

Chemically Reactive Darcy-Forchheimer Flow of Cross Fluid by an Inclined Plate with Heat Source

Darapuneni Purna Chandar Rao^{1*}, Swaminathan Thiagarajan², VajhaSrinivas Kumar³

¹Research Scholar, Department of Mathematics, JNTUH College of Engineering, JNTU Kukatpally, Hyderabad - 500085, Telangana, India. dprao@matrusri.edu.in

²Department of Mathematics, Matrusri Engineering College, Saidabad, Hyderabad – 500059, Telangana, India. drst_rajana@yahoo.co.in

³Department of Mathematics, JNTUH College of Engineering, JNTU Kukatpally, Hyderabad - 500085, Telangana, India. vajhasrinu@gmail.com

Abstract : *In this study, we investigated a dissipative Darcy-Forchheimer flow of Cross fluid over an inclined plate with chemical reaction. Appropriate similarity transformations are used to convert governing equations as a set of ordinary differential equations. Transmuted equations are solved with the combination of shooting and Runge-Kutta fourth order procedures. Important results are elucidated using graphs. We observed that magnetic field parameter diminishes fluid velocity and skin friction coefficient. It is detected that the raise in Eckert number and heat source parameters leads to enhance the fluid temperature. And also, it is observed that the fluid velocity minifies with the raise in Forchheimer number and porosity number. Further, we noticed that the Schmidt number and chemical reaction parameters are useful to enhance to the mass transfer rate.*

Keywords: *Inclined plate, Cross fluid, Darcy-Forchheimer, Chemical reaction, Porosity parameter, Magnetic field.*

1. Introduction

Many researchers examined various Newtonian and non-Newtonian laminar fluid flows past inclined plate because of its (inclined plate) usages in several industrial areas such as iron removal, chemical processing. Khademi et al. [1] conducted an analysis on the mixed convective flow of nanofluid by an inclined plate with magnetic field. They observed that the Prandtl number minimizes the fluid

temperature. Asjad et al. [2] used Laplace transform method to find the solution for convective heat transfer of CNTs nanofluids over an inclined plate. Idowu and Falodun [3] studied a non-Newtonian nanofluid flow through an inclined porous plate with cross-diffusion effects. They discovered that the Schmidt number minimizes the fluid concentration. Adesanya et al. [4] scrutinized the gravity-driven flow of an incompressible fluid over an inclined plate. Suganya et al. [5] proposed a mathematical model for a free convective flow of MHD fluid by an inclined plate with magnetic field. They found the increment in fluid velocity with larger buoyancy ratio parameter. Dharmaiyah et al. [6] detected that the radiation parameter ameliorates the fluid temperature in their work on the dissipative flow of MHD fluid through an inclined porous plate with chemical reaction and radiation. Omamoke et al. [7] inspected radiative convective flow of MHD fluid by an inclined plate with perturbation technique and discovered that the magnetic field parameter minimizes the fluid velocity. Reddy and Sreedevi [8] numerically investigated a chemically reactive flow of nanofluid over an inclined plate with radiation and thermophoresis. Ilias et al. [9] applied finite difference scheme to resolve the mathematical model of an unsteady MHD nanofluid flow through an inclined plate with magnetic field and noticed that the nanoparticles volume fraction parameter raises the skin friction coefficient. Saqib et al. [10] elucidated blood-CNT's nanofluid by an inclined porous plate. They found that the fractional parameter lessens the fluid temperature. Goud et al. [11] discussed a natural convective flow of MHD fluid by an inclined porous plate with Joule heating. Kumar et al. [12] offered analytical solution using perturbation technique to resolve the equations in the investigation of the chemically reactive Casson fluid flow by an inclined porous plate with cross-diffusion effects. Islam et al. [13] reported a problem of steady and mixed convective flow of MHD fluid over a vertical plate with Hartmann number and heat source parameter. Yusuf et al. [14] examined the mixed convective flow of Williamson nanofluid over an inclined plate with gyrotactic microorganisms and detected that the Peclet number ameliorates the microorganism's transfer rate. Vijayaraghavan et al. [15] numerically analysed an unsteady natural convective flow of Casson fluid by an inclined plate. Dash and Mishra [16] discussed micropolar fluid flow over an inclined plate with chemical reaction and heat source/sink and observed that Soret number enhances the fluid

concentration. Recently, Varghese and Pandaand [17] contributed to the work on the fluid flow by an inclined plate.

Inspired by the aforesaid literature, in this paper, we have examined the mixed convectiveflow of Cross fluid through an inclined plate with magnetic field. We have incorporated viscous dissipationin energy equation to scrutinize the heat transfer and chemical reaction in diffusion equation to analyse the mass transfer. Runge-Kutta fourth order based shooting technique is applied to resolve the transmuted equations. Outcomes are exhibited via graphs for two instances i.e., suction and injection.

2. Formulation

The flow considered is an incompressible, mixed convective, two dimensional Cross fluid flow through an inclined (stretching) plate with viscous dissipation and chemical reaction. Darcy-Forchheimer model is used in momentum equation. We presume that plate is inclined with an angle α and elongated with velocity $u_w(x) = cx$. Flow pattern is displayed in Fig. 1. Temperatures and concentrations of the surface and ambient are denoted by T_w, T_∞ and C_w, C_∞ . Further, we neglected induced magnetic field.

With these assumptions, governing equations for this problem are given as:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left[\frac{1 + (1-n) \left(\gamma \left(\frac{\partial u}{\partial y} \right) \right)^n}{\left(1 + \left(\gamma \left(\frac{\partial u}{\partial y} \right) \right)^n \right)^2} \right] + (g \beta_T (T - T_\infty) + g \beta_C (C - C_\infty)) \cos \alpha \quad (2)$$

$$-\frac{\nu}{K} u - Fu^2 - \frac{\sigma B_0^2}{\rho} u$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left[\frac{1 + (1-n) \left(\gamma \left(\frac{\partial u}{\partial y} \right) \right)^n}{\left(1 + \left(\gamma \left(\frac{\partial u}{\partial y} \right) \right)^n \right)^2} \right] \left(\frac{\partial u}{\partial y} \right)^2 + \frac{1}{\rho C_p} Q_0 (T - T_\infty) \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - k_0 (C - C_\infty)^m \quad (4)$$

with the conditions

$$\left. \begin{aligned} u = u_w(x), v = -v_w, T = T_w, C = C_w \text{ at } y = 0 \\ u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (5)$$

u, v are velocity constituents in x, y directions, ρ fluid density, ν kinematic viscosity, σ electrical conductivity, γ fluid relaxation time, β_T thermal expansion volumetric coefficient, g gravity acceleration, K permeability of porous medium, $F \left(= \frac{C_b}{x\sqrt{K}} \right)$ inertia coefficient, B_0 magnetic field intensity, C_b drag coefficient, T fluid temperature, k thermal conductivity, C_p specific heat capacity, Q_0 (dimensional) heat source parameter, μ dynamic viscosity, D_m molecular diffusivity, k_0 chemical reaction parameter, m order of chemical reaction, v_w permeability of the porous surface.

Using the similarity transformations

$$\eta = \sqrt{\frac{c}{\nu}} y, u = cx f'(\eta), v = -\sqrt{c\nu} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad (6)$$

continuity equation is satisfied and equations (2) - (4) are changed as:

$$\frac{1 + (1-n)(We f'')^n}{[1 + (We f'')^n]^2} f''' + ff'' + \delta(\theta + \delta^* \phi) \cos \alpha - (\Lambda + M) f' - f'^2 (1 + Fr) = 0 \quad (7)$$

$$\frac{1}{Pr} \theta'' + f \theta' + \frac{1 + (1-n)(We f'')^n}{[1 + (We f'')^n]^2} E_c f'^2 + H \theta = 0 \quad (8)$$

$$\frac{1}{Sc} \phi'' + f \phi' - \Gamma \phi^n = 0 \quad (9)$$

and conditions in (5) are altered as

$$\left. \begin{aligned} f(0) = S, f'(0) = 1, \theta(0) = 1, \phi(0) = 1 \text{ i.e., at } \eta = 0 \\ f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad (10)$$

Weissenberg number We , local Reynold's number Re_x , local thermal Grashoff number Gr , mixed convection parameter δ , local diffusion Grashoff number Gc ,

ratio of the concentration to the thermal buoyancy forces δ^* , Eckert number E_c , Prandtl number Pr , Porosity parameter λ , Heat source parameter H , Forchheimer number Fr , Schmidt number Sc , Chemical reaction parameter Γ , Suction/injection coefficient S are indicated as:

$$\left. \begin{aligned} We &= c\gamma(\text{Re}_x)^{1/2}, \text{Re}_x = \frac{xu_w}{\nu}, Gr = \frac{g\beta_T(T_w - T_\infty)x^3}{\nu^2}, \delta = \frac{Gr}{\text{Re}_x^2}, \\ Gr &= \frac{g\beta_c(C_w - C_\infty)x^3}{\nu^2}, \delta^* = \frac{Gc}{Gr}, E_c = \frac{u_w^2}{C_p(T_w - T_\infty)}, Pr = \frac{\mu C_p}{k}, \\ \lambda &= \frac{\nu}{Kc}, H = \frac{Q^*}{c(\rho C_p)}, Fr = \frac{C_b}{\sqrt{K}}, Sc = \frac{\nu}{D_m}, \Gamma = \frac{k_0(C_w - C_\infty)^{n-1}}{b}, S = \frac{v_w}{\sqrt{cv}} \end{aligned} \right\}.$$

Sherwood number Sh_x , Nusselt number Nu_x , surface friction drag C_{fx} are defined as:

$$Sh_x = \frac{xS_w}{D_m(C_w - C_\infty)} \Big|_{y=0}, Nu_x = \frac{xq_w}{k(T_w - T_\infty)} \Big|_{y=0}, Cf_x = \frac{\tau_w}{\frac{1}{2}\rho u_w^2} \Big|_{y=0} \quad (11)$$

where (wall shear stress) $\tau_w = \tau_{xy} \Big|_{y=0} = \left[\mu \frac{\frac{\partial u}{\partial y}}{1 + \left(\gamma \left(\frac{\partial u}{\partial y} \right)^n \right)} \right]$, (heat flux) $q_w = -k \frac{\partial T}{\partial y}$,

(mass flux) $s_w = -D_m \frac{\partial C}{\partial y}$.

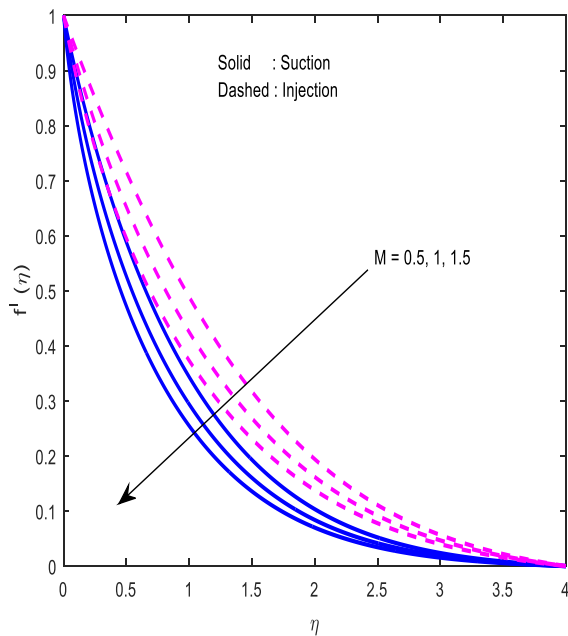
By using (6), we can write (11) in non-dimensional form as

$$(\text{Re}_x)^{-1/2} Sh_x = -\phi'(0), (\text{Re}_x)^{-1/2} Nu_x = -\theta'(0), (\text{Re}_x)^{1/2} Cf_x = 2 \frac{f''(0)}{1 + (Wef''(0))^n}.$$

3. Results and Discussion

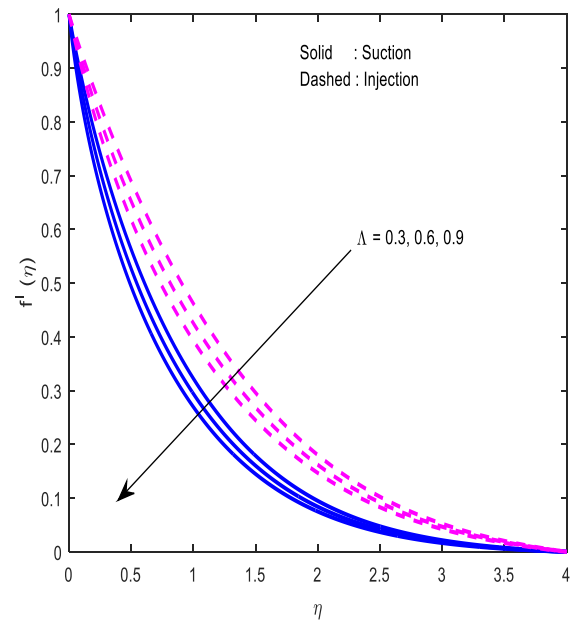
We used Runge-Kutta 4th order based shooting technique to solve transmuted ordinary differential equations [7-9] with the conditions [10]. Generally, when we apply magnetic field on the fluid flow, particles comprehend Lorentz force, which leads to the fall in velocity of the fluid (Fig. 1). Fig. 2 exhibit that larger porosity parameter lowers velocity profile because of the amelioration in fluid viscosity. Raise in Forchheimer number diminishes the fluid velocity (Fig. 3). Fluid particles get more

energy owing to electromagnetic field. So, raise in M leads to the amelioration in fluid temperature (Fig. 4). Figs. 5&6 elucidated that E_c and H are helpful in escalating fluid temperature. Increase in magnetic field parameter diminuite the friction factor (Fig. 7) and we observe the same outcome upon the impact of Forchheimer number on frictin factor (Fig. 8). Since E_c and H assists to enhance thermal boundary layer thickness, E_c and H are useful to lower the heat transfer rate (Figs. 9& 10). Fig. 11 and 12 exhibited the fact that the chemical reaction and



enhance to the mass transfer rate.

Fig. 1 Consequence of M on velocity



Schmidt numbers are useful to

Fig. 2 Consequence of Λ on

velocity profile

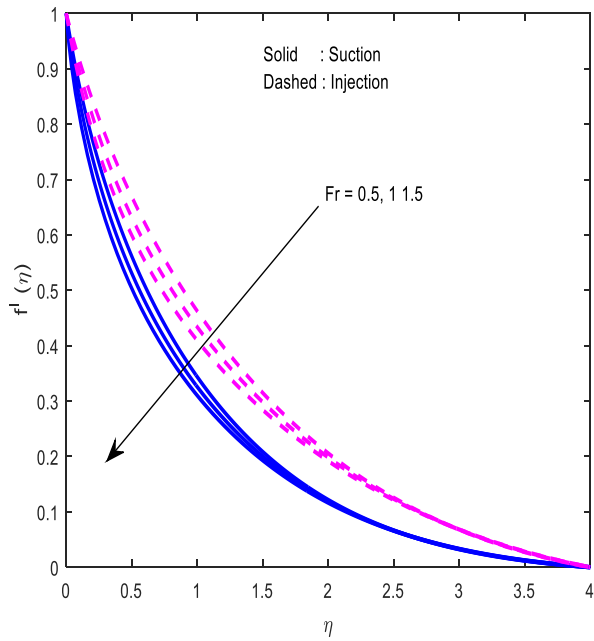


Fig. 3 Consequence of Fr on velocity profile

velocity profile

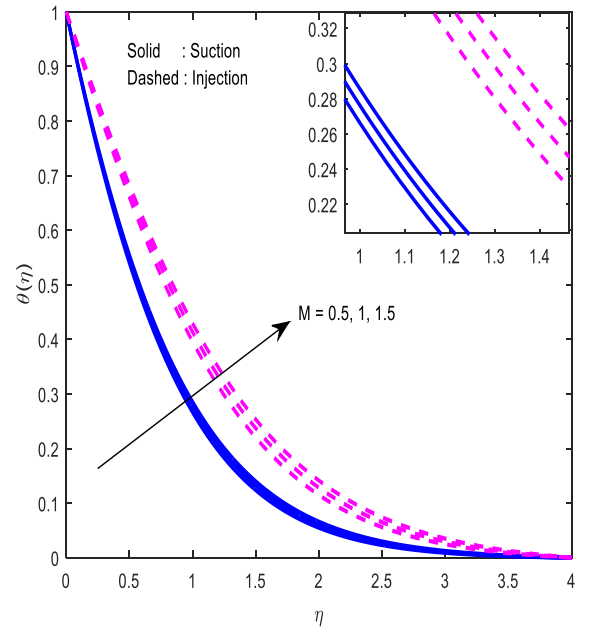


Fig. 4 Consequence of M temperature profile

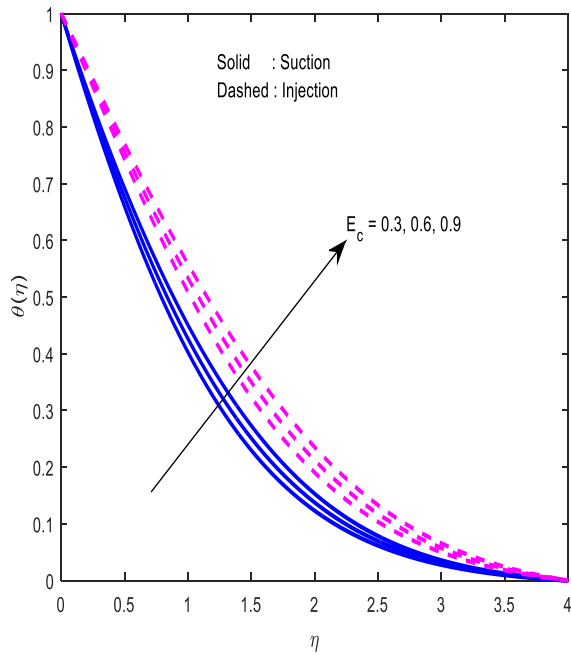


Fig. 5 Consequence of E_c on temperature profile

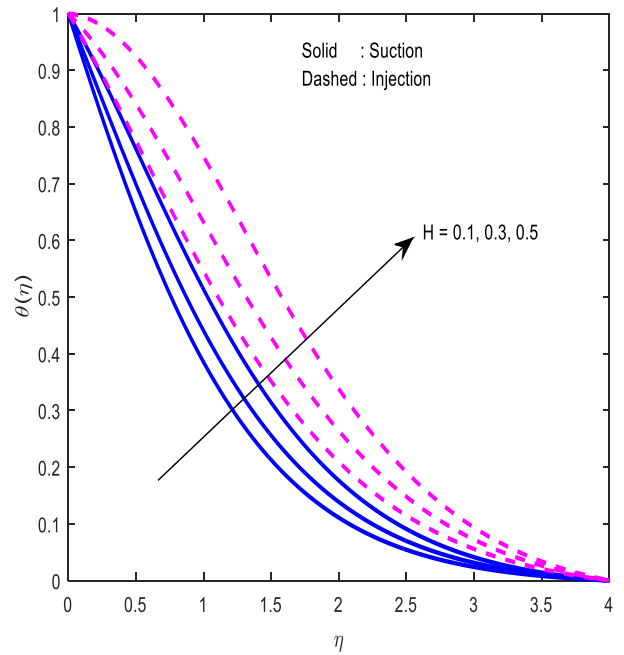


Fig. 6 Consequence of H on temperature profile

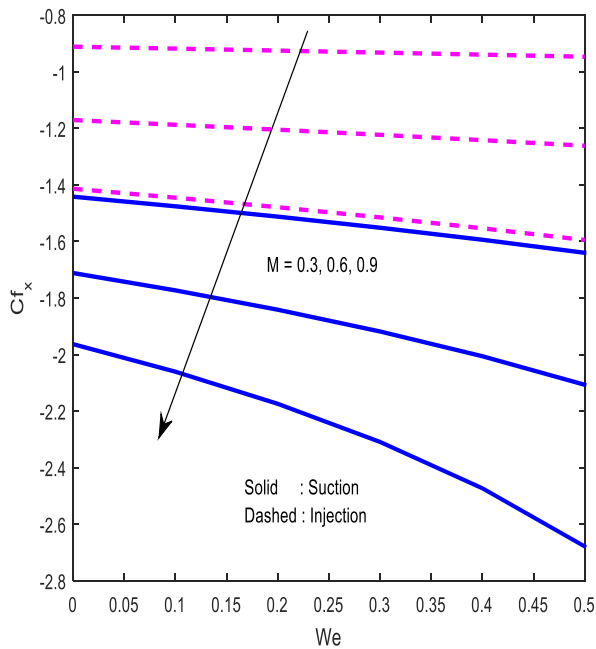


Fig. 7 Consequence of M on skin friction Coefficient

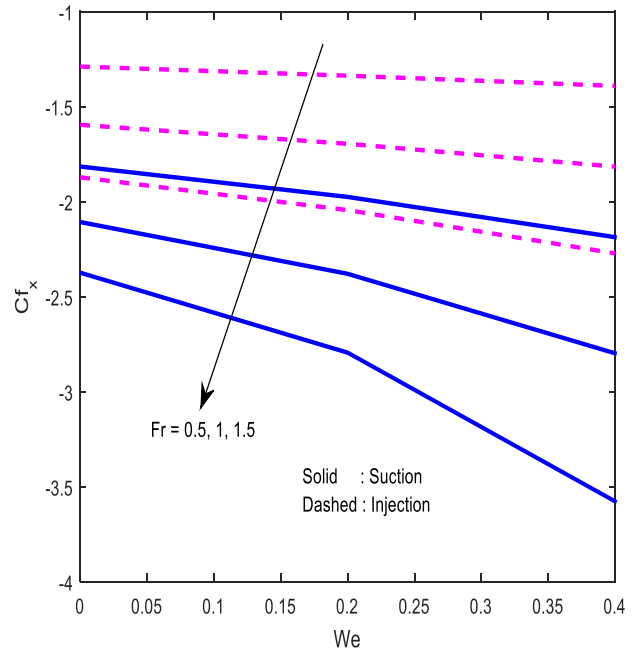


Fig. 8 Consequence of Fr on friction factor

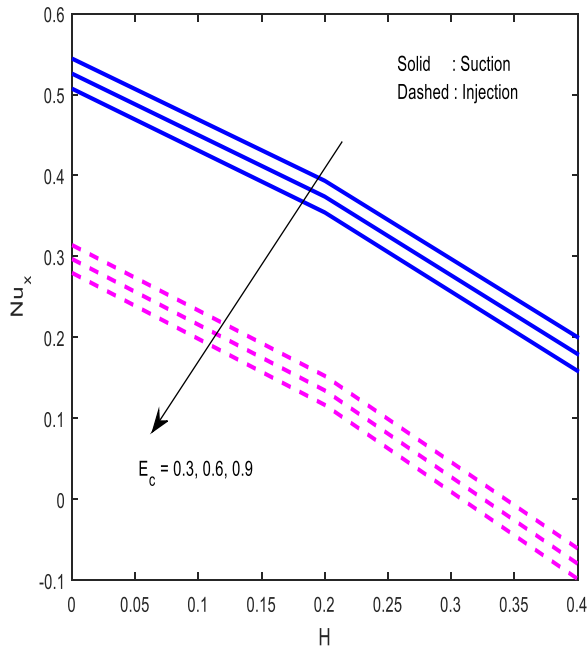


Fig. 9 consequence of E_c on Nusselt numberon Nusselt number

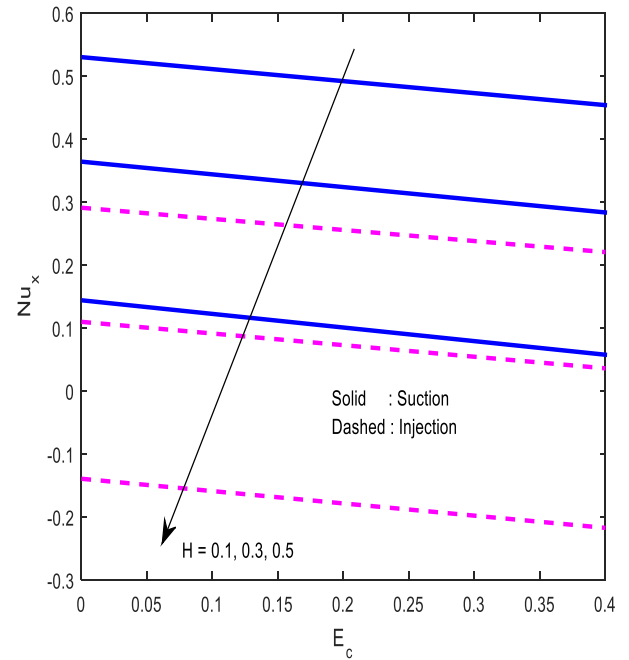


Fig. 10 Consequence of H

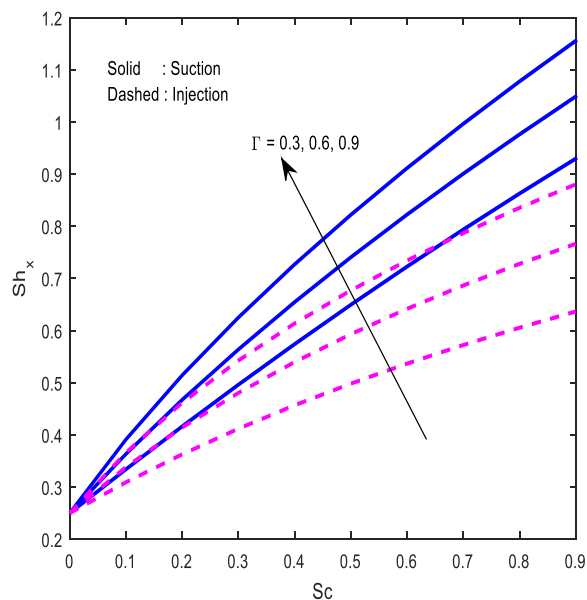


Fig. 11 Consequence of Γ on Shewood number on

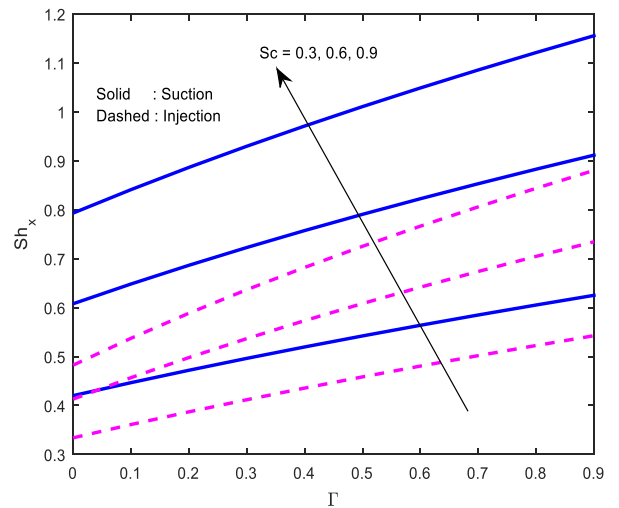


Fig. 12 Consequence of Sc Sherwood number

4. Conclusion

Main outcomes of the present work are mentioned below:

- Magnetic field parameter minimizes velocity of the fluid and enhances temperature.
- Eckert number escalates fluid temperature.
- Porosity parameter lowers velocity of the fluid.
- Forchheimer number lessens the friction factor.
- Heat transfer rate minifies with larger heat source parameter.
- Friction factor minifies with larger magnetic field parameter.
- Chemical reaction minimizes the fluid concentration and ameliorates the mass transfer rate.

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