

$$f(u_iu_{i+1}) = \begin{cases} -1 & ; 3 \le i \le m+2 \\ 1 & ; otherwise \end{cases}$$

$$f(u_iu'_i) = \begin{cases} 1 & ; 3 \le i \le m+2 \\ -1 & ; otherwise \end{cases}$$

m	Edge Condition	Vertex Condition
$m \ge 1$	$e_f(1) = m+3 = e_f(-1)$	$v_f(1) = m+2 = v_f(-1)$ $v_f(2) = 1 = v_f(-2)$

Hence, $K_{3,m} \odot K_1$ Graph is $H_2 -$ cordial.
Example 2.12 $K_{3,4} \odot K_1$ *Graph* is $H_2 -$ cordial shown in Figure 6.

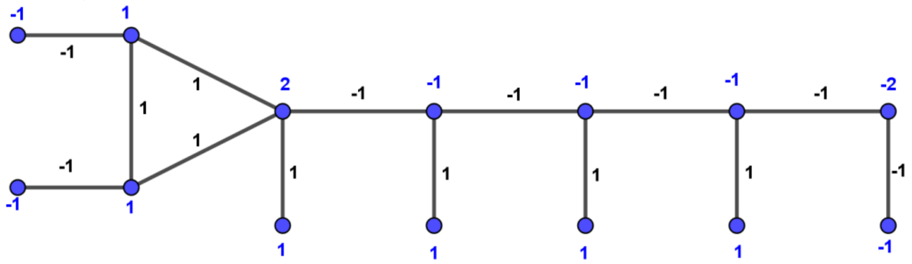

Figure 6: $K_{3,4} \odot K_1$

Theorem 2.13 Ladder graph $L_{n,1} (n \ge 4)$ is H_2 –cordial if n is even.
Proof: Let $V = \{u_i, v_i : 1 \le i \le n\}$ and $E = \{u_iu_{i+1}, v_iv_{i+1}, 1 \le i \le n-1\} \cup \{u_iv_i : 1 \le i \le n\}$ are vertex and edge set of ladder graph $L_{n,1}$.
Consider a function $f: E \to \{-1,1\}$ defined as

$$f(u_iu_{i+1}) = f(v_iv_{i+1}) = \begin{cases} 1 & ; 1 \le i \le \frac{n}{2} \\ -1 & ; \frac{n}{2} \le i \le n-1 \end{cases}$$

$$f(u_1v_1) = 1, f(u_nv_n) = -1$$

$$f(u_iv_i) = \begin{cases} -1 & ; 2 \le i \le \frac{n}{2}+1 \\ 1 & ; \frac{n}{2}+2 \le i \le n-1 \end{cases}$$

n	Edge Condition	Vertex Condition
n is even	$e_f(1) = \frac{3n-2}{2} = e_f(-1)$	$v_f(1) = n-2 = v_f(-1)$ $v_f(2) = 2 = v_f(-2)$

Hence, $L_{n,1}$ Graph is $H_2 -$ cordial if n is even.
Example 2.14 $L_{4,1}$ Graph is $H_2 -$ cordial shown in Figure 7.

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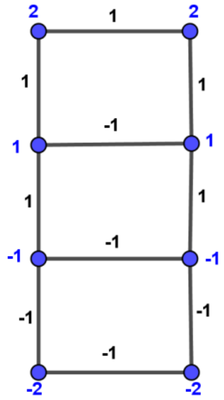

Figure 7: $L_{4,1}$

Theorem 2.15 Ladder graph $L_{n,1}$ is H_3 -cordial.

Proof: Let $V = \{u_i, v_i : 1 \leq i \leq n\}$ and $E = \{u_i u_{i+1}, v_i v_{i+1}, 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\}$ are vertex and edge set of ladder graph $L_{n,1}$.

Case 1: If n is even then by Theorem 2.13 $L_{n,1}$ is H_2- cordial. Therefore it is H_3- cordial.

Case 2: If $n = 2$, then

Consider a function $f: E \to \{-2, -1, 1, 2\}$ defined as

$f(u_1v_1) = 2,$

$f(u_2v_2) = -2,$

$f(u_1u_2) = 1,$

$f(v_1v_2) = -1.$

Case 3: If $n = 3$, then

Consider a function $f: E \to \{-2, -1, 1, 2\}$ defined as

$f(u_1v_1) = 2,$

$f(u_2v_2) = 1,$

$f(u_3v_3) = -2,$

$f(u_iu_{i+1}) = 1; i = 1,2,$

$f(v_iv_{i+1}) = -1; i = 1,2.$

Case 4: If $n \geq 5$,then

Consider a function $f: E \to \{-1, 1\}$ defined as

$$f(u_iu_{i+1}) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n-3}{2} \\ -1 & ; \frac{n-1}{2} \leq i \leq n-1 \end{cases}$$

$$f(v_iv_{i+1}) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n+1}{2} \\ -1 & ; \frac{n+3}{2} \leq i \leq n-1 \end{cases}$$

$f(u_1v_1) = 1,$

$f(u_nv_n) = -1,$

$$f(u_iv_i) = \begin{cases} -1 & ; 2 \leq i \leq \frac{n+1}{2} \\ 1 & ; \frac{n+3}{2} \leq i \leq n-1 \end{cases}$$

n	Edge Condition	Vertex Condition
$n = 2$	$e_f(1) = 1 = e_f(-1)$	$v_f(1) = 1 = v_f(-1)$

	$e_f(2) = 1 = e_f(-2)$	$v_f(3) = 1 = v_f(-3)$
$n = 3$	$e_f(1) = 3, e_f(-1) = 2$ $e_f(2) = 1 = e_f(-2)$	$v_f(1) = 1, v_f(-1) = 2$ $v_f(3) = 2, v_f(-3) = 1$
n is odd	$e_f(1) = \frac{3n-3}{2}, e_f(-1) = \frac{3n-1}{2}$	$v_f(1) = n-2, v_f(-1) = n-3$ $v_f(2) = 2 = v_f(-2)$ $v_f(3) = 0, v_f(-3) = 1$

In each case, the graph satisfies the condition $|e_f(\mathrm{i}) - e_f(-\mathrm{i})| \leq 1$ and $|v_f(i) - v_f(-i)| \leq 1$.

Hence, $L_{n,1}$ Graph is $H_3 -$ cordial.

Example 2.16 $L_{5,1}$ Graph is $H_3 -$ cordial shown in Figure 8.

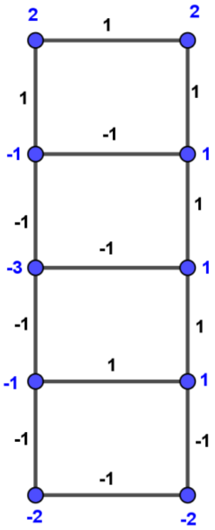

Figure 8: $L_{5,1}$

Theorem 2.17 The graph $L_{n,1} \odot K_1$ is H_3 -cordial.

Proof Let $V = \{u_i, v_i, u'_i, v'_i : 1 \leq i \leq n\}$ and $E = \{u_i u_{i+1}, v_i v_{i+1}, 1 \leq i \leq n-1\} \cup \{u_i v_i, u_i u'_i, v_i v'_i : 1 \leq i \leq n\}$ are vertex and edge set of ladder graph $L_{n,1} \odot K_1$.

Consider a function $f: E \to \{-2, -1, 1, 2\}$ defined as

$f(u_1 v_1) = 1, f(u_n v_n) = -1, f(u_i v_i) = (-1)^i 2 ; 2 \leq i \leq n-1$

$f(u_i u_{i+1}) = 1; 1 \leq i \leq n-1$

$f(v_i v_{i+1}) = -1; 1 \leq i \leq n-1$

$f(u_i u'_i) = -1; 1 \leq i \leq n$

$f(v_i v'_i) = 1; 1 \leq i \leq n$

$n \geq 2$	Edge Condition	Vertex Condition
n is even	$e_f(1) = 2n = e_f(-1)$ $e_f(2) = \frac{n-2}{2} = e_f(-2)$	$v_f(1) = \frac{3n+2}{2} = v_f(-1)$ $v_f(3) = \frac{n-2}{2} = v_f(-3)$
n is odd	$e_f(1) = 2n = e_f(-1)$ $e_f(2) = \frac{n-1}{2}, e_f(-2) = \frac{n-3}{2}$	$v_f(1) = \frac{3n+3}{2}, v_f(-1) = \frac{3n+1}{2}$ $v_f(3) = \frac{n-1}{2}, v_f(-3) = \frac{n-3}{2}$

Hence, $L_{n,1} \odot K_1$ Graph is $H_3 -$ cordial.

Example 2.18 $L_{5,1} \odot K_1$ Graph is $H_3 -$ cordial shown in Figure 9.

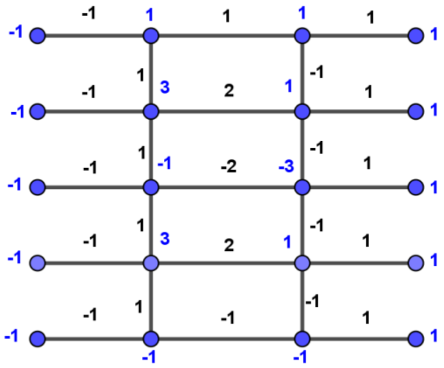

Figure 9 : $L_{5,1} \odot K_1$

Theorem 2.19 Comb ($P_n \odot K_1$) $(n \geq 2)$ is $H_3 -$ cordial.

***Proof*:** Let P_n be the path $u_1, u_2, \dots, u_n$. The graph $P_n \odot K_1$ is obtained by adding edge to each vertex .Let $V = \{u_i, u'_i : 1 \leq i \leq n\}$ and $E = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i u'_i : 1 \leq i \leq n\}$ are vertex and edge set of graph $P_n \odot K_1$.

Consider a function $f: E \rightarrow \{-2, -1, 1, 2\}$ defined as

$f(u_i u_{i+1}) = (-1)^{i+1}.2\,;\, 1 \leq i \leq n-1$

$f(u_i u'_i) = (-1)^i\,;\; 1 \leq i \leq n$

$n \geq 2$	Edge Condition	Vertex Condition
n is even	$e_f(1) = \frac{n}{2} = e_f(-1)$ $e_f(2) = \frac{n}{2}, e_f(-2) = \frac{n-2}{2}$	$v_f(1) = n, v_f(-1) = n-1$ $v_f(3) = 1, v_f(-3) = 0$
n is odd	$e_f(1) = \frac{n-1}{2}, e_f(-1) = \frac{n+1}{2}$ $e_f(2) = \frac{n-1}{2} = e_f(-2)$	$v_f(1) = n, v_f(-1) = n-1$ $v_f(3) = 0, v_f(-3) = 1$

Hence, $P_n \odot K_1$ is $H_3 -$ cordial.

Example 2.20 $P_6 \odot K_1$ is $H_3 -$ cordial shown in Figure 10.

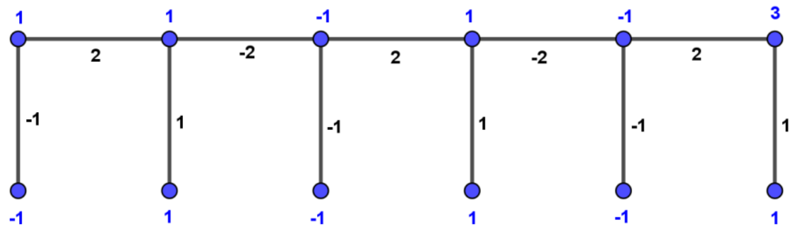

Figure 10: $P_6 \odot K_1$

Theorem 2.21 The $P_n \odot mK_1$ is $H_3 -$ cordial$(n \geq 3)$.

Proof: Let P_n be a cycle with vertices $u_1, u_2, \dots u_n$. $P_n \odot mK_1$ is obtained from path P_n by attaching $m -$ pendant edge to each vertex. Let $V = \{u_i, u_{ij} : 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i u_{ij} : 1 \leq i \leq n, 1 \leq j \leq m\}$ are vertex and edge set of graph $P_n \odot mK_1$.

Consider a function $f: E \longrightarrow \{-2, -1, 1, 2\}$ defined as

Case 1: If m is even, then

$$f(u_i u_{i+1}) = \begin{cases} 1 & ; 1 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1 \\ 2 & ; i = \left\lceil \frac{n}{2} \right\rceil \\ -1 & ; Otherwise \end{cases}$$

$f(u_i u_{ij}) = (-1)^j ; 1 \le i \le n, 1 \le j \le m.$

$m,n \ge 3$	Edge Condition	Vertex Condition
m is even	$e_f(1) = \frac{n(m+1)-2}{2} = e_f(-1)$ $e_f(2) = 1, e_f(-1) = 0$	$v_f(1) = \frac{mn+4}{2}, v_f(-1) = \frac{mn+2}{2}$ $v_f(2) = \frac{n-4}{2} = v_f(-2)$ $v_f(3) = 1, v_f(-3) = 0$

Case 2: If m is odd, then

$f(u_i u_{i+1}) = (-1)^{i+1}.2 ; 1 \le i \le n-1$

$f(u_i u_{i1}) = (-1)^i ; 1 \le i \le n$

$f(u_i u_{ij}) = (-1)^j ; 1 \le i \le n, 2 \le j \le m.$

$n \ge 3, m$ is odd	Edge Condition	Vertex Condition
n is even	$e_f(1) = \frac{mn}{2} = e_f(-1)$ $e_f(2) = \frac{n}{2}, e_f(-2) = \frac{n-2}{2}$	$v_f(1) = \frac{(m+1)n}{2},$ $v_f(-1) = \frac{(m+1)n-2}{2}$ $v_f(3) = 1, v_f(-3) = 0$
n is odd	$e_f(1) = \frac{mn-1}{2}, e_f(-1) = \frac{mn+1}{2}$ $e_f(2) = \frac{n-1}{2} = e_f(-2)$	$v_f(1) = \frac{(m+1)n}{2},$ $v_f(-1) = \frac{(m+1)n-2}{2}$ $v_f(3) = 0, v_f(-3) = 1$

Hence, $P_n \odot mK_1$ is $H_3 -$ cordial.

Example 2.22 $P_4 \odot 4K_1$ is H_3 −cordial shown in Figure 11.

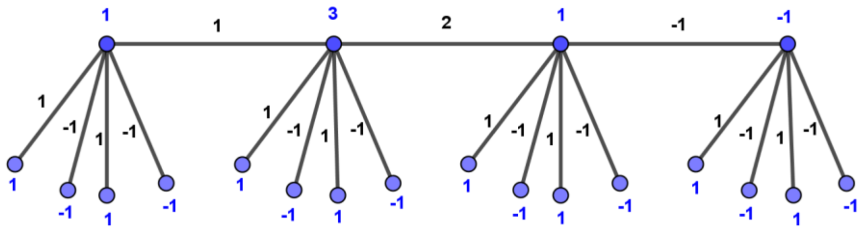

Figure 11: $P_4 \odot 4K_1$

Theorem 2.23 Crown $C_n \odot K_1$ is $H -$ cordial.

Proof: Let C_n be a cycle with vertices $u_1, u_2, \ldots u_n$ with $u_{n+1} = u_1$. $C_n \odot K_1$ is obtained from cycle C_n by attaching pendant edge to each vertex. Let $V = \{u_i, u'_i : 1 \le i \le n\}$ and $E = \{u_i u_{i+1}, u_i u'_i : 1 \le i \le n, u_{n+1} = u_1\}$ are vertex and edge set of graph $C_n \odot K_1$. Consider a function $f: E \longrightarrow \{-1,1\}$ defined as

$f(u_i u_{i+1}) = 1 ; 1 \leq i \leq n$
$f(u_i u'_i) = -1 ; 1 \leq i \leq n$

$n \geq 3$	Edge Condition	Vertex Condition
n	$e_f(1) = n = e_f(-1)$	$v_f(1) = n = v_f(-1)$

Hence, $C_n \odot K_1$ is $H -$ cordial.
Example 2.24 $C_5 \odot K_1$ is $H -$cordial shown in Figure 12.

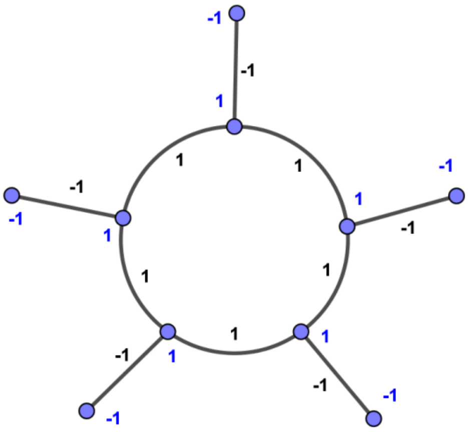

Figure 12: $C_5 \odot K_1$

Theorem 2.25 The $C_n \odot mK_1$ is $H -$ cordial if m is odd $(n \geq 3)$.
Proof: Let C_n be a cycle with vertices $u_1, u_2, \dots u_n$ with $u_{n+1} = u_1$. $C_n \odot mK_1$ is obtained from cycle C_n by attaching $m -$ pendant edge to each vertex. Let $V = \{u_i, u_{ij} : 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E = \{u_i u_{i+1}, u_i u_{ij} : 1 \leq i \leq n, 1 \leq j \leq m, u_{n+1} = u_1\}$ are vertex and edge set of graph $C_n \odot mK_1$.
Consider a function $f : E \longrightarrow \{-1,1\}$ defined as
$f(u_i u_{i+1}) = 1 ; 1 \leq i \leq n$
$f(u_i u_{i1}) = -1 ; 1 \leq i \leq n$
$f(u_i u_{ij}) = (-1)^j ; 1 \leq i \leq n, 2 \leq j \leq m.$

m	Edge Condition	Vertex Condition
m is odd	$e_f(1) = \frac{n(m+1)}{2} = e_f(-1)$	$v_f(1) = \frac{n(m+1)}{2} = v_f(-1)$

Hence, $C_n \odot mK_1$ is $H -$ cordial.
Example 2.26 $C_5 \odot 3K_1$ is $H -$cordial shown in Figure 13.

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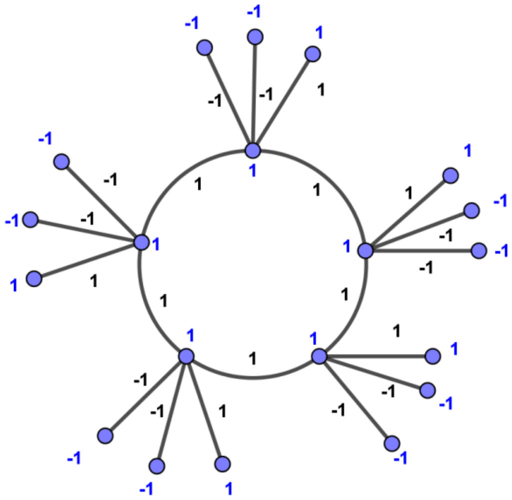

Figure 13: $C_5 \odot 3K_1$

Theorem 2.27 The $C_n \odot mK_1$ is H_3- cordial if m is even($n \geq 4$).

Proof: Let C_n be a cycle with vertices $u_1, u_2, \dots u_n$ with $u_{n+1} = u_1$. $C_n \odot mK_1$ is obtained from cycle C_n by attaching $m-$ pendant edge to each vertex. Let $V = \{u_i, u_{ij} : 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E = \{u_i u_{i+1}, u_i u_{ij} : 1 \leq i \leq n, 1 \leq j \leq m, u_{n+1} = u_1\}$ are vertex and edge set of graph $C_n \odot mK_1$.

Consider a function $f: E \longrightarrow \{-2, -1, 1, 2\}$ defined as

$$f(u_i u_{i+1}) = \begin{cases} 1 & ; 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1 \\ 2 & ; i = \left\lceil \frac{n}{2} \right\rceil \\ -1 & ; \left\lceil \frac{n}{2} \right\rceil + 1 \leq i \leq n-1 \end{cases}$$

$f(u_n u_1) = -2$,

$f(u_i u_{ij}) = (-1)^j; 1 \leq i \leq n, 1 \leq j \leq m.$

m is even	Edge Condition	Vertex Condition
n is odd	$e_f(1) = \frac{n(m+1)-1}{2}$, $e_f(-1) = \frac{n(m+1)-3}{2}$ $e_f(2) = 1 = e_f(-2)$	$v_f(1) = \frac{nm+2}{2} = v_f(-1)$ $v_f(2) = \frac{n-3}{2}, v_f(-2) = \frac{n-5}{2}$ $v_f(3) = 1 = v_f(-3)$
n is even	$e_f(1) = \frac{n(m+1)-2}{2} = e_f(-1)$ $e_f(2) = 1 = e_f(-2)$	$v_f(1) = \frac{nm+2}{2} = v_f(-1)$ $v_f(2) = \frac{n-4}{2} = v_f(-2)$ $v_f(3) = 1 = v_f(-3)$

Hence, $C_n \odot mK_1$ is H_3- cordial.

Example 2.28 $C_4 \odot 4K_1$ is H_3- cordial shown in Figure 14.

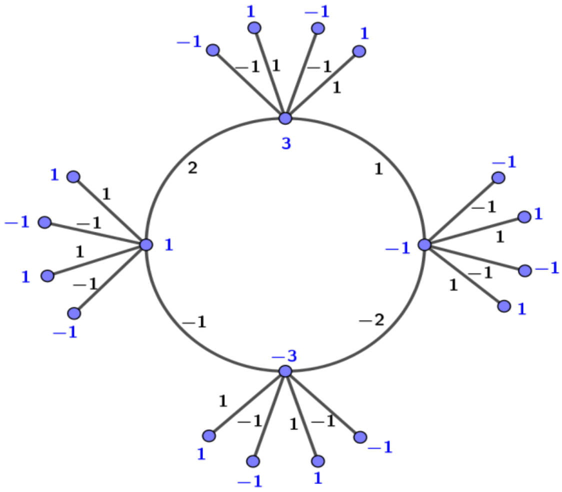

Figure 14: $C_4 \odot 4K_1$

Theorem 2.29 Circular ladder graph CL_n , $n \geq 3$ is $H-$ cordial if n is even.

Proof: Let $= \{u_i, v_i, 1 \leq i \leq n\}$ and $E = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_i : 1 \leq i \leq n$ with $u_{n+1} = u_1, v_{n+1} = v_1\}$ are vertex and edge set of graph CL_n .

Consider function $f: E \longrightarrow \{-1,1\}$ defined as

$f(u_i u_{i+1}) = (-1)^{i+1}$; $1 \leq i \leq n$,

$f(v_i v_{i+1}) = (-1)^{i+1}$; $1 \leq i \leq n$,

$f(u_i v_i) = (-1)^{i+1}$; $1 \leq i \leq n$.

$n \geq 3$	Edge Condition	Vertex Condition
n is even	$e_f(1) = \frac{3n}{2} = e_f(-1)$	$v_f(1) = n = v_f(-1)$

Hence, CL_n is $H-$ cordial if n is even.

Example 2.30 CL_6 is H_3- cordial shown in Figure 15.

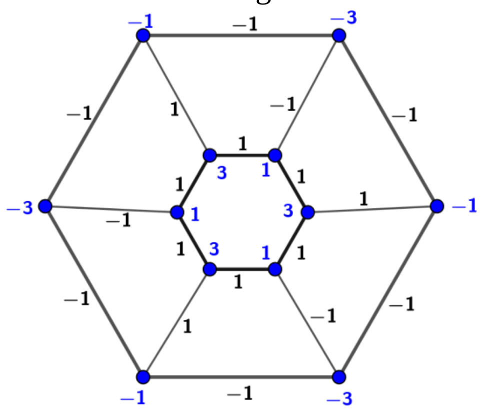

Figure 15: CL_6

Theorem 2.31 Circular ladder graph CL_n , $n \geq 3$ is H_3- cordial.

Proof: Let $= \{u_i, v_i, 1 \leq i \leq n\}$ and $E = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_i : 1 \leq i \leq n$ with $u_{n+1} = u_1, v_{n+1} = v_1\}$ are vertex and edge set of graph CL_n .

Case 1: If n is even then it is $H-$ cordial with $|f(v)| = 1$.Therefore it is H_2- cordial and also H_3- cordial.

Case 2: If n is odd then

Consider function $f: E \longrightarrow \{-1,1\}$ defined as

$f(u_i u_{i+1}) = 1$; $1 \leq i \leq n$,

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$f(v_i v_{i+1}) = -1\,; 1 \le i \le n,$
$f(u_i v_i) = (-1)^{i+1}\,; 1 \le i \le n.$

$n \ge 3$	Edge Condition	Vertex Condition
n is odd	$e_f(1) = \frac{3n+1}{2}, e_f(-1) = \frac{3n-1}{2}$	$v_f(1) = \frac{n-1}{2}, v_f(-1) = \frac{n+1}{2}$ $v_f(3) = \frac{n+1}{2}, v_f(-3) = \frac{n-1}{2}$

Hence, CL_n is $H_3 -$ cordial.

Example 2.32 CL_5 is $H_3 -$ cordial shown in Figure 16.

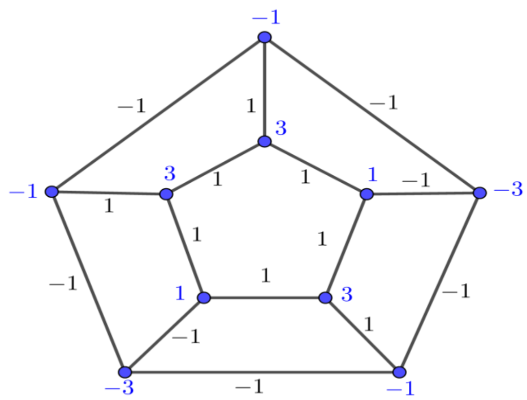

Figure 16: CL_5

3. CONCLUSION

In this paper we have proved that H- graph, Kite graph, Ladder graph ,Comb, Crown and $T_n \odot K_1, H \odot K_1$, $K_{3,m} \odot K_1$, $L_{n,1} \odot K_1$ graph are $H_K -$ cordial labeling.

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