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Proposing Buckingham Expression for Occurrence of Breaking Waves

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Abstract: This study aims to investigate all parameters that effect occurrence of breaking waves, these parameters selected according to physical phenomenon behavior and Buckingham theory procedure for proposing an expression that used to predict breaking waves performance. The results, mathematical dimension analysis with non-dimensional groups, shows that the parameters that have a major effect on the free surface phase at the braking moment can be summarized as: the depth of water (d), the height of wave (H), the length of wave (L) and the slope of bed (m) when compared with the other parameters of fluid properties and wave characteristics.

Key words: Breaking waves, Buckingham theory, Dimensional analysis

1. INTRODUCTION

The basic principles of this study focused on breaking wave behavior, the most important parameters of wave that traveled from the sea side to the shoreline can be seen in figure (1). where the parameter H represent wave height that measured from the lowest point in the trough to the highest point in the crest, L represent length of the distance between two successive crests for individual wave, a represent wave amplitude that can be measured from the sea water level to the maximum height of crest and d represent the water depth from sea water level to the sea bed.

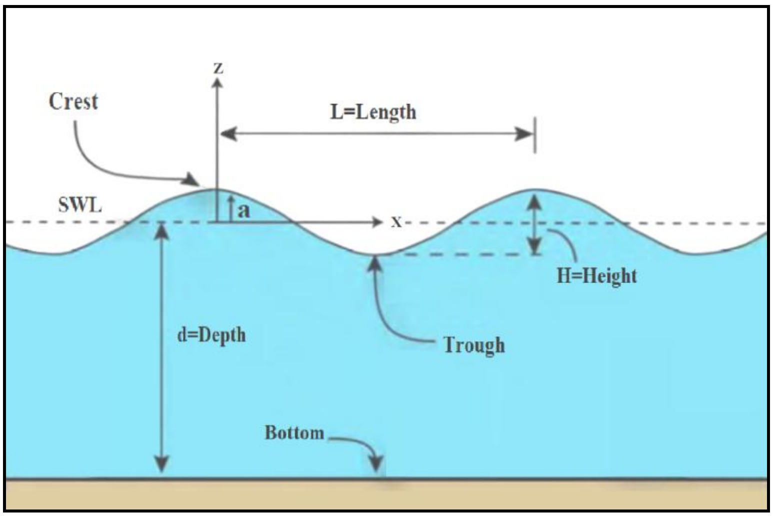

Figure (1): parameters of progressive wave, (Douglass and Krolak, 2008).

From figure (1), it can be seen that the amplitude (a) is equal to one half of the wave length as present in equation (1).

$$a = \frac{1}{2} H \quad \text{Eq. 1}$$

Small amplitude wave theory concluded that the wave length L is a function of wave period T and water depth d, as shown in equation (2), where the wave period T is defined as the time that wave crest spends to move from one position to another. When the depth of water is equal or more than one half the length of wave, in this case, deep water, the wave length L is a function of wave period T only as shown in equation (3), (Douglass and Krolak, 2008).

$$L = \frac{g\,T^2}{2\pi} tanh\left(\frac{2\pi\ d}{L}\right) \quad \text{Eq. 2}$$

$$L = \frac{g\,T^2}{2\pi} \quad \text{Eq. 3}$$

Where: g represents the gravity acceleration; m/sec^2.

Many studies tried to realize the occurrence of wave breaking for different types of slops such as Stokes in 1847, these studies conducted for the slopes between (1:100 to 1:50) for irregular waves, as a results, the occurrence of wave breaking determined based on wave velocity and progression speed, according to these studies, two indices have been used to express the wave breaking occurrence, the first one is the wave steepness ($\frac{H_i}{L_o}$) which is the ratio between the incident wave height; H_i and the wave length; in deep water L_o, this index is usually used in deep water, while the other one is ($\frac{H_i}{d}$) which is the ratio between the incident wave height; H_i and the water depth; d, used in shallow water (McCowan, 1894).

McCowan, 1894, investigate and defined the wave breaking criteria of shallow water for horizontal bottom to be take place when the ratio of wave height to water depth equals 0.78 as presented in equation (4):

$$\frac{H_b}{d_b} = 0.78 \quad \text{Eq. 4}$$

Where the subscribe symbol (b) refer to the inception of wave breaking state.

Miche, 1944, introduced a theoretical condition equation for breaking of waves that depends on hyperbolic tangent of wave angle, the equation applied in deep water to gave breaker index equal to 0.88, so that, this condition increased the limit that defined by McCowan, 1894, as expressed in equation (5):

$$\frac{H_b}{L_b} = 0.142 \tanh\left(\frac{2\pi\ d_b}{L_b}\right) \quad \text{Eq.5}$$

Le Méhauté and Koh, 1967, tried to understand the breaking wave behavior that reach to shoreline with an angle and to obtain breaking wave characteristics by proposing equation (6). This equation should satisfy the limits of bed slope; $\frac{1}{50} < m < \frac{1}{5}$ and the wave steepness ratio $\frac{H_o}{L_o}$.

$$\frac{H_b}{H_o} = 0.76\ m^{\frac{1}{7}} \left(\frac{H_o}{L_o}\right)^{-0.25} \quad \text{Eq.6}$$

Where: m represent the bed slope of shoreline.

Weggel, 1972, studied the breaking occurrence of regular waves taking in to account the bed slope in shallow water. Therefore, the researcher developed an equation (7) which is considered commonly used in the designing of sloped shorelines. Its worth to mention that, this equation can be applied for horizontal shorelines; zero sloped, to provide breaker waves index equal to 0.78, locks like similar to the McCowan, 1894, index.

$$\frac{H_b}{d_b} = \frac{\frac{1.5}{(1+e^{-19.5m})}}{\left(1+43.75\ (1-e^{19m})\left(\frac{d_b}{gT^2}\right)\right)} \qquad \text{Eq.7}$$

Goda, 1974, analyzed the laboratory data achieved by (Mitsuyasu, 1962 and Goda, 1964) in Japan to propose a new wave pressure formula for composite type of breakwater and summarized equation (8) for predicting braking wave index, this equation showed that when bed slope increase causes an increase in deep water braking index.

$$\frac{H_b}{L_o} = 0.17\left[1 - e^{\left(-1.5\frac{\pi\ d_b}{L_o}\left(1+15\ m^{\frac{4}{3}}\right)\right)}\right] \qquad \text{Eq.8}$$

Ostendorf and Madsen, 1979, suggested a modification on the parameters of Miche, 1944, equation to be produce equation (9) with best fit equal to 0.8, after that, the researchers presented two different equations (10 and 11), derived from Miche, 1944 equation, take in to account the effect of bed slope with two different ranges.

$$\frac{H_b}{L_b} = 0.142\tanh\left(0.91\ \frac{2\pi\ d_b}{L_b}\right) \qquad \text{Eq.9}$$

$$\frac{H_b}{L_b} = 0.14\tanh\left((0.8+5m)\frac{2\pi\ d_b}{L_b}\right) \qquad \text{for } m \leq 0.1 \qquad \text{Eq.10}$$

$$\frac{H_b}{L_b} = 0.14\tanh\left(0.13\ \frac{2\pi\ d_b}{L_b}\right) \qquad \text{for } m > 0.1 \qquad \text{Eq.11}$$

Moore, 1982, quoted from (Del Vita, 2016), studied the braking waves in shallow water and used the same parameters of Weggel, 1972 to build up a new braking wave index with linear relationships, this index include wave steepness ratio and bed slope as shown equation (12).

$$\frac{H_b}{d_b} = \frac{1.5}{(1+e^{-19.5m})} - 0.082 + 43.75\ (1-e^{19m})\ \left(\frac{H_o}{L_o}\right)^{\frac{4}{5}} \qquad \text{Eq.12}$$

Kamphuis, 1991, attempted to study the regular and irregular wave conditions and developed two new equations, these equations have exponential form with bed slope parameter and used for recognizing the breaking wave index. Equation (13 and 14) presented for regular and irregular wave conditions, respectively.

$$\frac{H_b}{L_b} = 0.127\ e^{4m}\tanh\left(\frac{2\pi\ d_b}{L_b}\right) \qquad \text{Eq.13}$$

$$\frac{H_b}{L_b} = 0.095\ e^{4m}\tanh\left(\frac{2\pi\ d_b}{L_b}\right) \qquad \text{Eq.14}$$

Smith and Kraus, 1991, conducted experimental research for submerged artificial reefs and bars breakwater, large number of experiments done under regular and irregular wave conditions. Linear relationship produced for breaking wave index in shallow water as shown in equation (15). The limits for applying this equation should be satisfied.

$$\frac{H_b}{d_b} = \frac{1.12}{1+e^{-60m}} + 5\ (1-e^{-43})\left(\frac{H_o}{L_o}\right) \qquad \text{Eq.15}$$

Where: $0.0007 \leq \frac{H_o}{L_o} \leq 0.0921$ and $\frac{1}{80} \leq m \leq \frac{1}{10}$

Hara et al., 1992, used numerical experiments to analyses the occurrence of wave breaking behavior for impermeable conventional breakwater, according to the results of the regression analyses, the researchers assumed that the breaking wave doesn't depend on the bed slope and wave steepness only, as in Iribarren number, but also depend on breakwater height h_c and crest width B, therefore the modifications on Iribarren number made to propose Iribarren number for submerged breakwater as presented in equation (16).

$$\xi_s = \left(\frac{B}{d} + \frac{h_c/d}{3.5 \tan\alpha}\right)^{0.2} \frac{h_c/d}{(H/d)^{0.4}} \quad \text{Eq.16}$$

Hattori and Sakai, 1994, carried out a laboratory experiments on permeable submerged breakwater and illustrated that the different porosities (0 to 0.52) of breakwater effect on breaking wave occurrence phenomenon, as a rustles, the offshore currents over submerged breakwater have effect on the breaking wave, offshore sea currents defined by the breaking wave index as present in equation (17), which depends on the Iribarren number for submerged breakwater, that proposed by (Hara et al., 1992).

$$\frac{H_b}{L_b} = \left(1 - 0.12\,\frac{R_c}{d_b} - 0.6\,\xi\right)\left(\frac{B}{5\,d_b}\right)^3 \frac{d}{L_o}\,\xi_s \quad \text{Eq.17}$$

Kawasaki and Iwata, 1998, demonstrate numerically that crest width considered an importance parameter in determining breaking wave index, a study of impermeable rectangular submerged breakwater conducted, as a results, breaking wave index decreasing when the increasing of crest width and with decreasing of relative depth. After that, Kawasaki and Iwata, 2001, assumed that breaking wave index depends on submergence and incident wave heights, a study of impermeable trapezoidal submerged type of breakwater showed that the bed slope of breakwater and its side slope have insignificant effect on breaking wave index.

Rattanapitikon and Shibayama, 2000, used 24 of existing equation to investigate the height of breaking waves according to experimental data of 574 cases, the results showed that equations with bed slope ranged between $0 \leq m \leq 0.07$ can be predicted with reasonable acceptance and bed slope ranged between $0.1 \leq m \leq 0.44$ predicted with lower confidence level. Therefore, the researchers proposed two new equations (18 and 19) for wave breaking index based on linear wave theory with regular wave condition. This equation available for bed slope ranged between $0 \leq m \leq 0.44$ and $0.001 \leq \frac{H}{L} \leq 0.1$ as shown:

$$\frac{H_b}{d_b} = 0.17\,\frac{L_o}{d_b}\left(1 - e^{\left(\frac{\pi d_b}{L_o}(16.21\,m^2 - 7.07m - 1.55)\right)}\right) \quad \text{Eq.18}$$

$$\frac{H_b}{L_b} = 0.14 \tanh\left[\frac{2\pi\, d_b}{L_b}(-11.21\,m^2 - 5.01m - 0.91)\right] \quad \text{Eq.19}$$

Rattanapitikon et al., 2003, developed a new wave breaking index equation (20) derived from re-analysis of existing laboratory results, for 695 cases collected from laboratory data. The relationship between breaking wave and deepwater steepness can be expressed by power form. Overall, equation (20) showed an agreement prediction under various experimental conditions.

$$\frac{H_b}{L_b} = (-11.21\,m^2 - 5.01m - 0.91\,)\left(\frac{H_o}{L_o}\right)^{0.35} \quad \text{Eq.20}$$

Camenen and Larson, 2007, concluded new breaking waves index equation (21) by following the same way of (Rattanapitikon and Shibayama, 2000) and (Rattanapitikon et al., 2003), but with different number of studied cases; 524, this equation used for wide range of hydraulic conditions.

$$\frac{H_b}{d_b} = C_1 \tanh\left(\left[0.87 + C_2 \sin\left\{\frac{\pi}{2}\left(\frac{m}{m_{max}}\right)^{\alpha}\right\}\right]\pi\left[\frac{H_o}{L_o}\right]\right) \quad \text{Eq.21}$$

Where: $C_1 = 0.248\left(\frac{H_o}{L_o}\right)^{-0.5}$, $C_2 = 0.32 + 14\left(\frac{H_o}{L_o}\right)$, $m_{max} = 0.1 + 1.6\left(\frac{H_o}{L_o}\right)$

$\alpha = 1 + 14\left(\frac{H_o}{L_o}\right) \quad if\ m \leq m_{max}$ and $\alpha = -1 - 20\left(\frac{H_o}{L_o}\right) \quad if\ m > m_{max}$

Yao et al., 2013, conducted a series of laboratory tests in a wave flume to explain the behavior of submerged reef breakwater, as a result, the ratio of the submergence to the wave height considered an important factor to describe wave breaking index as present in equation (22), top to that, the results showed that the influence of bed slope seems not important according to the studied experimental conditions.

$$\frac{H_b}{d_b} = \frac{Y_1 - Y_2}{2}\left\{tanh\left[\frac{\alpha}{1.4}\left(1.4 - \frac{R_c}{H_o}\right)\right] + \frac{Y_1 + Y_2}{Y_1 - Y_2}\right\} \qquad \text{Eq.22}$$

This equation can be applied when $0 \leq \frac{R_c}{H_o} \leq 2.8$ therefore, Y equal to Y_1 at $\frac{R_c}{H_o} = 0$, while Y equal to Y_2 at $\frac{R_c}{H_o} = 2.8$. The best fitting curves demonstrated that: $Y_1 = 1.07$, $Y_2 = 0.61$ and $\alpha = 3.24$, so that equation (20) modified to equation (23) as shown:

$$\frac{H_b}{d_b} = 0.23\ tanh\left(3.23 - 2.31\frac{R_c}{H_o}\right) + 8.4 \qquad \text{Eq.23}$$

Chiang et al., 2017, studied sediment transport mechanisms in coastal engineering under nonlinear wave effects, for theoretical basis considering Stokes 2nd theory, the researchers made a combination between (Le Méhauté and Koh, 1967) equation and (Goda, 1974) equation to produce equation (24) as shown:

$$\frac{d_b}{L_o} = -\frac{\ln\left[1 - 4.47(m)^{\frac{1}{7}}\left(\frac{H_o}{L_o}\right)^{0.75}\right]}{1.5\pi\left(1 + 15\ m^{\frac{4}{3}}\right)} \qquad \text{Eq.24}$$

Buckingham Theory Application

The dimensional analysis considering one of most mathematical approach accustomed explore in details of hydraulic problems or phenomenon that having different affected physical measurements to create a affiliation by recognizing their fundamental dimensions. In this article, dimensional analyses used to analyze the waves field at the certain breaking moment. The parameters characterized breaking waves behavior are shown in **Tables 1**

Table 1: Parameters characterizing the breaking waves characteristics.

No.	sample	Define parameters	Units	Dimensions
1	ρ	Density of the fluid	kg/m^3	ML^{-3}
2	μ	Dynamic viscosity of the fluid	Kg/(m*s)	M/(L*T)
3	g	Gravitational acceleration	m/s^2	LT^{-2}
4	H	Height of wave	m	L
5	d	Depth of water	m	L
6	L	Length of wave	m	L
7	V	Wave velocity	m/s	LT^{-1}
8	m	Bed slope	-	-

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The variables that affect the breaking waves behavior can be briefed as follow:
f $(\rho, \mu, V, H, g, L, M, d) = 0$ Eq.25

Buckingham π-theorem applied to found the dimensionless groups and by identifying the repeated variables that should signify flow characteristics and fluid properties as ρ, V, H, The dimensional analysis of equation (25) give:

$MLT = f_1(\rho, V, H, g)$

$MLT = f_2(\rho, V, H, \mu)$

$MLT = f_3(\rho, V, H, m)$

$MLT = f_4(\rho, V, H, L)$

$MLT = f_5(\rho, V, H, d)$

The resulted of dimensionless groups will be

$$\left(\frac{H}{L}, \frac{H}{d}, m, \frac{\rho V H}{\mu}, \frac{V^2}{g H}\right)$$

The terms $(\frac{\rho V H}{\mu}, \frac{V^2}{g H})$ represent Froude Number and Reynolds Number, respectively. The influence of these two terms can be neglected in wave braking condition because the effect of viscosity is small at open channel problems and the effect of gravity is including in wave length at wave condition state , therefore, the following relationship in equations (26 and 27) dominated :

$\frac{H}{L} = f(\frac{H}{d}, m)$ Eq.26

$\frac{H}{L} = f(\frac{H}{d}, m)$ Eq.26

2. CONCLUSIONS

Waves behavior investigated taken in to account the breaking conditions and the effects of all parameters to produce an expression that govern this physical phenomenon, using Buckingham theory as a mathematical dimension analysis technique with non-dimensional groups, shows that the parameters that have a major effect on the free surface phase at the braking moment can be summarized as: the depth of water (d), the height of wave (H), the length of wave (L) and the slope of bed (m) when compared with the other parameters of fluid properties and wave characteristics.

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