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Fixed PoinTheorems For Weakly Compatible Hybrid Maps In Intuitionistic Fuzzy Metric Spaces

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Abstract: We prove some common fixed point theorems for set – valued and single valued mappings in intuitionistic fuzzy metric and intuitionistic fuzzy 2– metric space. An Extension of common fixed point theorems for weakly compatible Hybrid mappings in intuitionistic fuzzy metric space.

Keywords: Intuitionistic fuzzy metric spaces, Compatible mappings, Common fixed point.
AMS Subject Classification: 47H10, 54H25.

1. INTRODUCTION:

The Theory of Fuzzy sets was introduced by Zadeh [17] in 1965. Many authors have introduced the concept of Fuzzy metric spaces in different ways the concept of compatible mapping has been investigated initially by Jungck [7] by which the nation of commuting and weakly computing mapping is generalized. Expanded the nation of compatible mapping fuzzy metric space and proved common fixed point theorems in set valued mapping. Further, many mathematicians used different conditions on self-mappings and several fixed point theorems for contractions in Fuzzy metric spaces. After words defined a new property which contains some common fixed point theorems under hybrid contractive conditions. Atanassov [2] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [16]. In 2004, Park [12] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et al. [1] Using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorm as a generalization of fuzzy metric space due to Kramosil and Michalek [10]. Gahler [3] introduced and studied the concept of 2-metric spaces in a series of his papers. Iseki et. al. [6] investigated, for the first time, contraction type mappings in 2-metric spaces. In 2002 Sharma [14] introduced the concept of fuzzy 2- metric spaces. Mursaleen et. al. [11] introduced the concept of intuitionistic fuzzy 2-metric space. In this paper, we prove some common fixed point theorems for set valued and single valued mappings in intuitionistic fuzzy metric space and fuzzy 2 – metric spaces. Also generalization and extension a unique common fixed point for four hybrid mappings in intuitionistic fuzzy 2 –metric spaces.

2. PRELIMINARIES:

Definition 2.1:

A 5-tuple (X, $\mathcal{M}$, ℵ, *, ◊) is called a intuitionistic fuzzy 2- metric space (shortly, *IF2M-space*) if X is an arbitrary set, * is a continuous t-norm, ◊ is a continuous t-conorm and $\mathcal{M}$ and ℵ are fuzzy sets on $X^3 \times [0, \infty]$ satisfying the following conditions:
For each $x, y, z, u \in X$ and $t_1, t_2, t_3 > 0$.

a) $\mathcal{M}(x, y, z, t) + \aleph(x, y, z, t) \leq 1$,
b) $\mathcal{M}(x, y, z, 0) = 0$,
c) $\mathcal{M}(x, y, z, t) = 1$, if at least two of x , y, z of X are equal,
d) $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$, where p is a permutation function,
e) $\mathcal{M}(x, y, z, t_1 + t_2 + t_3) \geq \mathcal{M}(x, y, u, t_1) * \mathcal{M}(x, u, z, t_2) * \mathcal{M}(u, y, z, t_3)$
f) $\mathcal{M}(x, y, z, .): [0, \infty) \to [0, 1]$ is left continuous,
g) $\aleph(x, y, z, 0) = 1$,
h) $\aleph(x, y, z, t) = 0$, if at least two of x, y, z of X are equal,
i) $\aleph(x, y, z, t) = \aleph(p\{x, y, z\}, t)$, where p is a permutation function,
j) $\aleph(x, y, z, t_1 + t_2 + t_3) \leq \aleph(x, y, u, t_1) \lozenge \aleph(x, u, z, t_2) \lozenge \aleph(u, y, z, t_3)$,
k) $\aleph(x, y, z, .): [0, \infty) \to [0, 1]$ is right continuous.

In this case ($\mathcal{M}$, ℵ) is called an intuitionistic fuzzy 2-metric on X. The function $\mathcal{M}$ (x, y, z, t) and ℵ (x, y, z, t) denote the degree of nearness and the degree of non-nearness between x, y and z with respect to t, respectively

Definition 2.2:

A sequence $\{x_n\}$ in a *IF2M-space* (X, $\mathcal{M}$, ℵ, *, ◊) is said to be converge to x in X iff $\lim_{n\to\infty} \mathcal{M}(x_n, x, a, t) = 1$ and $\lim_{n\to\infty} \aleph(x_n, x, a, t) = 0$, for all $a \in X$ and $t > 0$.

Definition 2.3:

Let (X, $\mathcal{M}$, ℵ, *, ◊) be a *IF2M-space*. A sequence $\{x_n\}$ in X is called Cauchy sequence, iff $\lim_{n\to\infty} \mathcal{M}(x_{n+p}, x_p, a, t) = 1$ and $\lim_{n\to\infty} \aleph(x_{n+p}, x_p, a, t) = 0$, for all $a \in X$ and $p > 0$, $t > 0$.

Definition 2.4:

A *IF2M-space* (X, $\mathcal{M}$, ℵ, *, ◊) is said to be complete iff every Cauchy sequence in X is convergent in X.

Definition 2.5:

The mappings $I : X \to X$ and $F : X \to B(X)$ are weakly compatible it they commute at coincidences points, i.e. for each $u \in X$ such that $Fu = \{ Iu \}$. We have $FIu = IFu$.
Not that the equation $Fu = \{ Iu \}$ implies that Fu is singleton.

Definition 2.6:

The mappings $I: X \to X$ and $F: X \to B(X)$ are R – weakly commuting if, for all , $t > 0$, $\mathcal{M}(FI, IFx, t) \geq \mathcal{M}(Fx, Ix, \frac{t}{R})$ and $\aleph(FI, IFx, t) \leq \aleph(Fx, Ix, \frac{t}{R})$ *such that*

$x \in$ X , $IFx \in$ B(X). R – Weakly commuting is weakly compatible but the converse is not true.

3. MAIN RESULTS

In the following we denote the set of all non-empty bounded closed subsets of X by CB(X).

Theorem- 3.1:

Let S and T be two self-mappings of a Intuitionistic Fuzzy Metric Space (X, $\mathcal{M}$, ℵ, *, ◊) and A, B : X → CB(X) set – valued mappings satisfying following conditions :

(3.1.1) A(X) ⊆ S(X) and B(X) ⊆ T(X)

(3.1.2) $\{A, T\}$ and $\{B, S\}$ are weakly compatible pairs.

(3.1.3) $q\,\mathcal{M}\,(Ax, By, t) \geq a\,\mathcal{M}\,(Tx, Sy, t) + b\,\mathcal{M}\,(Tx, Ax, t) + c\,\mathcal{M}\,(By, Ty, t)$
$+ d\,\mathcal{M}\,(Sy, By, t) + max\{\,\mathcal{M}\,(Ax, Sy, t), \mathcal{M}\,(By, Tx, t)\}$,

(3.1.4) $q\,\aleph\,(Ax, By, t) \leq a\,\aleph\,(Tx, Sy, t) + b\,\aleph\,(Tx, Ax, t) + c\,\aleph\,(By, Ty, t) + d\,\aleph\,(Sy, By, t)$
$+ min\{\aleph\,(Ax, Sy, t), \aleph\,(By, Tx, t)\}$,

$for\ all\ x, y \in$ X $where\ a, b, c, d \geq 0\ with\ 0 < q < a + b + c + d < 1$ and if the range of one of the mappings A, B, S and T have a unique common fixed point.

Proof:

Let x_0 be an arbitrary point in X. We chose a point x_1 in X, such that $Sx_1 \in Ax_0$. For this point x_1 there exist a point x_2 in X such that $Tx_2 \in Bx_1$ and so on. Inductively, we can define a sequence $\{z_n\}$ in X such that

$Sx_{2n+1} \in Ax_{2n} = z_{2n}\,, Tx_{2n+2} \in Bx_{2n+1} = z_{2n+1}$, for all $n = 0,1,2, ...$

We will prove that $\{z_n\}$ is Cauchy sequence.Using inequality (3.1.3) and (3.1.4), we obtain

$$q\,\mathcal{M}\,(z_{2n}, z_{2n+1}, t) = q\,\mathcal{M}\,(Ax_{2n}, Bx_{2n+1}, t)$$
$$\geq a\,\mathcal{M}(Tx_{2n}, Sx_{2n+1}, t) + b\,\mathcal{M}(Tx_{2n}, Ax_{2n}, t)$$
$$+ c\,\mathcal{M}(Bx_{2n+1}, Tx_{2n+1}, t) + d\,\mathcal{M}\,(Sx_{2n+1}, Bx_{2n+1}, t)$$
$$+ max\{\,\mathcal{M}\,(Ax_{2n}, Sx_{2n+1}, t), \mathcal{M}\,(Bx_{2n+1}, Tx_{2n}, t)\}$$
$$\geq a\,\mathcal{M}(z_{2n-1}, z_{2n}\ t) + b\,\mathcal{M}(z_{2n-1}, z_{2n}, t) + c\,\mathcal{M}(z_{2n+1}, z_{2n}, t)$$
$$+ d\,\mathcal{M}\,(z_{2n}, z_{2n+1}, t) + max\{\,\mathcal{M}\,(z_{2n}, z_{2n}, t), \mathcal{M}$$
$$(z_{2n+1}, z_{2n-1}, t)\}$$

Then $\mathcal{M}\,(z_{2n}, z_{2n+1}, t)(q - c - d) \geq (a + b + 1)\,\mathcal{M}(z_{2n-1}, z_{2n}\ t)$

$$\mathcal{M}\,(z_{2n}, z_{2n+1}, t) \geq \frac{a+b+1}{q-c-d}\,\mathcal{M}(z_{2n-1}, z_{2n}\ t)$$

$\mathcal{M}\,(z_{2n}, z_{2n+1}, t) \geq k\,\mathcal{M}(z_{2n-1}, z_{2n}\ t)$, where $k = \frac{a+b+1}{q-c-d} > 1$.

$$q\,\aleph\,(z_{2n}, z_{2n+1}, t) = q\,\aleph\,(Ax_{2n}, Bx_{2n+1}, t)$$
$$\leq a\,\aleph(Tx_{2n}, Sx_{2n+1}, t) + b\,\aleph(Tx_{2n}, Ax_{2n}, t)$$
$$+ c\,\aleph(Bx_{2n+1}, Tx_{2n+1}, t) + d\,\aleph\,(Sx_{2n+1}, Bx_{2n+1}, t)$$
$$+ min\{\,\aleph\,(Ax_{2n}, Sx_{2n+1}, t), \aleph\,(Bx_{2n+1}, Tx_{2n}, t)\}$$
$$\leq a\,\aleph(z_{2n-1}, z_{2n}\ t) + b\,\aleph(z_{2n-1}, z_{2n}, t) + c\,\aleph(z_{2n+1}, z_{2n}, t)$$
$$+ d\,\aleph\,(z_{2n}, z_{2n+1}, t) + max\{\,\aleph\,(z_{2n}, z_{2n}, t), \aleph\,(z_{2n+1}, z_{2n-1}, t)\}.$$

Then $\aleph\,(z_{2n}, z_{2n+1}, t)(q - c - d) \leq (a + b + 1)\,\aleph(z_{2n-1}, z_{2n}\ t)$

$$\aleph\,(z_{2n}, z_{2n+1}, t) \leq \frac{a+b+1}{q-c-d}\,\aleph(z_{2n-1}, z_{2n}\ t)$$

$\aleph\,(z_{2n}, z_{2n+1}, t) \leq k\,\aleph(z_{2n-1}, z_{2n}\ t)$, where $k = \frac{a+b+1}{q-c-d} > 1$.

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Since $k > 1$.
We obtain $\mathcal{M}(z_{2n+1}, z_{2n}, t) > \mathcal{M}(z_{2n}, z_{2n-1}, t)$ and $\aleph(z_{2n+1}, z_{2n}, t) < \aleph(z_{2n}, z_{2n-1}, t)$.
Similarly, $\mathcal{M}(z_{2n+2}, z_{2n+1}, t) > \mathcal{M}(z_{2n+1}, z_{2n}, t)$ and $\aleph(z_{2n+2}, z_{2n+1}, t) < \aleph(z_{2n+1}, z_{2n}, t)$.
Now for any positive integer p
$\mathcal{M}(z_n, z_{n+p}, t) \geq \mathcal{M}(z_n, z_{n+1}, \frac{t}{p}) * \mathcal{M}(z_{n+1}, z_{n+2}, \frac{t}{p}) * \ldots * \mathcal{M}(z_{n+p-1}, z_{n+p}, \frac{t}{p})$
as $n \to \infty$ $we\ get$ $\mathcal{M}(z_n, z_{n+p}, t) \to 1 * 1 * \ldots * 1 \to 1$,
$\aleph(z_n, z_{n+p}, t) \geq \aleph(z_n, z_{n+1}, \frac{t}{p}) \lozenge \aleph(z_{n+1}, z_{n+2}, \frac{t}{p}) \lozenge \ldots \lozenge \aleph(z_{n+p-1}, z_{n+p}, \frac{t}{p})$
as $n \to \infty$ $we\ get$ $\aleph(z_n, z_{n+p}, t) \to 0 \lozenge 0 \lozenge \ldots \lozenge 0 \to 0$.
Hence $\{ z_n \}$ is a Cauchy sequence.
Suppose that SX is complete therefore by the above , $\{ Sx_{2n+1}\}$ is a Cauchy sequence and hence $Sx_{2n+1} \to z = Sv$ for some $v \in$ X.
Hence $z_n \to z$ and the subsequences $Tx_{2n+1}, Tx_{2n+2}, Ax_{2n}$ and Bx_{2n+1} converge to z.
We shall prove that z = $Sv \in Bv$ by (3.1.3) and (3.1.4), we have
$q\,\mathcal{M}(Ax_{2n}, Bv, t) \geq a\,\mathcal{M}(Tx_{2n}, Sv, t) + b\,\mathcal{M}(Tx_{2n}, Ax_{2n}, t) + c\,\mathcal{M}(Bv, Tv, t)$
$+ \mathcal{M}(Sv, Bv, t) + \max\{ \mathcal{M}(Ax_{2n}, Sv, t), \mathcal{M}(Bv, Tx_{2n}, t)\}$
as $n \to \infty$ we obtain
$q\,\mathcal{M}(z, Bv, t) \geq a\,\mathcal{M}(z, z, t) + b\,\mathcal{M}(z, z, t) + c\,\mathcal{M}(Bv, z, t) + d\,\mathcal{M}(z, Bv, t)$
$+ max\{ \mathcal{M}(z, z, t), \mathcal{M}(Bv, z, t)\}$
$q\,\mathcal{M}(z, Bv, t) - c\,\mathcal{M}(Bv, z, t) - d\,\mathcal{M}(z, Bv, t) \geq a\,\mathcal{M}(z, z, t) + b\,\mathcal{M}(z, z, t)$
$+ max\{ \mathcal{M}(z, z, t), \mathcal{M}(Bv, z, t)\}$
$\mathcal{M}(z, Bv, t)(q - c - d) \geq (a + b + 1)\,\mathcal{M}(z, z, t)$
$\mathcal{M}(z, Bv, t) \geq \frac{(a+b+1)}{(q-c-d)} > 1$,
$q\aleph(Ax_{2n}, Bv, t) \leq a\aleph(Tx_{2n}, Sv, t) + b\aleph(Tx_{2n}, Ax_{2n}, t) + c\aleph(Bv, Tv, t) + d\aleph(Sv, Bv, t)$
$+ min\{\aleph(Ax_{2n}, Sv, t), \aleph(Bv, Tx_{2n}, t)\}$
as $n \to \infty$ we obtain
$q\aleph(z, Bv, t) \leq a\aleph(z, z, t) + b\aleph(z, z, t) + c\aleph(Bv, z, t) + d\aleph(z, Bv, t)$
$+ min\{ \aleph(z, z, t), \aleph(Bv, z, t)\}$
$q\aleph(z, Bv, t) - c\aleph(Bv, z, t) - d\aleph(z, Bv, t) \leq a\aleph(z, z, t) + b\aleph(z, z, t)$
$+ min\{\aleph(z, z, t), \aleph(Bv, z, t)\}$

$\aleph(z, Bv, t)(q - c - d) \leq 0$, $\aleph(z, Bv, t) \leq 0$.
Which yields $\{z\} = \{Sv\} = Bv$.
Since B(X) $\subseteq$ T(X) thus there exist $u \in$ X such that $\{Tu\} = Bv = \{z\} = \{Sv\} = Tv$.
Now if $Au \neq Bv$ we get,
$q\,\mathcal{M}(Au, Bv, t) \geq a\,\mathcal{M}(Tu, Sv, t) + b\,\mathcal{M}(Tu, Au, t) + c\,\mathcal{M}(Bv, Tv, t)$
$+ d\,\mathcal{M}(Sv, Bv, t) + max\{ \mathcal{M}(Au, Sv, t), \mathcal{M}(Bv, Tu, t)\}$.
$q\,\mathcal{M}(Au, z, t) \geq a\,\mathcal{M}(z, z, t) + b\,\mathcal{M}(z, Au, t) + c\,\mathcal{M}(z, z, t) + d\,\mathcal{M}(z, z, t)$
$+ max\{ \mathcal{M}(Au, z, t), \mathcal{M}(z, z, t)\}$
$\mathcal{M}(Au, z, t) \geq \frac{(a+c+d+1)}{(q-b)} > 1$,
$q\aleph(Au, Bv, t) \leq a\aleph(Tu, Sv, t) + b\aleph(Tu, Au, t) + c\aleph(Bv, Tv, t)$
$+ d\aleph(Sv, Bv, t) + min\{\aleph(Au, Sv, t), \aleph(Bv, Tu, t)\}$.
$q\aleph(Au, z, t) \leq a\aleph(z, z, t) + b\aleph(z, Au, t) + c\aleph(z, z, t) + d\aleph(z, z, t)$
$+ min\{\aleph(Au, z, t), \aleph(z, z, t)\}$

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$\aleph\ (Au, z, t) \leq 0$,
Which yields $Au = \{z\} = \{Tu\} = \{Sv\} = Tv = Bv$.
Since $Au = \{Tu\}$ and $\{A, T\}$ is weakly compatible $ATv = TAv$ $gives$ $Az = \{Tz\}$.
On using (3.1.3) and (3.1.4) we obtain,

$$q\ \mathcal{M}\ (Az, Bv, t) \geq a\ \mathcal{M}\ (Tz, Sv, t) + b\ \mathcal{M}\ (Tz, Az, t) + c\ \mathcal{M}\ (Bv, Tv, t) + d\ \mathcal{M}\ (Sv, Bv, t) + max\{\ \mathcal{M}\ (Az, Sv, t), \mathcal{M}\ (Bv, Tz, t)\}$$

$$q\ \mathcal{M}\ (Az, z, t) \geq a\ \mathcal{M}\ (Tz, z, t) + b\ \mathcal{M}\ (z, Az, t) + c\ \mathcal{M}\ (z, z, t) + d\ \mathcal{M}\ (z, z, t) + max\{\ \mathcal{M}\ (Az, z, t), \mathcal{M}\ (z, z, t)\}$$

$$q\aleph\ (Az, Bv, t) \leq a\aleph\ (Tz, Sv, t) + b\aleph\ (Tz, Az, t) + c\aleph\ (Bv, Tv, t) + d\aleph\ (Sv, Bv, t) + min\{\aleph\ (Az, Sv, t), \aleph\ (Bv, Tz, t)\}$$

$$q\aleph\ (Az, z, t) \leq a\aleph\ (Tz, z, t) + b\aleph\ (z, Az, t) + c\aleph\ (z, z, t) + d\aleph\ (z, z, t) + min\ \{\aleph\ (Az, z, t), \aleph\ (z, z, t)\}.$$

Hence $Az = \{z\} = \{Tz\}$. Similarly $Bz = \{z\} = \{Sz\}$, where $\{B, S\}$ is weakly compatible.
Then $Az = \{Tz\} = \{z\} = \{Sz\} = Bz$. z is the common fixed point of A, B, S and T .
To see z is unique. Suppose that $p \neq z$ such that $Ap = \{Tp\} = \{p\} = \{Sp\} = Bp$.
On using (3.1.3) and (3.1.4) we get

$$q\ \mathcal{M}\ (Az, Bp, t) \geq a\ \mathcal{M}\ (Tz, Sp, t) + b\ \mathcal{M}\ (Tz, Az, t) + c\ \mathcal{M}\ (Bp, Tp, t) + d\ \mathcal{M}\ (Sp, Bp, t) + max\{\ \mathcal{M}\ (Az, Sp, t), \mathcal{M}\ (Bp, Tz, t)\}$$

$$\mathcal{M}\ (z, p,\ t) \geq \frac{(b + c + d)}{(q - a - 1)} > 1$$

$$q\aleph\ (Az, Bp, t) \leq a\aleph\ (Tz, Sp, t) + b\aleph\ (Tz, Az, t) + c\aleph\ (Bp, Tp, t) + d\aleph\ (Sp, Bp, t) + \min\{\aleph\ (Az, Sp, t), \aleph\ (Bp, Tz, t)\}$$

$\aleph\ (z, p, t\ ) \leq 0$. Which is impossible $z\ =\ p$.
Then A, B, S and T have a unique common fixed point.

Theorem 3.2:

Let S and T be two self-mappings of a *IF2M-space* (X, $\mathcal{M}$, ℵ, *, ◊) and A, B : X → CB(X) set valued mappings satisfying following conditions:
(3.2.1) A(X) ⊆ S(X) and B(X) ⊆ T(X).
(3.2.2) $\{A, T\}$ and $\{B, S\}$ are weakly compatible pairs.
(3.2.3) $q\ \mathcal{M}\ (Ax, By, w, t) \geq a\ \mathcal{M}\ (Tx, Sy, w, t) + b\ \mathcal{M}\ (Tx, Ax, w, t) + c\ \mathcal{M}\ (Ay, Ty, w, t) + d\ \mathcal{M}\ (Sy, By, w, t) + max\ \{M(Ax, Sy, w, t), M(By, Tx, w, t)\}$
(3.2.4) $q\aleph\ (Ax, By, w, t) \leq a\aleph\ (Tx, Sy, w, t) + b\aleph\ (Tx, Ax, w, t) + c\aleph\ (Ay, Ty, w, t) + d\aleph\ (Sy, By, w, t) + min\ \{\aleph\ (Ax, Sy, w, t), \aleph\ (By, Tx, w, t)\}$.
For all x, y, w ∈ X, where $a, b, c, d\ \geq\ 0$ with $0\ <\ q\ <\ a + b + c + d\ < 1$. If the range of one of the mappings A, B, S and T have a unique common fixed point.

Proof:

We can define a sequence $\{z_n\}$ in X such that
$Sx_{2n+1} \in Ax_{2n} = z_{2n}\ , Tx_{2n+2} \in Bx_{2n+1} =\ z_{2n+1}$, for all $n = 0,1,2, ...$
We will prove that $\{z_n\}$ is Cauchy sequence.Using inequality (3.1.3) and (3.1.4), we obtain

$$\begin{aligned} q\ \mathcal{M}\ (z_{2n},\ z_{2n+1}, w, t) &=\ q\ \mathcal{M}\ (Ax_{2n},\ Bx_{2n+1}, w,\ t) \\ &\geq a\ \mathcal{M}(Tx_{2n}, Sx_{2n+1}, w, t\ ) + b\ \mathcal{M}(Tx_{2n}, Ax_{2n}, w, t) \\ &\quad + c\ \mathcal{M}(Bx_{2n+1}, Tx_{2n+1}, w, t)\ + d\ \mathcal{M}\ (Sx_{2n+1}\ , Bx_{2n+1}\ , w, t) \\ &\quad + max\{\ \mathcal{M}\ (Ax_{2n},\ Sx_{2n+1}, w,\ t), \mathcal{M}\ (Bx_{2n+1}, Tx_{2n}, w, t)\} \\ &\geq a\ \mathcal{M}(z_{2n-1}, z_{2n}\ , w, t\ ) + b\ \mathcal{M}(z_{2n-1}, z_{2n}, w, t) \end{aligned}$$

$$+ c\,\mathcal{M}(z_{2n+1}, z_{2n}, w, t) + d\,\mathcal{M}\,(z_{2n}, z_{2n+1}\,, w, t) + max\{\,\mathcal{M}\,(z_{2n},\; z_{2n},\; w, t), \mathcal{M}\,(z_{2n+1}, z_{2n-1}, w, t)\}$$

Then $\mathcal{M}\,(z_{2n},\; z_{2n+1}, w, t)(q - c - d) \geq (a + b + 1)\,\mathcal{M}(z_{2n-1}, z_{2n}\,, w, t\,)$

$$\mathcal{M}\,(z_{2n},\; z_{2n+1}, w, t)\; \geq \frac{a+b+1}{q-c-d}\,\mathcal{M}(z_{2n-1}, z_{2n}, w, t\,)$$

$$\mathcal{M}\,(z_{2n},\; z_{2n+1}, w, t)\; \geq k\,\mathcal{M}(z_{2n-1}, z_{2n}, w, t\,),\text{ where } k = \frac{a+b+1}{q-c-d} > 1.$$

$$q\,\aleph\,(z_{2n},\; z_{2n+1}, w, t) \;=\; q\,\aleph\,(Ax_{2n},\; Bx_{2n+1}, w,\; t)$$
$$\leq\; a\,\aleph(Tx_{2n}, Sx_{2n+1}, w, t\,) + b\,\aleph(Tx_{2n}, Ax_{2n}, w, t) + c\,\aleph(Bx_{2n+1}, Tx_{2n+1}, w, t) \;+ d\,\aleph\,(Sx_{2n+1}\,, Bx_{2n+1}\,, w, t) + min\{\,\aleph\,(Ax_{2n},\; Sx_{2n+1}, w,\; t), \aleph\,(Bx_{2n+1}, Tx_{2n}, w, t)\}$$
$$\leq a\,\aleph(z_{2n-1}, z_{2n}\,, w, t\,) + b\,\aleph(z_{2n-1}, z_{2n}, w, t) + c\,\aleph(z_{2n+1}, z_{2n}, w, t) + d\,\aleph\,(z_{2n}, z_{2n+1}\,, w, t) + max\{\,\aleph\,(z_{2n},\; z_{2n}, w,\; t), \aleph\,(z_{2n+1}, z_{2n-1}, w, t)\}$$

Then $\aleph\,(z_{2n},\; z_{2n+1}, w, t)(q - c - d) \leq (a + b + 1)\,\aleph(z_{2n-1}, z_{2n}\,, w, t\,)$

$$\aleph\,(z_{2n},\; z_{2n+1}, w, t)\; \leq \frac{a+b+1}{q-c-d}\,\aleph(z_{2n-1}, z_{2n}\,, w, t\,)$$

$$\aleph\,(z_{2n},\; z_{2n+1}, w, t)\; \leq k\,\aleph(z_{2n-1}, z_{2n}\,, w, t\,),\text{ where } k = \frac{a+b+1}{q-c-d} > 1.$$

Since $k > 1$.
We obtain $\mathcal{M}(z_{2n+1}, z_{2n}, w, t) > \mathcal{M}(z_{2n}, z_{2n-1}, w, t)$ and
$\aleph(z_{2n+1}, z_{2n}, w, t) < \aleph(z_{2n}, z_{2n-1}, w, t)$.
Similarly, $\mathcal{M}(z_{2n+2}, z_{2n+1}, w, t) > \mathcal{M}(z_{2n+1}, z_{2n}, w, t)$ and
$\aleph(z_{2n+2}, z_{2n+1}, w, t) < \aleph(z_{2n+1}, z_{2n}, w, t)$.
Now for any positive integer p

$$\mathcal{M}\,(z_n, z_{n+p}, w, t) \geq\; \mathcal{M}\,(z_n, z_{n+1}, w, \tfrac{t}{p}) * \mathcal{M}\,(z_{n+1}, z_{n+2}, w, \tfrac{t}{p})$$
$$* \ldots * \;\mathcal{M}\,(z_{n+p-1}, z_{n+p}, w, \tfrac{t}{p})$$

as $n \to \infty$ we get $\mathcal{M}\,(z_n, z_{n+p}, w, t) \to 1 * 1 * \ldots * 1 \to 1,$

$$\aleph\,(z_n, z_{n+p}, w, t) \geq\; \aleph\,(z_n, z_{n+1}, w, \tfrac{t}{p})\,\Diamond\,\aleph\,(z_{n+1}, z_{n+2}, w, \tfrac{t}{p})$$
$$\Diamond \ldots \Diamond\; \aleph\,(z_{n+p-1}, z_{n+p}, w, \tfrac{t}{p})$$

as $n \to \infty$ we get $\aleph\,(z_n, z_{n+p}, w, t) \to 0 \Diamond 0 \Diamond \ldots \Diamond 0 \to 0.$
Hence $\{\,z_n\,\}$ is a Cauchy sequence.
Suppose that SX is complete therefore by the above , $\{\,Sx_{2n+1}\}$ is a Cauchy sequence and hence $Sx_{2n+1} \to z = Sv$ for some $v \in$ X.
Hence $z_n \to z$ and the subsequences $Tx_{2n+1}, Tx_{2n+2}, Ax_{2n}$ and Bx_{2n+1} converge to z.
We shall prove that z = $Sv \in Bv$ by (3.1.3) and (3.1.4), we have

$$q\,\mathcal{M}(Ax_{2n}, Bv, w, t) \geq a\,\mathcal{M}(Tx_{2n}, Sv, w, t) + b\,\mathcal{M}(Tx_{2n}, Ax_{2n}, w, t) + c\,\mathcal{M}(Bv, Tv, w, t) + d\,\mathcal{M}(Sv, Bv, w, t) + \max\{\,\mathcal{M}(Ax_{2n}, Sv, w, t), \mathcal{M}(Bv, Tx_{2n}, w, t)\}$$

as $n \to \infty$ we obtain

$$q\,\mathcal{M}(z, Bv, w, t) \geq a\,\mathcal{M}(z, z, w, t) + b\,\mathcal{M}(z, z, w, t) + c\,\mathcal{M}(Bv, z, w, t) + d\,\mathcal{M}(z, Bv, w, t) + max\{\,\mathcal{M}(\,z, z, w, t)\,,\; \mathcal{M}(Bv, z, w, t)\}$$

$$q\,\mathcal{M}\,(z, Bv, w, t) - c\,\mathcal{M}\,(Bv, z, w, t) - \;d\,\mathcal{M}\,(z, Bv, w, t) \geq a\,\mathcal{M}(z, z, w, t) + \;b\,\mathcal{M}\,(z, z, w, t) + max\{\,\mathcal{M}\,(z, z, t), \mathcal{M}\,(Bv, z, t)\}$$

$$\mathcal{M}\,(z, Bv, w, t)(q - c - d) \geq (a\; +\; b\; +\; 1)\,\mathcal{M}\,(z, z, w, t)$$

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$\mathcal{M}(z, Bv, w, t) \geq \frac{(a+b+1)}{(q-c-d)} > 1$,

$q\aleph(Ax_{2n}, Bv, w, t) \leq a\aleph(Tx_{2n}, Sv, w, t) + b\aleph(Tx_{2n}, Ax_{2n}, w, t) + c\aleph(Bv, Tv, w, t) + d\aleph(Sv, Bv, w, t) + min\{\aleph(Ax_{2n}, Sv, w, t), \aleph(Bv, Tx_{2n}, w, t)\}$

as $n \to \infty$ we obtain

$q\aleph(z, Bv, w, t) \leq a\aleph(z, z, w, t) + b\aleph(z, z, w, t) + c\aleph(Bv, z, w, t) + d\aleph(z, Bv, w, t) + min\{\aleph(z, z, w, t), \aleph(Bv, z, w, t)\}$

$q\aleph(z, Bv, w, t) - c\aleph(Bv, z, w, t) - d\aleph(z, Bv, w, t) \leq a\aleph(z, z, w, t) + b\aleph(z, z, w, t) + min\{\aleph(z, z, w, t), \aleph(Bv, z, w, t)\}$

$\aleph(z, Bv, w, t)(q - c - d) \leq 0$, $\aleph(z, Bv, w, t) \leq 0$.

Which yields $\{z\} = \{Sv\} = Bv$.

Since $B(X) \subseteq T(X)$ thus there exist $u \in X$ such that $\{Tu\} = Bv = \{z\} = \{Sv\} = Tv$.

Now if $Au \neq Bv$ we get,

$q\,\mathcal{M}(Au, Bv, w, t) \geq a\,\mathcal{M}(Tu, Sv, w, t) + b\,\mathcal{M}(Tu, Au, w, t) + c\,\mathcal{M}(Bv, Tv, w, t) + d\,\mathcal{M}(Sv, Bv, w, t) + max\{\mathcal{M}(Au, Sv, w, t), \mathcal{M}(Bv, Tu, w, t)\}$.

$q\,\mathcal{M}(Au, z, w, t) \geq a\,\mathcal{M}(z, z, w, t) + b\,\mathcal{M}(z, Au, w, t) + c\,\mathcal{M}(z, z, w, t) + d\,\mathcal{M}(z, z, w, t) + max\{\mathcal{M}(Au, z, w, t), \mathcal{M}(z, z, w, t)\}$

$\mathcal{M}(Au, z, w, t) \geq \frac{(a+c+d+1)}{(q-b)} > 1$,

$q\aleph(Au, Bv, w, t) \leq a\aleph(Tu, Sv, w, t) + b\aleph(Tu, Au, w, t) + c\aleph(Bv, Tv, w, t) + d\aleph(Sv, Bv, w, t) + min\{\aleph(Au, Sv, w, t), \aleph(Bv, Tu, w, t)\}$.

$q\aleph(Au, z, w, t) \leq a\aleph(z, z, w, t) + b\aleph(z, Au, w, t) + c\aleph(z, z, w, t) + d\aleph(z, z, w, t) + min\{\aleph(Au, z, w, t), \aleph(z, z, w, t)\}$.

$\aleph(Au, z, w, t) \leq 0$,

Which yields $Au = \{z\} = \{Tu\} = \{Sv\} = Tv = Bv$.

Since $Au = \{Tu\}$ *and* $\{A, T\}$ is weakly compatible $ATv = TAv$ *gives* $Az = \{Tz\}$.

On using (3.1.3) and (3.1.4) we obtain,

$q\,\mathcal{M}(Az, Bv, w, t) \geq a\,\mathcal{M}(Tz, Sv, w, t) + b\,\mathcal{M}(Tz, Az, w, t) + c\,\mathcal{M}(Bv, Tv, w, t) + d\,\mathcal{M}(Sv, Bv, w, t) + max\{\mathcal{M}(Az, Sv, w, t), \mathcal{M}(Bv, Tz, w, t)\}$

$q\,\mathcal{M}(Az, z, w, t) \geq a\,\mathcal{M}(Tz, z, w, t) + b\,\mathcal{M}(z, Az, w, t) + c\,\mathcal{M}(z, z, w, t) + d\,\mathcal{M}(z, z, w, t) + max\{\mathcal{M}(Az, z, w, t), \mathcal{M}(z, z, w, t)\}$

$q\aleph(Az, Bv, w, t) \leq a\aleph(Tz, Sv, w, t) + b\aleph(Tz, Az, w, t) + c\aleph(Bv, Tv, w, t) + d\aleph(Sv, Bv, w, t) + min\{\aleph(Az, Sv, w, t), \aleph(Bv, Tz, w, t)\}$

$q\aleph(Az, z, w, t) \leq a\aleph(Tz, z, w, t) + b\aleph(z, Az, w, t) + c\aleph(z, z, w, t) + d\aleph(z, z, w, t) + min\{\aleph(Az, z, w, t), \aleph(z, z, w, t)\}$.

Hence $Az = \{z\} = \{Tz\}$. Similarly $Bz = \{z\} = \{Sz\}$, where $\{B, S\}$ is weakly compatible.

Then $Az = \{Tz\} = \{z\} = \{Sz\} = Bz$. z is the common fixed point of A, B, S and T.

To see z is unique. Suppose that $p \neq z$ such that $Ap = \{Tp\} = \{p\} = \{Sp\} = Bp$.

On using (3.1.3) and (3.1.4) we get

$q\,\mathcal{M}(Az, Bp, w, t) \geq a\,\mathcal{M}(Tz, Sp, w, t) + b\,\mathcal{M}(Tz, Az, w, t) + c\,\mathcal{M}(Bp, Tp, w, t)$

$+ d\,\mathcal{M}\,(Sp, Bp, w, t) + max\{\,\mathcal{M}\,(Az, Sp, w, t), \mathcal{M}\,(Bp, Tz, w, t)\}$

$\mathcal{M}\,(z,\ p,\ w, t) \geq \frac{(b + c + d)}{(q - a - 1)} > 1.$

$q\aleph\,(Az, Bp, w, t) \leq a\aleph\,(Tz, Sp, w, t) + b\aleph\,(Tz, Az, w, t) + c\aleph\,(Bp, Tp, w, t) + d\aleph\,(Sp, Bp, w, t)$

$+ min\{\aleph\,(Az, Sp, w, t), \aleph\,(Bp, Tz, w, t)\}$

$\aleph\,(z, p, w, t) \leq 0$. Which is impossible $z = p$.

Then A, B, S and T have a unique common fixed point.

4. CONCLUSION:

It is also used in intuitionistic fuzzy 3- metric spaces other type of metric**.** Also in integral metric spaces type in intuitionistic fuzzy 2 - metric spaces and intuitionistic fuzzy 3 - metric spaces.

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