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Cordial Labeling In Some Special Graphs

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Abstract: A bijection $f : V(G) \to \{0,1\}$ ***is called binary vertex labeling of*** $G(V,E)$ ***and*** $f(v)$ ***is called the label of the vertex*** $v \in G$ ***under*** f ***.For an edge*** $e = uv$***, the induced edge labeling*** $f^* : E(G) \to \{0,1\}$ ***is given by*** $f^*(e) = |f(u) - f(v)|$***. Let*** $v_f(0), v_f(1)$ ***be the number of vertices of G having labels 0 and 1 respectively under f and*** $e_f(0), e_f(1)$ ***be the number of edges having labels 0 and 1 respectively under*** f^****.A graph*** $G(V,E)$ ***is cordial if it admits cordial labelling.In this paper, we investigate the cordiallabeling for some special graphs like Golomb graph,Wagner graph, Herschel graph, Petersen graph,etc. .***

Keywords: Binary labeling, cordial labeling and cordial Graph.

1. INTRODUCTION

Let $G(V,E)$ graph of order $|V(G)| = p|$ and size $|E(G)| = q$. A labeling of a graph is a map that carries the graph elements to the set of positive integers. If the domain is the set of vertices the labeling is called vertex labeling. If the domain is the set of edges, then we called about edge labeling. If the labels are assigned to both vertices and edges then the labeling is called total labeling.

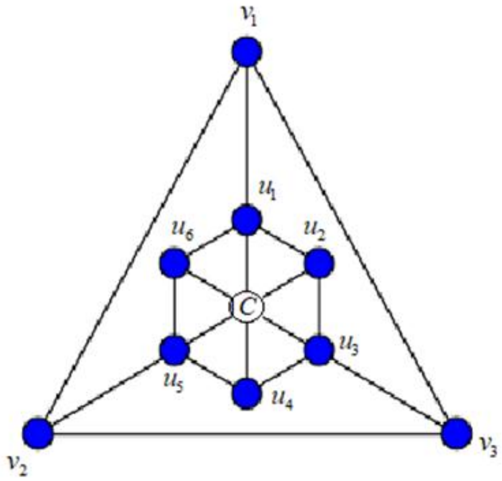
Definition 1.1: Let $G(V,E)$ be a graph. A mapping $f : V(G) \to \{0,1\}$ is called binary vertex labeling of $G(V,E)$ and $f(v)$ is called the label of the vertex $v \in G$ under f .

For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_f(0), e_f(1)$ be the number of edges having labels 0 and 1 respectively under f^*.A graph $G(V,E)$ is cordial if it admits cordial labelling.

2. MAIN RESULTS

In this segmentwe examine thecordial labelling of several special graphs alikeBidiakis cube,Durer graph,Golomb graph and etc.

Golomb Graph: In Golomb graph is a graph with 10 vertex and 18 edges.



Remarks:

1. In the Golomb graph, 6 vertices forms a inner Hexagon ,3 vertices forms the outer triangle and the remaining vertex C lies in the Centre of the Hexagon and triangle.
2. The 3 vertices of the triangle is adjacent to only one vertices on the Hexagon.

Theorem 2.1 Let $G(V,E)$ is a Golombgraph. Then the Golombgraph is a cordial graph.

Proof: Let $G(V,E)$ is a Golombgraph. In Golombgraph there are 10 vertices and 18 edges in Golombgraph. Construct the mapping $f:V \rightarrow \{0,1\}$ as follows.

$$f(c)=0,\ f(u_i)=\begin{cases}0, & for\ i=3,4\\ 1, & for\ i=1,2,5,6\end{cases} \text{ and } f(v_i)=\begin{cases}0, & for\ i=1,3\\ 1, & for\ i=2\end{cases}$$

We note that from the above mapping in the Golombgraph, the number of edges labeled 1 or 0 is $e_{f^*}(1)=9$ and $e_{f^*}(0)=9$. It satisfies the condition $|e_{f^*}(1)-e_{f^*}(0)| \le 1$. Hence the Golombgraph is a cordial graph.

Illustration:

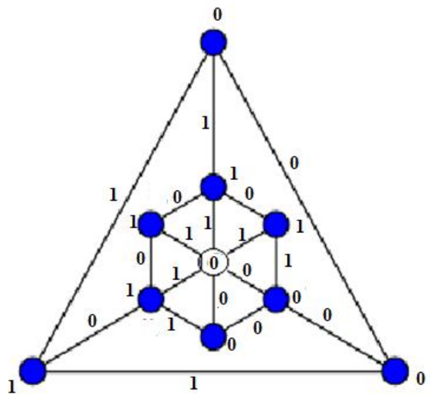


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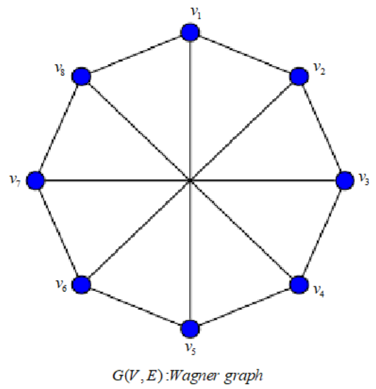
Figure 2.1: Cordial labeling of Golombgraph

In the Golombgraph, the number of edges labeled 1 or 0 is $e_{f*}(1)=9$ and $e_{f*}(0)=9$. It satisfies the condition $\left|e_{f*}(1)-e_{f*}(0)\right|\leq 1$. Hence the Golombgraph is a cordial graph.

Wagner graph: The Wagner graph is a 3- regular with 8 vertices and 12 edges



Theorem 2.2: Let $G(V,E)$ is a Wagner graph . Then the Wagner graph is a cordial graph.

Proof: Let $G(V,E)$ is a Wagner graph .In Wagner graph there are 8 vertices and 12 edges in Wagner graph . Construct the mapping $f:V\to\{0,1\}$ as follows.

$$f(v_i)=\begin{cases}0, & for\ i=1,2,4,8\\ 1, & for\ i=3,5,6,7\end{cases}$$

We note that from the above mapping in the Wagner graph , the number of edges labeled 1 or 0 is $e_{f*}(1)=6$ and $e_{f*}(0)=6$. It satisfies the condition $\left|e_{f*}(1)-e_{f*}(0)\right|\leq 1$. Hence the Wagner graph is a cordial graph.

Illustration:

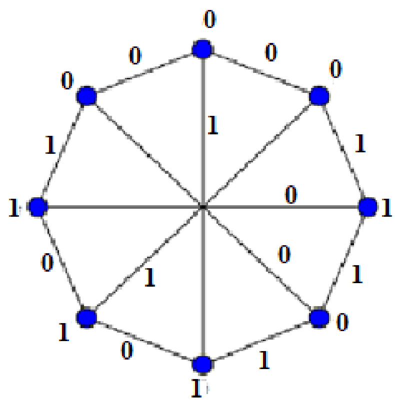
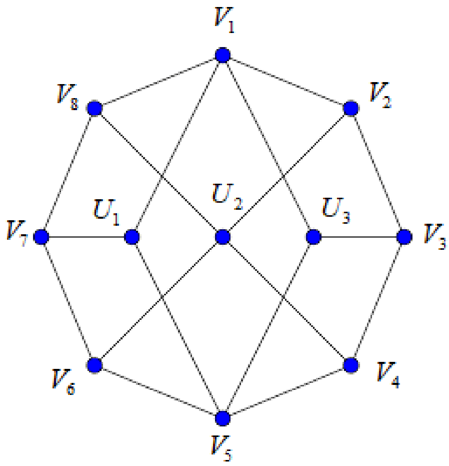


Figure 2.2: Cordial labeling of Wagner graph

In the Wagner graph, the number of edges labeled 1 or 0 is $e_{f*}(1)=6$ and $e_{f*}(0)=6.$ It satisfies the condition $|e_{f*}(1)-e_{f*}(0)|\leq 1$. Hence the Wagner graph is a cordial graph.

Herschel graph: The Herschel graph is the smallest non Hamiltonian polyhedral graph. It is the unique such graph on 11 nodes and 18 edges.



$G(V,E)$:**Herschel graph**

Remarks:

1. Herschel graph consist outer and inner region with 11 and 3 vertices respectively.
2. The outer region form aOctagon and inner vertices not adjacent with each other vertices.

Theorem 2.3: Let $G(V,E)$ is a Herschelgraph. Then the Herschelgraph is a cordial graph.

Proof: Let $G(V,E)$ is a Herschelgraph .In Herschelgraph there are 11 vertices and 18 edges in Herschelgraph . Construct the mapping $f:V\rightarrow\{0,1\}$ as follows.

$$f(u_i)=\begin{cases}0, \ for\ i=2,3\\1, \ for\ i=1\end{cases} \text{ and } f(v_i)=\begin{cases}0, \ for\ i=1,2,5,6\\1, \ for\ i=3,4,7,8\end{cases}$$

We note that from the above mapping in the Herschelgraph, the number of edges labeled 1 or 0 is $e_{f*}(1)=9$ and $e_{f*}(0)=9.$ It satisfies the condition $|e_{f*}(1)-e_{f*}(0)|\leq 1$. Hence the Herschelgraph is a cordial graph.

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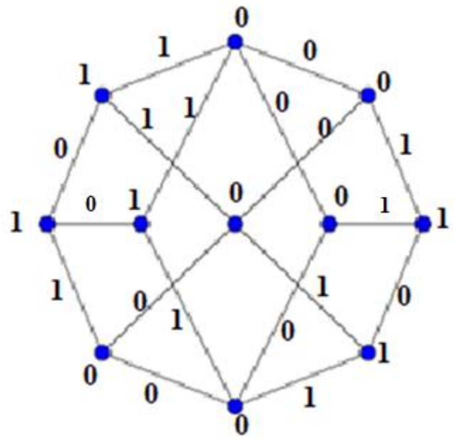
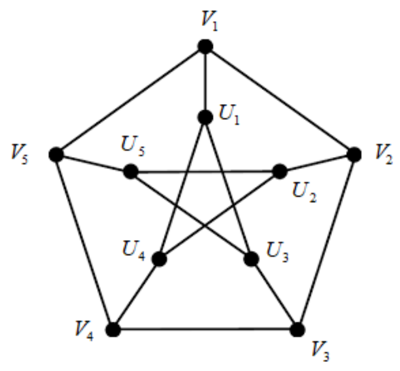


Illustration:

Figure 2.3: Cordial labeling of Wagner graph

In the Herschelgraph, the number of edges labeled 1 or 0 is $e_{f^*}(1)=9$ and $e_{f^*}(0)=9$. It satisfies the condition $|e_{f^*}(1)-e_{f^*}(0)|\le 1$. Hence the Herschelgraph is a cordial graph.

Petersen graph :In the mathematical field of graph theory, the Petersen graph is an undirected graph with 10 vertices and 15 edges.



G(V,E):Petersen graph

Remarks:

1. Petersen graph is 3 regular graph.
2. Petersen graph consist outer and inner region with 5 vertices each. The outer region form a pentagon and inner region forms a star.
3. One vertices of the pentagon adjacent with at most one vertices in the inner region

Theorem 2.4: Let $G(V,E)$ is a Petersen graph . Then the Petersen graph is a cordial graph.

Proof: Let $G(V,E)$ is a Petersen graph .In Petersen graph there are 10 vertices and 15 edges in Petersen graph . Construct the mapping $f:V\to\{0,1\}$ as follows.

$$f(u_i)=\begin{cases}0, \; for\ i=1,3,4\\ 1, \; for\ i=2,5\end{cases} \text{ and } f(v_i)=\begin{cases}0, \; for\ i=4,5\\ 1, \; for\ i=1,2,3\end{cases}$$

We note that from the above mapping in the Petersen graph, the number of edges labeled 1 or 0 is $e_{f^*}(1)=8$ and $e_{f^*}(0)=7$. It satisfies the condition $|e_{f^*}(1)-e_{f^*}(0)|\le 1$. Hence the Petersen graph is a cordial graph.

Illustration:

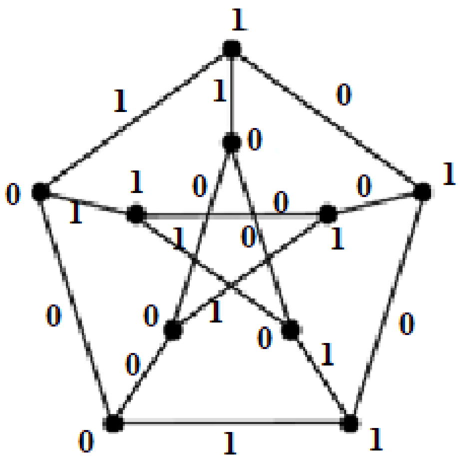
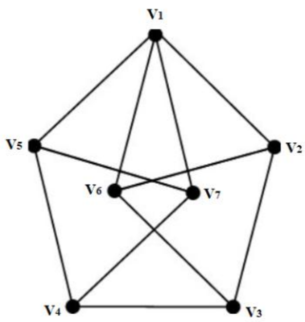


Figure 2.5: Cordial labeling of Petersen graph

In the Petersen graph, the number of edges labeled 1 or 0 is $e_{f^*}(1)=8$ and $e_{f^*}(0)=7$. It satisfies the condition $|e_{f^*}(1)-e_{f^*}(0)|\le 1$. Hence the Petersen graph is a cordial graph.

Moser spindle: In graph theory, a branch of mathematics, the Moser spindle (also called the Mosers' spindle or Moser graph) is an undirected graph, named after mathematicians Leo Moser and his brother William, with 7 vertices and 11 edges.



Theorem 2.5: Let $G(V,E)$ is a Mosergraph. Then the Moser graph is a cordial graph.

Proof: Let $G(V,E)$ is a Mosergraph .In Moser graph there are 7 vertices and 11 edges in Moser graph . Construct the mapping $f:V \rightarrow \{0,1\}$ as follows.

$$f(v_i) = \begin{cases} 0, \ for\ i = 1,2,5,6 \\ 1, \ for\ i = 3,4,7 \end{cases}$$

We note that from the above mapping in the Mosergraph , the number of edges labeled 1 or 0 is $e_{f^*}(1) = 5$ and $e_{f^*}(0) = 6$. It satisfies the condition $|e_{f^*}(1) - e_{f^*}(0)| \leq 1$. Hence the Moser graph is a cordial graph.

Illustration:

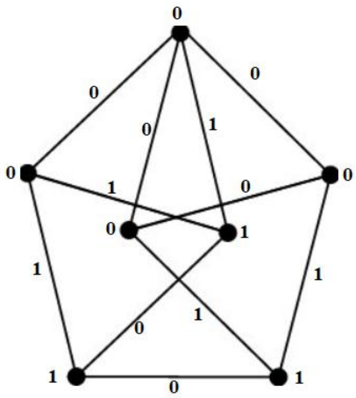


Figure 2.5: Cordial labeling of Petersen graph

In the Mosergraph , the number of edges labeled 1 or 0 is $e_{f^*}(1) = 5$ and $e_{f^*}(0) = 6$. It satisfies the condition $|e_{f^*}(1) - e_{f^*}(0)| \leq 1$. Hence the Moser graph is a cordial graph.

3. CONCLUSION:

In this paper, we investigate the cordiallabeling for some special graphs like Golomb graph,Wagner graph, Herschel graph, Petersen graph,etc. In future,investigate the various cordiallabeling for some special graphs.

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