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Group Modelling of the causal influence of personal beliefs on Self-esteem using Fuzzy Relational Map based on Linguistic Interval-valued Pythagorean fuzzy sets

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Abstract – One of the highly challenging tasks in modelling complex systems that involve experts' opinion is quantifying the uncertainty. Linguistic expressions of human perception that describe the causal relationship among concepts in a fuzzy relational map has to be captured with proper tools in order to construct efficient models. Fuzzy sets characterise the uncertainty and subjectivity involved in the modelling process. In order to include and accommodate imprecise information efficiently several extensions of fuzzy sets have been introduced. Pythagorean fuzzy sets is one of the extensions that is more effective in including uncertainty and vagueness. In this paper, a new approach of constructing fuzzy relational map based on Linguistic Interval Valued Pythagorean Fuzzy Sets is proposed. The linguistic interval-valued Pythagorean fuzzy relational map is used to analyse the influence of self-beliefs on the characteristics of self-esteem.

Index terms – Fuzzy relational map, Pythagorean fuzzy set, linguistic term, interval-valued, aggregation operators, self-beliefs, self-esteem

1. INTRODUCTION

Fuzzy Relational Map is a soft computing technique to analyse the complex and reasoning problems that deals with high level uncertainty. In most of the real-life situations, linguistic terms are used to describe the influence of one factor on another. Representing the linguistic terms using real numbers is inadequate as it failed to include the uncertainty and ambiguity in human insight. The fuzzy sets introduced by L. A. Zadeh in 1965 are capable of representing the vagueness and imprecision in the user provided data with the help of membership grades. In order to make an efficient use of fuzzy concept several non-standard second order fuzzy sets have been introduced since the introduction of fuzzy sets. Intuitionistic fuzzy sets (IFS), proposed by Atanassov in 1985, is an extension of fuzzy set which included the non-membership degree of an element belonging to a set.

The intuitionistic fuzzy sets satisfy the limiting condition that the sum of the membership and non-membership degrees of an element is less than or equal to 1. Consequently, many generalised Intuitionistic and interval-valued intuitionistic fuzzy sets are proposed such that the enhanced extensions of fuzzy sets that capture the uncertainty in the form of membership, non-membership and hesitancy degrees of elements belonging to a particular set. A new class of non-standard fuzzy subsets called Pythagorean fuzzy set (PFS) was introduced by Yager in 2013 [1]. The Pythagorean fuzzy sets satisfy the condition that sum of the squares of membership and non-membership degrees is less than or equal to 1. The membership grades associated with Pythagorean fuzzy set allows some uncertainty in assigning the membership degrees and it provides a type of imprecise membership grades generally referred to as type-2.

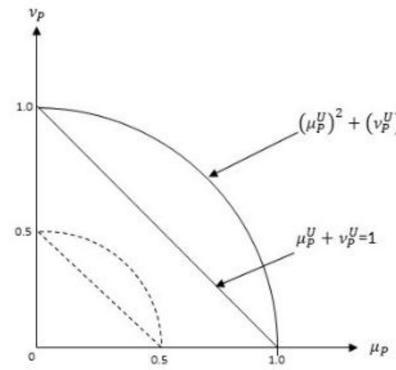

Figure 1: Constraints of IVPFN and IVIFN

The intuitionistic membership degrees are all points under the line $x + y \leq 1$ and the Pythagorean membership degrees are all points under the curve $x^2 + y^2 \leq 1$ [1]. The difference between Intuitionistic fuzzy sets (IFS) and Pythagorean fuzzy sets (PFS) is their corresponding constraints which is show in Figure-1. The space of Pythagorean membership grades is greater than that of intuitionistic membership grades. Every intuitionistic membership grade is also a Pythagorean membership grade but not all Pythagorean membership grades are intuitionistic membership grades. Consequently, Pythagorean fuzzy sets can be used in situations where intuitionistic fuzzy sets cannot be used. [2].

Pythagorean Fuzzy Sets – Preliminaries

Let X be a fixed set. A Pythagorean Fuzzy Set (PFS) [1] is of the form

$P = \{\langle x, \ (\mu_P(x), \nu_P(x))\rangle | x \in X\})$ (1)

where $\mu_P(x), \nu_P(x): X \to [0,1]$ are the degree of membership and non-membership of the element $x \in X$ respectively with a condition that for $x \in X$, $0 \leq \mu_P^2(x) + \nu_P^2(x) \leq 1$.

For every P (PFS) in X, $\pi_P(x) = \sqrt{1 - \left[(\mu_P(x))^2 + \left(\nu_P(x)\right)^2\right]}$, is called the degree of hesitation of $x \in X$ to P.

A pair $\mathcal{P} = (\mu_P, \nu_P)$ where $(\mu_P, \nu_P) \in [0,1]$
with $0 \leq \mu_P^2 + \nu_P^2 \leq 1$ is called a Pythagorean Fuzzy Number (PFN) [3].

Peng and Yang introduced interval-valued Pythagorean fuzzy set (IVPFS) [4] in 2015. An interval-valued Pythagorean fuzzy set is defined as

$P = \{\langle x, ([\mu_P^L(x), \mu_P^U(x)], [\nu_P^L(x), \nu_P^U(x)])\rangle | x \in X\}$ (2)

Where $0 \leq \mu_P^L \leq \mu_P^U \leq 1$, $0 \leq \nu_P^L \leq \nu_P^U \leq 1$
and $0 \leq (\mu_P^U)^2 + (\nu_P^U)^2 \leq 1$ for all $x \in X$.

A pair $P = ([\mu_P^L, \mu_P^U], [\nu_P^L, \nu_P^U])$ is called interval-valued Pythagorean fuzzy number (IVPFN) with $[\mu_P^L, \mu_P^U], [\nu_P^L, \nu_P^U] \in [0,1]$ and $(\mu_P^U)^2 + (\nu_P^U)^2 \leq 1$, where $[\mu_P^L, \mu_P^U], [\nu_P^L, \nu_P^U]: X \to [0,1]$ are the degree of membership and non-membership of the element $x \in X$ respectively with a condition that for $x \in X$, $0 \leq \left(\mu_P^U(x)\right)^2 + (\nu_P^U(x))^2 \leq 1$. The degree of hesitant membership is defined as

$$\pi_P(x) = [\pi_P^L(x), \pi_P^U(x)] = \begin{bmatrix} \sqrt{1 - \left[\left(\mu_P^U(x)\right)^2 + \left(\nu_P^U(x)\right)^2\right]}, \\ \sqrt{1 - \left[(\mu_P^L(x))^2 + \left(\nu_P^L(x)\right)^2\right]} \end{bmatrix}$$

Some of the basic operations on interval-valued Pythagorean fuzzy number are as follows [4]. Let $P_1 = ([\mu_1^L, \mu_1^U], [\nu_1^L, \nu_1^U])$ and $P_2 = ([\mu_2^L, \mu_2^U], [\nu_2^L, \nu_2^U])$ be two IVPFNs.

$$P_1 \oplus P_2 = \left(\begin{bmatrix} \sqrt{\left(\mu_1^L(x)\right)^2 + \left(\mu_2^L(x)\right)^2 - \left(\mu_1^L(x)\right)^2 \cdot \left(\mu_2^L(x)\right)^2}, \\ \sqrt{\left(\mu_1^U(x)\right)^2 + \left(\mu_2^U(x)\right)^2 - \left(\mu_1^U(x)\right)^2 \cdot \left(\mu_2^U(x)\right)^2} \end{bmatrix}, \\ [\nu_1^L(x) \cdot \nu_2^L(x), \nu_1^L(x) \cdot \nu_2^L(x)] \right) \quad (3)$$

$$P_1 \otimes P_2 = \left(\begin{array}{c} [\mu_1^L(x) \cdot \mu_2^L(x), \mu_1^L(x) \cdot \mu_2^L(x)], \\ \begin{bmatrix} \sqrt{\left(\nu_1^L(x)\right)^2 + \left(\nu_2^L(x)\right)^2 - \left(\nu_1^L(x)\right)^2 \cdot \left(\nu_2^L(x)\right)^2}, \\ \sqrt{\left(\nu_1^U(x)\right)^2 + \left(\nu_2^U(x)\right)^2 - \left(\nu_1^U(x)\right)^2 \cdot \left(\nu_2^U(x)\right)^2} \end{bmatrix} \end{array} \right) \quad (4)$$

The linguistic variable (LV) is defined as $S = \{s_t | t = 0, 1, \ldots \ldots, h\}$ with odd cardinality be a linguistic term set (LTS) where s_h has the following characteristics [5].

1) $s_k \leq s_t \Leftrightarrow k \leq t$
2) Negation $(s_k) = s_{h-k}$
3) $max\,\{s_k, s_t\} = s_{\max(k,t)}$
4) $min\,\{s_k, s_t\} = s_{\min(k,t)}$

Linguistic interval-valued Pythagorean fuzzy set (LIVPFS) was introduced as a generalisation of Linguistic Pythagorean fuzzy set by Garg in 2020 [6]. A LIVPFS is defined as

$$\mathcal{P} = \left\{ \langle x, \begin{pmatrix} \left[s_{\mu_{\mathcal{P}}^L}(x), s_{\mu_{\mathcal{P}}^U}(x)\right], \\ \left[s_{\nu_{\mathcal{P}}^L}(x), s_{\nu_{\mathcal{P}}^U}(x)\right] \end{pmatrix} \rangle \middle| x \in X \right\}$$

(5)

where the pairs represent the interval-valued membership and non-membership degree such that $(\mu_P^U)^2 + (\nu_P^U)^2 \leq h$ for all $x \in X$.

A pair $\mathcal{P} = \left(\left[s_{\mu_{\mathcal{P}}^L}, s_{\mu_{\mathcal{P}}^U}\right], \left[s_{\nu_{\mathcal{P}}^L}, s_{\nu_{\mathcal{P}}^U}\right]\right)$ of the membership degrees is called as linguistic interval-valued Pythagorean fuzzy number (LIVPFN). For convenience, LIVPFN is denoted

as $\mathcal{P} = ([s_a, s_b], [s_c, s_d])$ where $a, b, c, d \in [0, h]$ and $b^2 + d^2 \leq h^2$. Some of the basic operations on Linguistic interval-valued Pythagorean fuzzy number are given below [6].
Let $\mathcal{P}_1 = \left([s_{a_1}, s_{b_1}], [s_{c_1}, s_{d_1}]\right)$ and $\mathcal{P}_2 = \left([s_{a_2}, s_{b_2}], [s_{c_2}, s_{d_2}]\right)$ be two IVPFNs.

$$\mathcal{P}_1 \oplus \mathcal{P}_2 = \left(\begin{array}{c} \left[s_{h\sqrt{1-\left(1-\frac{a_1^2}{h^2}\right)\left(1-\frac{a_2^2}{h^2}\right)}}, s_{h\sqrt{1-\left(1-\frac{b_1^2}{h^2}\right)\left(1-\frac{b_2^2}{h^2}\right)}} \right], \\ \left[s_{h\left(\frac{c_1 c_2}{h^2}\right)}, s_{h\left(\frac{d_1 d_2}{h^2}\right)} \right] \end{array} \right) \quad (6)$$

$$\mathcal{P}_1 \otimes \mathcal{P}_2 = \left(\begin{array}{c} \left[s_{h\left(\frac{a_1 a_2}{h^2}\right)}, s_{h\left(\frac{b_1 b_2}{h^2}\right)} \right], \\ \left[s_{h\sqrt{1-\left(1-\frac{c_1^2}{h^2}\right)\left(1-\frac{c_2^2}{h^2}\right)}}, s_{h\sqrt{1-\left(1-\frac{d_1^2}{h^2}\right)\left(1-\frac{d_2^2}{h^2}\right)}} \right] \end{array} \right) \quad (7)$$

Let $\{\mathcal{P}_i\}$ be a collection of 'n' LIVPFNs. The linguistic interval-valued Pythagorean fuzzy ordered weighted average operator (LIVPFOWA) [6] is used to aggregate the linguistic terms. The LIVPFOWA operator is a map $LIVPFOWA: \Omega^n \to \Omega$ defined by

$$LIVPFOWA(\mathcal{P}_1, \mathcal{P}_2 \dots, \mathcal{P}_n) = \oplus_{i=1}^{n} w_i \mathcal{P}_{\sigma(i)} \quad (8)$$

where $\mathcal{P}_{\sigma(i)}$ is the i^{th} largest LIVPFN and $\sum_{i=1}^{n} w_i = 1$. For a collection of 'n' LIVPFNs $\mathcal{P} = \left([s_{a_i}, s_{b_i}], [s_{c_i}, s_{d_i}]\right)$ the aggregated value is also given by

$$\left(\begin{array}{c} \left[s_{h\sqrt{1-\prod_{i=1}^{n}\left(1-\left(\frac{a_{\sigma(i)}^2}{h^2}\right)\right)^{w_i}}}, s_{h\sqrt{1-\prod_{i=1}^{n}\left(1-\left(\frac{a_{\sigma(i)}^2}{h^2}\right)\right)^{w_i}}} \right], \\ \left[s_{h\left(\prod_{i=1}^{n}\left(\frac{c_{\sigma(i)}}{h}\right)^{w_i}\right)}, s_{h\left(\prod_{i=1}^{n}\left(\frac{d_{\sigma(i)}}{h}\right)^{w_i}\right)} \right] \end{array} \right) \quad (9)$$

Let $A = ([\mu_A^L(x),\ \mu_A^U(x)], [\nu_A^L(x),\ \nu_A^U(x)])$ and $B = ([\mu_B^L(x),\ \mu_B^U(x)], [\nu_B^L(x),\ \nu_B^U(x)])$ be two interval valued Pythagorean fuzzy sets. Then the weighted normalised Euclidean distance [7] between two interval valued Pythagorean fuzzy sets A and B is given by

$$D(A, B) = \sqrt{\frac{1}{2n} \sum_{i=1}^{n} w_i \left(\begin{array}{c} \left| \left(\mu_A^L(c_i)\right)^2 - \left(\mu_B^L(c_i)\right)^2 \right|^2 \\ + \left| \left(\mu_A^U(c_i)\right)^2 - \left(\mu_B^U(c_i)\right)^2 \right|^2 \\ + \left| \left(\nu_A^L(c_i)\right)^2 - \left(\nu_B^L(c_i)\right)^2 \right|^2 \\ + \left| \left(\nu_A^L(c_i)\right)^2 - \left(\nu_B^L(c_i)\right)^2 \right|^2 \end{array} \right)} \quad (10)$$

Pythagorean Fuzzy Relational Map Model

Fuzzy Relational Map (FRM) is a generalisation of Fuzzy Cognitive Map (FCM) that can model the causal influence among the concepts [8]. The difference between FRM and FCM is that FRM models the influence between two disjoint sets while FCM discusses the influence among the concepts of the same set that are concurrently active. Like FCMs, FRMs also efficient in modelling complex and highly non-linear systems. An FRM is dynamical structure that is represented quantifiable concepts from two disjoint sets and the causal links between different concepts.

In FRMs, the state of nodes in domain space is represented by a state vector d^k with an instantaneous state value d_i^{k+1} for each concept, where k denotes the index of iteration. The concept value for the next iteration k+1 can be calculated as follows.

$$d_i^{k+1} = \boldsymbol{f}\left(d_i^k + \sum_{j=1}^{n} r_j^k \cdot w_{ji}\right) \qquad (11)$$

where $r_j^k = \boldsymbol{f}\left(\sum_{i=1}^{m} d_i^k \cdot {w_{ji}}^T\right)$ where $d_i^k, i = 1, \dots \dots, m$ and $r_j^k, j = 1, \dots \dots, n$ are the concepts in domain space and range space respectively, w_{ji} is the strength of the influence of the concept d_i^k on r_j^k and f is a non-linear activation function such as sigmoid or hyperbolic type of function.

The conventional FRMs use real numbers to represent the membership value of an element which may be inadequate to express the uncertainty and in reality, may lose some of the information. The linguistic terms in the place of real numbers are much more capable of capturing all the information provided by the stakeholders. Interval valued Pythagorean fuzzy sets are more sophisticated to take into account the membership, non-membership and hesitancy degrees of elements. Applying the addition and multiplication operators for IVPFS from (3) and (4), the inference in conventional FRM defined by (10) can be reformulated as follows:

$$d_i^{k+1} = \{[\mu_P^L(d), \mu_P^U(d)], [\nu_P^L(x), \nu_P^U(d)]\}_{\boldsymbol{i}}^{\boldsymbol{k+1}}$$
$$= \boldsymbol{f}\left(\begin{array}{c} \{[\mu_P^L(d), \mu_P^U(d)], [\nu_P^L(d), \nu_P^U(d)]\}_i^k \\ \oplus \left(\begin{array}{c} \oplus_{j=1}^{n} \{[\mu_P^L(r), \mu_P^U(r)], [\nu_P^L(r), \nu_P^U(r)]\}_j^k \\ \otimes\{[\mu_P^L(w), \mu_P^U(w)], [\nu_P^L(w), \nu_P^U(w)]\}_{ji} \end{array}\right) \end{array}\right) \qquad (12)$$

Where $\{[\mu_P^L(r), \mu_P^U(r)], [\nu_P^L(r), \nu_P^U(r)]\}_j^k$

$$= \boldsymbol{f}\left(\left(\begin{array}{c} \oplus_{j=1}^{n} \{[\mu_P^L(r), \mu_P^U(r)], [\nu_P^L(r), \nu_P^U(r)]\}_j^k \\ \otimes\{[\mu_P^L(w), \mu_P^U(w)], [\nu_P^L(w), \nu_P^U(w)]\}_{ji}^T \end{array}\right)\right)$$

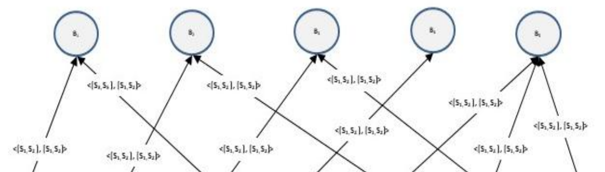

Figure 2: An illustrative example of LIVPFRM

Description of the Problem

Self-esteem is one of the important psychological constructs that has become a household word in the recent past [9]. Self-esteem is a primary factor in the building and maintenance of social and emotional well-being. In simple words, Self-esteem is about feeling lovable and competent [10]. A child who has a healthy level of self-esteem is more likely to achieve at her full potential and form successful relationships than those who suffer from acute lack of self-worth [10]. Parents and teachers believe that children with healthy level of self-esteem can perform much better in academics and in other academic related activities. Perceived reactions from others, particularly significant others, is an important element of self-esteem.

Several researches have demonstrated that boosting self-esteem have no impact on their performance, instead one's beliefs about themselves have important consequences regardless of the underlying realities [9]. Lipton (2015) explains in his book 'The Biology of Belief', "Thoughts,

Notation	Factors of Domain Space	Notation	Factors of Range Space
B_1	I matter	S_1	Confident
B_2	I am competent	S_2	Resilient
B_3	I can stand it	S_3	Responsible
B_4	I can trust others	S_4	Optimistic
B_5	I am good enough	S_5	Secure
		S_6	Involved
		S_7	Healthy relationship

Table I: Factors of LIVPFRM

the mind's energy, directly influence how the physical brain controls the body's physiology". People's personal beliefs shape their actions in many important ways and these actions in turn shape their social reality and that of the people around them [11]. Hence, it can be said that there is a constant and consistent interaction between personal beliefs and self-esteem. In this paper, a study is conducted to demonstrate the relationship between the traits of self-esteem and personal beliefs using fuzzy relational map based of linguistic interval valued Pythagorean fuzzy sets.

Self-esteem is basically a combination of several specific characteristics of an individual. Many a times, an individual may be unaware of these characteristics, but they may keep assigning negative or positive values to each characteristic and add them together and this results in a general evaluation of the self [12]. Hence, these characteristics determine the level of 'healthy' self-esteem that plays an important role in the performance of an individual. Out of a list of constituent characteristics, the experts chose certain characteristics as important factors that describe self-esteem and self-beliefs that influence these characteristics [12], [14]. The factors of the domain and range space of the FRM model is given in Table-I.

Construction of LIVPFRM

The algorithm for construction and analysis of LIVPFRM is briefly described in the following steps.

Step 1: The factors that constitute the nodes of the domain space and range space of the LIVPFRM are chosen with the help of the experts. These variables of the problem taken for study are given in Table-I.

Step 2: The linguistic term set (LTS) is constructed with the linguistic assessments. The linguistic terms and their corresponding membership values is illustrated in Figure-3.

Step 3: The relationship between nodes is obtained from participants based on their domain knowledge. Using the interval-valued linguistic evaluations that describe the causal relations between concepts of graph-based Pythagorean FRM model is constructed. An illustrative example of LIVPFRM is represented in figure-3.

Step 4: Let $\omega_j > 0$ be the weight of each stakeholder such that $\sum_{j=1}^{n} \omega_j = 1$. The interval-valued linguistic evaluations of the stakeholders are aggregated using linguistic interval valued Pythagorean fuzzy ordered weighted average operator (LIVPFOWA) [6], [14], [15] method given by the equation (9).
Step 5: The membership values of the aggregated interval valued linguistic terms is calculated. The resulting values are interval valued Pythagorean fuzzy number.
Step 6: The values obtained in the above step is taken as the edge strength of causal relation between the variables of domain space and range space of FRM model. These causal values of the edges constitute the adjacency matrix of the interval valued Pythagorean FRM.

Step 7: The desired values of the concepts of the domain space given in Table-V were defined by the experts.
Step 8: Using the Initial state vector of the concepts from Table-VI and the edge strength from the relational matrix (Table-IV), the LIVPFRM were simulated using the formula given in equation (12) for each stakeholder until the steady state is reached. Hyperbolic tangent functions were used as activation functions. The resultant state vector values of the concepts of domain space obtained after several iterations (around 28 plus iterations) are presented in Table-VII.
Step 9: The weight vector of each concept provided by the experts is used to compare the output values with the desired values of the concepts using the weighted normalised Euclidean distance between the interval valued Pythagorean fuzzy sets using the formula given in equation (10)

A. Analysis of the problem using LIVPFRM

The stakeholders namely a teacher (P_1), a parent (P_2) and a student (P_3) are selected to participate in this group modelling process using FRM. Each of them was asked to construct an FRM based on linguistic interval valued Pythagorean fuzzy sets independently. These user-provided FRMs are used to construct the FRM and study the causal interference. The seven-point linguistic term set $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$ is adopted to describe the causal relations between the concepts. The stakeholders involved in the group modelling are asked to assign interval values of linguistic terms for each edge between the domain and range spaces of the FRM.

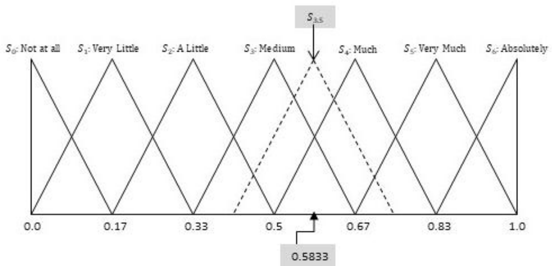

Figure 3: Linguistic terms

The relational matrices provided by the teacher (P_1), parent (P_2) and student (P_3) are $R^{(1)}$, $R^{(2)}$, and $R^{(3)}$ respectively. These relational matrices of FRM model, constructed with the LIVFLTS provided by the experts, is given in Table-II.

The weight information of the stakeholders is given by $\omega = (0.30, 0.45, 0.25)$. The Aggregated values of interval valued linguistic terms are obtained by applying the LIVPOWA method and it is given in Table-III. The corresponding membership degrees of the aggregated values of interval valued linguistic terms is given in Table-IV. At this stage the interval valued Pythagorean fuzzy numbers (IVPFN) denote the edge strength of FRM.

The desired values of the concepts of the domain space of FRM were defined by the experts in terms of interval valued Pythagorean fuzzy numbers (IVPFN) and it is given in Table-V. The initial values of the concepts of the domain space of FRM provided by the three participants who take part in the modelling process is given in Table-VI. The initial values of the concepts of the domain space of FRM provided by the participants is passed through the relational matrix using the operations on interval valued Pythagorean numbers defined in equation (3) & (4) is given in Table-VI.

2. RESULTS AND DISCUSSION

Using the weighted normalised Euclidean distances D_T, D_P and D_S for the participants is obtained, where $D_T = 0.0951, D_P = 0.1027$ and $D_S = 0.1152$. According to the final scores the appropriate assessment is done by the teacher (P_1) as $P_1 \succ P_2 \succ P_3$. From the steady state of the resultant vector, it can be inferred that all the nodes of the concepts in the output values attain the same state with respect to the membership value except for the non-membership values corresponding to the nodes $B_2, B_3,, B_4, B_5$. The suggests that these four beliefs are considered to be the most important in influencing the characteristics of self-esteem.

3. CONCLUSION

Problems associated with social or psychological sciences where human cognition is involved in assessing the factors are very challenging to model. Finding tools that would incorporate the complete view and understanding of the human perception is central to modelling adaptive complex systems such as social or psychological constructs. In such systems, linguistic terms that would accurately reflect the human perception would be of great help in constructing and analysing the complex systems. In this paper, the interval value of linguistic terms is incorporated with Pythagorean fuzzy sets in order to quantify the linguistic expressions of the participants in constructing the fuzzy relational map. By the very nature and operations of Pythagorean fuzzy sets, the LIVPFRM included more information as it allows more space and thus the uncertainty is reduced to a certain extent.

.		E_1	E_2	E_3	E_4	E_5	E_6	E_7
	B_1	([s_3, s_5], [s	([s_2, s_5], [s	([s_3, s_5], [s	([s_4, s_5], [s	([s_4, s_5], [s	([s_2, s_4], [s	([s_2, s_3], [s
	B_2	([s_3, s_5], [s	([s_2, s_3], [s	([s_4, s_5], [s	([s_4, s_5], [s	([s_1, s_2], [s	([s_2, s_3], [s	([s_2, s_4], [s
$R^{(1}$	B_3	([s_3, s_5], [s	([s_3, s_5], [s	([s_3, s_5], [s	([s_2, s_4], [s	([s_4, s_5], [s	([s_1, s_3], [s	([s_1, s_3], [s
	B_4	([s_2, s_4], [s	([s_2, s_4], [s	([s_2, s_3], [s	([s_1, s_3], [s	([s_2, s_4], [s	([s_3, s_5], [s	([s_4, s_5], [s
	B_5	([s_3, s_5], [s	([s_4, s_5], [s	([s_3, s_5], [s	([s_4, s_5], [s	([s_3, s_4], [s	([s_4, s_5], [s	([s_3, s_5], [s
	B_1	([s_3, s_5], [s	([s_3, s_4], [s	([s_3, s_4], [s	([s_3, s_4], [s	([s_2, s_5], [s	([s_3, s_5], [s	([s_2, s_3], [s
	B_2	([s_3, s_5], [s	([s_2, s_4], [s	([s_4, s_5], [s	([s_4, s_5], [s	([s_2, s_3], [s	([s_1, s_3], [s	([s_2, s_3], [s
$R^{(2}$	B_3	([s_2, s_4], [s	([s_3, s_5], [s	([s_2, s_5], [s	([s_2, s_3], [s	([s_3, s_5], [s	([s_2, s_4], [s	([s_2, s_3], [s
	B_4	([s_3, s_4], [s	([s_2, s_3], [s	([s_2, s_4], [s	([s_2, s_3], [s	([s_2, s_5], [s	([s_3, s_5], [s	([s_3, s_5], [s

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	B_5	$([s_3, s_5], [s$	$([s_4, s_5], [s$	$([s_2, s_4], [s$	$([s_4, s_5], [s$	$([s_4, s_5], [s$	$([s_3, s_5], [s$	$([s_4, s_5], [s$
	B_1	$([s_2, s_4], [s$	$([s_2, s_4], [s$	$([s_3, s_5], [s$	$([s_2, s_5], [s$	$([s_3, s_5], [s$	$([s_2, s_4], [s$	$([s_3, s_5], [s$
	B_2	$([s_4, s_5], [s$	$([s_4, s_5], [s$	$([s_2, s_5], [s$	$([s_3, s_4], [s$	$([s_4, s_5], [s$	$([s_2, s_5], [s$	$([s_3, s_5], [s$
$R^{(3}$	B_3	$([s_3, s_5], [s$	$([s_3, s_4], [s$	$([s_2, s_4], [s$	$([s_3, s_4], [s$	$([s_4, s_5], [s$	$([s_2, s_4], [s$	$([s_3, s_5], [s$
	B_4	$([s_2, s_4], [s$	$([s_1, s_3], [s$	$([s_2, s_4], [s$	$([s_4, s_5], [s$	$([s_3, s_5], [s$	$([s_3, s_5], [s$	$([s_4, s_5], [s$
	B_5	$([s_2, s_4], [s$	$([s_3, s_5], [s$	$([s_3, s_5], [s$	$([s_3, s_5], [s$	$([s_2, s_5], [s$	$([s_3, s_5], [s$	$([s_3, s_5], [s$

Table II: Relational matrix in terms of IVFLTS

	E_1	E_2	E_3	E_4	E_5	E_6	E_7
B_1	$([s_{2.7979}, s_{4.}$	$([s_{5.1496}, s_{5.}$	$([s_{5.5852}, s_{5.}$	$([s_{5.8456}, s_{5.}$	$([s_{5.8456}, s_{5.}$	$([s_{4.8330}, s_{5.}$	$([s_{4.8330}, s_{5.}$
	$[s_{2.7179}, s_{3.6}$	$[s_{0.0113}, s_{0.2}$	$[s_{0.0321}, s_{0.5}$	$[s_{0.0019}, s_{0.0}$	$[s_{0.0019}, s_{0.0}$	$[s_{0.0278}, s_{0.2}$	$[s_{0.0556}, s_{0.3}$
B_2	$([s_{3.3652}, s_{4.}$	$([s_{5.7407}, s_{5.}$	$([s_{5.9655}, s_{5.}$	$([s_{5.9100}, s_{5.}$	$([s_{5.2633}, s_{5.}$	$([s_{4.3835}, s_{5.}$	$([s_{5.3393}, s_{5.}$
	$[s_{3.1766}, s_{4.2}$	$[s_{0.0151}, s_{0.4}$	$[s_{0.0031}, s_{0.0}$	$[s_{0.0113}, s_{0.1}$	$[s_{0.0556}, s_{0.4}$	$[s_{0.0370}, s_{0.3}$	$[s_{0.0053}, s_{0.0}$
B_3	$([s_{2.7979}, s_{4.}$	$([s_{5.5852}, s_{5.}$	$([s_{5.1496}, s_{5.}$	$([s_{4.8330}, s_{5.}$	$([s_{5.9502}, s_{5.}$	$([s_{4.4364}, s_{5.}$	$([s_{5.2434}, s_{5.}$
	$[s_{2.4495}, s_{3.6}$	$[s_{0.0113}, s_{0.2}$	$[s_{0.0113}, s_{0.1}$	$[s_{0.0556}, s_{0.4}$	$[s_{0.0741}, s_{0.3}$	$[s_{0.0053}, s_{0.1}$	$[s_{0.0046}, s_{0.1}$
B_4	$([s_{2.3644}, s_{3.}$	$([s_{4.6362}, s_{5.}$	$([s_{4.8506}, s_{5.}$	$([s_{5.6482}, s_{5.}$	$([s_{5.2509}, s_{5.}$	$([s_{5.8286}, s_{5.}$	$([s_{5.9502}, s_{5.}$
	$[s_{3.4641}, s_{4.4}$	$[s_{0.0087}, s_{0.1}$	$[s_{0.0053}, s_{0.1}$	$[s_{0.0131}, s_{0.2}$	$[s_{0.0370}, s_{0.5}$	$[s_{0.0031}, s_{0.1}$	$[s_{0.0046}, s_{0.2}$
B_5	$([s_{2.7979}, s_{4.}$	$([s_{5.9100}, s_{5.}$	$([s_{5.5621}, s_{5.}$	$([s_{5.9100}, s_{5.}$	$([s_{5.8456}, s_{5.}$	$([s_{5.8767}, s_{5.}$	$([s_{0.8767}, s_{5.}$
	$[s_{2.4495}, s_{3.6}$	$[s_{0.0321}, s_{0.3}$	$[s_{0.0046}, s_{0.1}$	$[s_{0.0321}, s_{0.5}$	$[s_{0.0151}, s_{0.1}$	$[s_{0.0131}, s_{0.2}$	$[s_{0.0131}, s_{0.3}$

Table III: Aggregated Values of LIVPFRM

	E_1	E_2	E_3	E_4	E_5	E_6	E_7
B_1	([0.81, 0.97], [0.01, 0.06])	([0.81, 0.97], [0.01, 0.06])	([0.81, 0.97], [0.01, 0.06])	([0.97, 1.00], [0.00, 0.01])	([0.93, 0.99], [0.00, 0.04])	([0.81, 0.99], [0.00, 0.04])	([0.81, 0.97], [0.01, 0.06])
B_2	([0.89, 1.00], [0.00, 0.01])	([0.89, 1.00], [0.00, 0.01])	([0.89, 1.00], [0.00, 0.01])	([0.99, 1.00], [0.00, 0.02])	([0.88, 0.97], [0.01, 0.07])	([0.73, 1.00], [0.01, 0.06])	([0.89, 1.00], [0.00, 0.01])
B_3	([0.87, 1.00], [0.00, 0.03])	([0.87, 1.00], [0.00, 0.03])	([0.87, 1.00], [0.00, 0.03])	([0.81, 0.93], [0.01, 0.07])	([0.99, 1.00], [0.01, 0.06])	([0.74, 0.98], [0.00, 0.02])	([0.87, 1.00], [0.00, 0.03])
B_4	([0.99, 1.00], [0.00, 0.04])	([0.99, 1.00], [0.00, 0.04])	([0.99, 1.00], [0.00, 0.04])	([0.94, 0.94], [0.00, 0.04])	([0.88, 1.00], [0.01, 0.10])	([0.97, 1.00], [0.00, 0.02])	([0.99, 1.00], [0.00, 0.04])
B_5	([0.98, 1.00], [0.00, 0.06])	([0.98, 1.00], [0.00, 0.06])	([0.98, 1.00], [0.00, 0.06])	([0.99, 1.00], [0.01, 0.09])	([0.97, 1.00], [0.00, 0.02])	([0.98, 1.00], [0.00, 0.04])	([0.98, 1.00], [0.00, 0.06])

Table IV: Relational Matrix of Interval-Valued Pythagorean FRM

B_1	B_2	B_3	B_4	B_5
([0.30, 0.80], [0.00, 0.10])	([0.30, 0.90], [0.10, 0.25])	([0.60, 0.80], [0.25, 0.30])	([0.50, 1.00], [0.20, 0.50])	([0.30, 0.50], [0.20, 0.40])

Table V: Desired values of concepts of domain space

	B_1	B_2	B_3	B_4	B_5
P_1	([0.50, 0.80], [0.20, 0.50])	([0.60, 0.80], [0.10, 0.30])	([0.55, 0.80], [0.25, 0.30])	([0.75, 0.90], [0.25, 0.50])	([0.35, 0.75], [0.30, 0.50])
P_2	([0.35, 0.55], [0.10, 0.15])	([0.45, 0.60], [0.10, 0.20)	([0.40, 0.80], [0.15, 0.30])	([0.50, 0.90], [0.10, 0.25])	([0.35, 0.75], [0.10, 0.50])
P_3	([0.70, 0.90], [0.30, 0.50])	([0.75, 0.80], [0.20, 0.40])	([0.60, 0.85], [0.25, 0.45])	([0.60, 0.90], [0.20, 0.35])	([0.50, 0.75], [0.15, 0.35])

Table VI: Initial values of Concepts

	B_1	B_2	B_3	B_4	B_5
P_1	([0.87, 0.89], [0.00, 0.00])	([0.87, 0.89], [0.09, 0.21])	([0.87, 0.89], [0.17, 0.21])	([0.87, 0.89], [0.17, 0.22])	([0.88, 0.89], [0.19, 0.22])
P_2	([0.87, 0.89], [0.00, 0.00])	([0.87, 0.89], [0.09, 0.15])	([0.87, 0.89], [0.12, 0.18])	([0.87, 0.89], [0.09, 0.17])	([0.88, 0.89], [0.09, 0.20])
P_3	([0.87, 0.89], [0.00, 0.00])	([0.87, 0.89], [0.15, 0.20])	([0.87, 0.89], [0.16, 0.20])	([0.87, 0.89], [0.15, 0.19])	([0.88, 0.89], [0.12, 0.19])

Table VII: Final values of concepts

4. REFERENCES

[1] Yager. R. R. (2013), Pythagorean fuzzy subsets, in: Proceeding of The Joint IFSA World Congress and NAFIPS Annual Meeting, Edmonton, Canada, pp. 57–61.

[2] Yager. R. R. (2014), Pythagorean membership grades in multicriteria decision making, IEEE Trans Fuzzy Syst 22, 958–965.

[3] Peng, X., & Yang, Y. (2015). Some results for Pythagorean fuzzy sets. International Journal of Intelligent Systems, 30(11), 1133-1160.

[4] Peng, X., & Yang, Y. (2016), Fundamental properties of interval-valued Pythagorean fuzzy aggregation operators, Int. J. Intell. Syst., vol. 31, no. 5, pp. 444–487.

[5] F. Herrera and L. Martinez (2001), A model based on linguistic 2-tuples for dealing with multigranular hierarchical linguistic contexts in multi-expert decision-making, *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 31, no. 2, pp. 227-234, doi: 10.1109/3477.915345.

[6] Garg, H. Linguistic Interval-Valued Pythagorean Fuzzy Sets and Their Application to Multiple Attribute Group Decision-making Process. Cogn Comput **12,** 1313–1337 (2020). https://doi.org/10.1007/s12559-020-09750-4
[7] He, Y. (2021). A new distance measure of Pythagorean fuzzy sets based on matrix and and its application in medical diagnosis. arXiv preprint arXiv:2102.01538.
[8] Vasantha Kandasamy, W.B., and Smarandache, F. (2003), Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps, Xiquan, Phoenix, AZ, USA.
[9] Baumeister RF, Campbell JD, Krueger JI, Vohs KD (2003). Does High Self-Esteem Cause Better Performance, Interpersonal Success, Happiness, or Healthier Lifestyles? Psychol Sci Public Interest, 4(1):1-44. doi: 10.1111/1529-1006.01431.
[10] Plummer, D. (2007). Helping Children to Build Self-Esteem: A Photocopiable Activities Book Second Edition. Jessica Kingsley Publishers.
[11] Lipton, B. H. (2015). The Biology of Belief, Unleashing the power of Consciousness, Matter and Miracles. Carlsband, CA: Hay House
[12] Damian, Rodica & Robins, Richard. (2011). Self-Esteem Across the Lifespan: Issues and Interventions, edited by Mary H. Guindon. Journal of Women & Aging - J WOMEN AGING. 23. 177-179. 10.1080/08952841.2011.561147.
[13] Zeigler-Hill, V. (Ed.). (2013). *Current issues in social psychology. Self-esteem.* Psychology Press. https://doi.org/10.1093/obo/9780199828340-0124
[14] Delgado, M., et al. (1998). Combining numerical and linguistic information in group decision making. Information Sciences, 107(1–4), 177–194.
[15] Zhang, Xiaolu & Xu, Zeshui. (2014). Extension of TOPSIS to Multiple Criteria Decision Making with Pythagorean Fuzzy Sets. International Journal of Intelligent Systems. 29. 10.1002/int.21676.