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Fuzzy DEA Approach To Analysing Performance Efficiency Of Indian States & Union Territories In Demographic And Literacy

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Abstract: The purpose of this research is to evaluate the Literacy efficiency of Indian Statesusing Fuzzy Data Envelopment Approach. This approach suggests a framework for evaluating the best performing State & Union territories in education by using a combination of various input and output variables. For this study, Total Population (15-24 aged), Amount allocated from Total Budget and Total Expenditure are considered as aninput variablewhereas Literacy Rate and Gross Enrolment Ratio are considered as output variables.By using a hybrid learning procedure, the proposed Fuzzy Inference System can construct an input-output mapping based on the form of fuzzy if-then rules and stipulated input-output data pairs. This analysis compares the States in India and assists us in determining which is the finest in education.

Keywords: Fuzzy Inference System, Efficiency, Fuzzy Constant Returns to Scale model, Fuzzy Variable Returns to Scale model, Linear Regression model.

1. INTRODUCTION

Data Envelopment Analysis (DEA) is a technique that focuses on a specific application of linear programming. It was created for the purpose of evaluating the performance measurement.It's been used to assess the relative performance of a group of companies that produce a variety of identical outputs from a variety of identical inputs. The DEA was introduced by Charnes, Cooper, and Rhodes in 1978 [4]. Farrell [5] developed the DEA principles in 1957. It's a performance evaluation instrument for determining the relative effectiveness of decision-making units [DMUs] in organizations.Many articles have been published on the application of DEA in real-world situations. For multi-input, multi-output production functions, the framework has been used in a variety of industries.The best performing DMU is given a unit or 100 percent efficiency score, and the performance of other DMUs is rated between 0 and 100 percent in comparison to this best performance [9& 10].

2. LITERATURE REVIEW

According to a review of previous research literatures, DEA can be used in a variety of fields. In 2003, Saowanee Lertworasirikul et al.,[11] develops DEA models using imprecise data represented by fuzzy sets. They discussed an approach that transforms fuzzy DEA models into possibility DEA models by using possibility measures of fuzzy events (fuzzy constraints). A taxonomy and review of the fuzzy DEA methods were discussed by Adel Hatami et al.,[1] in 2011. Also, they present a classification scheme with four primary categories, namely, the tolerance approach, the a-level based approach, the fuzzy ranking approach and the possibility approach. Loganathan et al.,[7] converted the fractional programming problem in to a single objective linear programming problem in parametric form and introduced new fuzzy arithmetic and fuzzy ranking to obtain the optimal solution without converting to its equivalent crisp linear programming problem in 2019.

The researcher considered the nature of DEA applications and used them into the different way from than earlier research. As a result, the current study stands apart from all previous research. The researcher used Indian States and Union Territories as DMUs in this study to determine their efficiency in terms of demographics and literacy.

Data Collection and Selection of Input and Output Variables
For this research, the required data of Indian States& Union Territories have been carried based on the availability of reputed data from the website censusindia.gov.in for the years 1991, 2001, 2011.
Reviewing the literature on the application of Fuzzy DEA, different studies have used different combination of inputs and outputs. The current study considered three input variables and two output variables in order to hold an elaborate study. The variables under the study are presented below

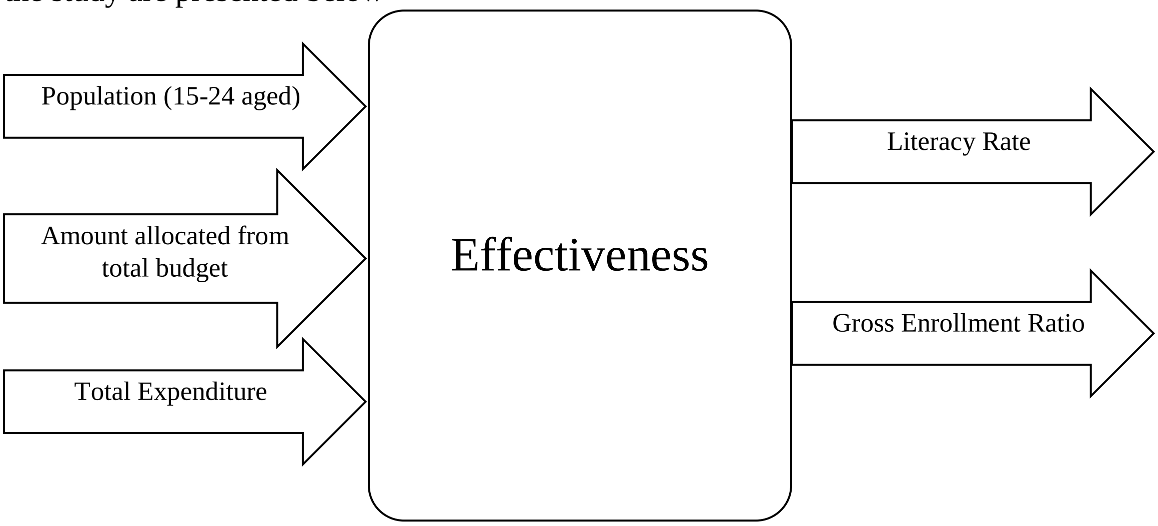


Figure: 1 Selected Input and Output Variables

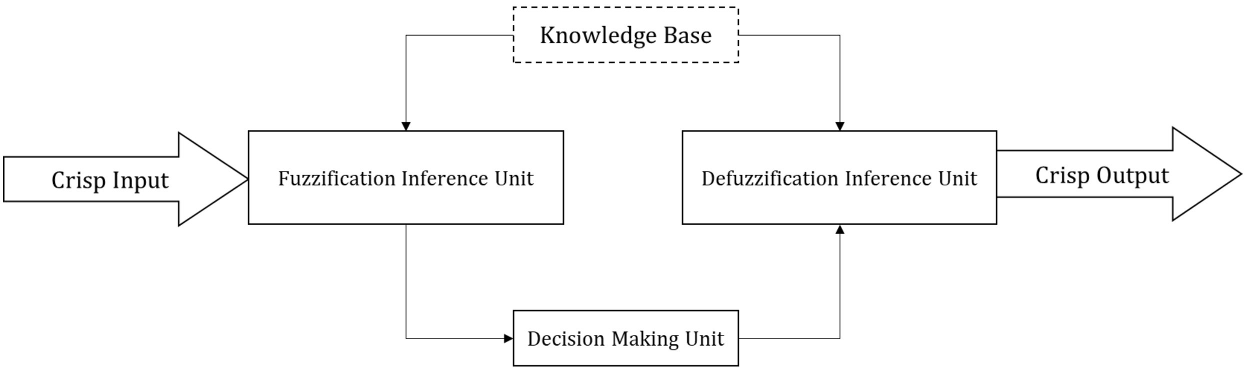
3. METHODOLOGY

Linear Regression Model

Due to insufficient data the researcher has used the linear regression model to predict the values for the year 2021.
Regression is a mathematical measure that refers the relationship between 2 variables. This is used to predict the expected value of 1 variable if the value of another is given. Among the 2 variables, one should be treated as an independent variable and the other as dependent.This relationship can be expressed in the form of a linear equation in 2 variables. Amongthe 2 variablesat a time can be treated as dependent on the other [8].
Let the linear equation representingthe data be

$$Y = aX + b$$

Fuzzy Inference System
A fuzzy inference system is composed of five functionalblocks:
1. A rule base containing a number of fuzzy if–then rules.
2. A database which defines the membership functions of the fuzzy sets used in thefuzzyrules.
3. A decision making unit which performs the inference operations on the rules.
4. A fuzzification inference which transforms the crisp inputs into degree of match withlinguistic values.
5. A defuzzification interface which transforms the fuzzy results of the inference into acrisp output.
Usually, the rule base and the database are jointly referred to as the knowledge base.Severaltypes of FIS have been proposed in the literature (Lee, 1990) [3]. It is due to thedifferences between the specification of the consequent part and the defuzzificationschemes [6].



Flowchart: 1 Fuzzy Inference System

4. MATHEMATICAL MODEL

Fractional DEA Program
Let there be N DMUs whose efficiencies have to be compared. Let us take one of the DMUs. Say the m^{th} DMU. And maximize its efficiency, according to the formula given above. Here the m^{th} DMU is the reference DMU [3, 10].
The mathematical problem is,

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$$Max\ E_m = \frac{\sum_{j=1}^{J} v_{jm} y_{jm}}{\sum_{i=1}^{I} u_{im} x_{im}}$$

Subject to the Constraints

$$0 \le \frac{\sum_{j=1}^{J} v_{jm} y_{jn}}{\sum_{i=1}^{I} u_{im} x_{in}} \le 1;\ n = 1,2,K,J$$

$$v_{jm}, u_{im} \ge 0;\ \ i = 1,2,K,I;\ \ j = 1,2,K$$

Where,
E_m is the efficiency of the m^{th} DMU,
Y_{ij} is the j^{th} output of the m^{th} DMU,
V_{jm} is the weight of that output,
X_{im} is i^{th} the input of the m^{th} DMU,
U_{jm} is the weight of that input, and
Y_{jn} and X_{in} are output j^{th} and i^{th} input, respectively, of the n^{th} DMU, n = 1,2,…,N.
Note that here n includes m.

The Fuzzy DEA principles:
The observed values in real-world problems are often imprecise or vague. Imprecise or vague data may be the result of unquantifiable, incomplete and non-obtainable information. Imprecise or vague data is often expressed with bounded intervals, ordinal (rank order) data or fuzzy numbers. In recent years, many researchers have formulated fuzzy DEA models to deal with situations where some of the input and output data are imprecise or vague [1].

Fuzzy Fractional DEA Program
Let there be n DMUs whose efficiencies have to be compared. Let us take one of the DMUs. Say the m^{th} DMU. And maximize its efficiency, according to the formula given above. Here the m^{th} DMU is the reference DMU. Where tilde represents the fuzzy values.
The mathematical problem is,

$$Max\ \tilde{E}_m = \frac{\sum_{j=1}^{J} v_{jm}\tilde{y}_{jm}}{\sum_{i=1}^{I} u_{im}\tilde{x}_{im}}$$

Subject to the Constraints

$$0 \le \frac{\sum_{j=1}^{J} v_{jm}\tilde{y}_{jm}}{\sum_{i=1}^{I} u_{im}\tilde{x}_{im}} \le 1;\quad \text{n} = 1,2,\dots,\text{k},\text{j}$$

$$v_{jm}, u_{im} \ge 0;\ \ i = 1,2,\dots\text{k},\text{i};\ \ j = 1,2,\dots k$$

Where,
$\tilde{E}_m$ is the efficiency of the m^{th} DMU,
$\tilde{v}_{ij}$ is the j^{th} fuzzy output of the m^{th} DMU,
y_{jm} is the weight of that output,
$\tilde{u}_{im}$ is i^{th} the fuzzy input of the m^{th} DMU,
x_{jm} is the weight of that input and
Y_{jn} and X_{in} are output j^{th} and i^{th} input, respectively, of the n^{th} DMU, n = 1, 2, …, N.

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Note that here n includes m.

Constant Returns to Scale & Variable Returns to Scale Model
The original CRS model was pertinent but to that expertise which is categorized by Constant Returns to Scale. The major promotion was extended by chance, and cooper (VRS) model to facilitate expertise that reveals the variable returns to scale. This study has used input-oriented DEA model, which emphasizes on the minimization of inputs and the maximization of outputs held at their current level and also the VRS model with varying returns to scale is believed.

General Form of F-CRS Model
The general form Output Maximization F-DEA [F-CRS] model can be represented in the form of Fuzzy Fractional Programming Model as follows:Here the general model is built to maximize the efficiency of the output variable:
$\tilde{v}_{jq} - j^{th}$fuzzy output value of theq^{th} DMU

$$y_{jq} - j^{th} \text{ output variable of the } q^{th} \text{ DMU}$$

$\tilde{u}_{iq} - i^{th}$fuzzy input value of theq^{th} DMU

$$x_{iq} - i^{th} \text{ input valueof the } q^{th} \text{ DMU}$$

$\tilde{E}_q -$ Efficiency of the q^{th} DMU

$$Max\ \ \tilde{E}_q = \frac{\sum_{j=1}^{m} \tilde{v}_{jq} y_{jq}}{\sum_{i=1}^{s} \tilde{u}_{iq} x_{iq}}$$

Subject to the constraints

$$\frac{\sum_{j=1}^{m} \tilde{v}_{jq} y_{jq}}{\sum_{i=1}^{s} \tilde{u}_{iq} x_{iq}} \leq 1; q = 1,2,\ldots,n$$

$$v_{jq}, y_{jq}, \tilde{u}_{iq}, \tilde{x}_{iq} \geq 0\ for\ all\ i = 1,2,..s; j = 1,2,..m, q = 1,2,\ldots n$$

Solving this Fuzzy Fractional Programming Problem directly is so tedious; hence the Fuzzy Fractional Programming model is changed into regular Fuzzy Linear Programming model as identified:

$$\text{Max}\, \tilde{E}_q = \sum_{j=1}^{m} \tilde{v}_{jq} y_{jq}$$

Subject to the constraints

$$\sum_{i=1}^{s} \tilde{u}_{iq} x_{iq} = 1$$

$$\sum_{j=1}^{m} \tilde{v}_{jq} y_{jq} - \sum_{i=1}^{s} \tilde{u}_{iq} x_{iq} \leq 0\,; \quad q = 1,2,\ldots n$$

$$\tilde{v}_{jq}, y_{jq}, \tilde{u}_{iq}, x_{iq} \geq 0\ for\ all\ i = 1,2,\ldots s; j = 1,2,..m, q = 1,2,\ldots n$$

The universal form of Input Minimization F-DEA [F-CRS] Linear Programming model can be interpreted as sticks with:

$$\text{Min}\, \tilde{E}_q = \sum_{i=1}^{s} \tilde{u}_{iq} x_{iq}$$

Subject to the constraints

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$$\sum_{j=1}^{m} \tilde{v}_{jq} y_{jq} = 1; \quad \sum_{j=1}^{m} \tilde{v}_{jq} y_{jq} - \sum_{i=1}^{s} \tilde{u}_{iq} x_{iq} \leq 0; \qquad q = 1,2, \dots, n$$
$$\tilde{v}_{jq}, y_{jq}, \tilde{u}_{iq}, x_{iq} \geq 0 \; for \; all \; i = 1,2,..s; j = 1,2,..m, q = 1,2, \dots n$$

General form of F-VRS Model

The DEA envelopment program for considering fuzzy variables return to scale is as follows [2]:

$$\text{Min}\, \theta_m$$

Subject to the Constraints

$$\tilde{Y}\lambda \geq \tilde{Y}_m; \quad \tilde{X}\lambda \leq \theta \tilde{X}_m$$
$$\sum_{n=1}^{N} \lambda_n = 1;$$
$$\lambda_n \geq 0; \quad \tilde{\theta}_m \text{free variable}$$

Trapezoidal Fuzzy Number

A Trapezoidal fuzzy number, which represented with four points as follows, $A = (a_1, a_2, a_3, a_4), a_i \in R$, This representation is interpreted as membership function

$$\mu_A = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & a_3 \leq x \leq a_4 \\ 0 & \text{Otherwise} \end{cases}$$

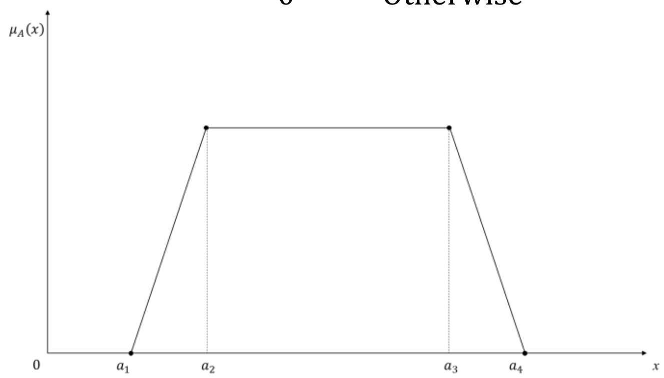

Figure: 2 Trapezoidal Fuzzy Number

Defuzzification

Since technical processes require clear control actions, a procedure which generates net values from one or several given fuzzy numbers.

Graded Mean Integration representation

Chen and Hseih propose graded mean integration representation for representing generalized fuzzy number [14, 15].

If the generalized fuzzy number $A = (a_1, a_2, a_3, a_4 : w)$, then the graded mean h-level is $\frac{h[L^{-1}(h)+R^{-1}(h)]}{2}$. Where $L^{-1}$and R^{-1} are inverse functions of L and R [14, 15].

And, the defuzzified value of the Fuzzy number A by the graded mean integration representation $\Re(A)$is defined as [14, 15]

$$\Re(A) = \frac{\int_0^h \left[\frac{L^{-1}(h)+R^{-1}(h)}{2}\right] dh}{\int_0^w h\,dh}$$

Where $h \in (0, w)$, and $0 < w \leq 1$.
If $A = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number. Chen and Hsieh have already found the general formulae of the representation of generalized pentagonal fuzzy number as follows [14, 15]:

$$\Re(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

For this study, the researcher used Graded Mean Integration representation method for defuzzification [14, 15].

Problem Formulation: Effectiveness
Fuzzy Constant Returns to Scale [Output Maximization]
Andaman and Nicobar Islands (1991)

$$Max\ E_{ANI} = \frac{73.02x_1 + 8.20x_2}{281x_3 + 5.00x_4 + 180539x_5}$$

Subject to the Constraints,

$$\frac{44.08x_1 + 5.72x_2}{66508x_3 + 1.40x_4 + 9884163x_5} \leq 1$$

$$\frac{41.59x_1 + 4.70x_2}{865x_3 + 12.80x_4 + 368690x_5} \leq 1$$

$$\frac{52.89x_1 + 6.23x_2}{22414x_3 + 1.10x_4 + 4403883x_5} \leq 1$$

$$\frac{37.49x_1 + 3.70x_2}{64531x_3 + 2.80x_4 + 11981670x_5} \leq 1$$

$$\frac{77.81x_1 + 5.72x_2}{642x_3 + 5.20x_4 + 396152x_5} \leq 1$$

$$\frac{42.91x_1 + 4.30x_2}{17615x_3 + 0.01x_4 + 4281165x_5} \leq 1$$

$$\frac{40.71x_1 + 4.59x_2}{138x_3 + 7.00x_4 + 29850x_5} \leq 1$$

$$\frac{71.20x_1 + 2.30x_2}{102x_3 + 9.90x_4 + 38604x_5} \leq 1$$

$$\frac{75.29x_1 + 8.17x_2}{9421x_3 + 6.60x_4 + 2853507x_5} \leq 1$$

$$\frac{75.51x_1 + 10.42x_2}{1170x_3 + 3.70x_4 + 660982x_5} \leq 1$$

$$\frac{61.29x_1 + 7.19x_2}{41310x_3 + 0.01x_4 + 8880771x_5} \leq 1$$

$$\frac{55.85x_1 + 6.36x_2}{16464x_3 + 10.40x_4 + 3169995x_5} \leq 1$$

$$\frac{63.86x_1 + 11.05x_2}{5171x_3 + 2.10x_4 + 1827795x_5} \leq 1$$

$$\frac{51.50x_1 + 6.17x_2}{7837x_3 + 4.50x_4 + 1429159x_5} \leq 1$$

$$\frac{41.39x_1 + 7.40x_2}{21844x_3 + 0.01x_4 + 3717689x_5} \leq 1$$

$$\frac{56.04x_1 + 5.65x_2}{44977x_3 + 0.70x_4 + 7808732x_5} \leq 1$$

$$\frac{89.81x_1 + 10.53x_2}{29099x_3 + 6.80x_4 + 7613447x_5} \leq 1$$

$$\frac{81.78x_1 + 10.78x_2}{52x_3 + 4.00x_4 + 46045x_5} \leq 1$$

$$\frac{44.67x_1 + 5.55x_2}{48566x_3 + 4.50x_4 + 8750554x_5} \leq 1$$

$$\frac{64.87x_1 + 7.95x_2}{78937x_3 + 0.90x_4 + 17061469x_5} \leq 1$$

$$\frac{59.89x_1 + 6.29x_2}{1837x_3 + 1.80x_4 + 675471x_5} \leq 1$$

$$\frac{49.10x_1 + 5.85x_2}{1775x_3 + 2.60x_4 + 559237x_5} \leq 1$$

$$\frac{82.26x_1 + 7.44x_2}{690x_3 + 0.80x_4 + 425093x_5} \leq 1$$

$$\frac{61.65x_1 + 6.86x_2}{1210x_3 + 2.50x_4 + 470325x_5} \leq 1$$

$$\frac{49.09x_1 + 5.16x_2}{31660x_3 + 1.00x_4 + 4445480x_5} \leq 1$$

$$\frac{74.74x_1 + 12.65x_2}{808x_3 + 5.50x_4 + 342782x_5} \leq 1$$

$$\frac{58.51x_1 + 7.28x_2}{20282x_3 + 8.30x_4 + 4986902x_5} \leq 1$$

$$\frac{38.55x_1 + 4.49x_2}{44006x_3 + 1.20x_4 + 7970482x_5} \leq 1$$

$$\frac{56.94x_1 + 4.82x_2}{406x_3 + 3.10x_4 + 238440x_5} \leq 1$$

$$\frac{62.66x_1 + 9.80x_2}{55859x_3 + 2.60x_4 + 12619123x_5} \leq 1$$

$$\frac{60.44x_1 + 8.07x_2}{2757x_3 + 0.60x_4 + 1033665x_5} \leq 1$$

$$\frac{40.71x_1 + 5.16x_2}{132062x_3 + 2.70x_4 + 20798452x_5} \leq 1$$

$$\frac{57.75x_1 + 9.20x_2}{7051x_3 + 0.01x_4 + 3813950x_5} \leq 1$$

$$\frac{57.70x_1 + 6.49x_2}{68078x_3 + 3.50x_4 + 13553003x_5} \leq 1$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

The corresponding F-LPP structure for the above problem can be written as follows,

$$Max\ E_{ADMC} = 73.02x_1 + 8.20x_2$$

Subject to the Constraints,

$$281x_3 + 5.00x_4 + 180539x_5 = 1$$

$$44.08x_1 + 5.72x_2 - 66508x_3 - 1.40x_4 - 9884163x_5 \leq 0$$

$$41.59x_1 + 4.70x_2 - 865x_3 - 12.80x_4 - 368690x_5 \leq 0$$

$$52.89x_1 + 6.23x_2 - 22414x_3 - 1.10x_4 - 4403883x_5 \leq 0$$

$$37.49x_1 + 3.70x_2 - 64531x_3 - 2.80x_4 - 11981670x_5 \leq 0$$

$$77.81x_1 + 5.72x_2 - 642x_3 - 5.20x_4 - 396152x_5 \leq 0$$

$$42.91x_1 + 4.30x_2 - 17615x_3 - 0.01x_4 - 4281165x_5 \leq 0$$

$$40.71x_1 + 4.59x_2 - 138x_3 - 7.00x_4 - 29850x_5 \leq 0$$

$$71.20x_1 + 2.30x_2 - 102x_3 - 9.90x_4 - 38604x_5 \leq 0$$

$$75.29x_1 + 8.17x_2 - 9421x_3 - 6.60x_4 - 2853507x_5 \leq 0$$

$$75.51x_1 + 10.42x_2 - 1170x_3 - 3.70x_4 - 660982x_5 \leq 0$$

$$61.29x_1 + 7.19x_2 - 41310x_3 - 0.01x_4 - 8880771x_5 \leq 0$$

$$55.85x_1 + 6.36x_2 - 16464x_3 - 10.40x_4 - 3169995x_5 \leq 0$$

$$63.86x_1 + 11.05x_2 - 5171x_3 - 2.10x_4 - 1827795x_5 \leq 0$$

$$51.50x_1 + 6.17x_2 - 7837x_3 - 4.50x_4 - 1429159x_5 \leq 0$$

$$41.39x_1 + 7.40x_2 - 21844x_3 - 0.01x_4 - 3717689x_5 \leq 0$$

$$56.04x_1 + 5.65x_2 - 44977x_3 - 0.70x_4 - 7808732x_5 \leq 0$$

$$89.81x_1 + 10.53x_2 - 29099x_3 - 6.80x_4 - 7613447x_5 \leq 0$$

$$81.78x_1 + 10.78x_2 - 52x_3 - 4.00x_4 - 46045x_5 \leq 0$$

$$44.67x_1 + 5.55x_2 - 48566x_3 - 4.50x_4 - 8750554x_5 \leq 0$$

$$64.87x_1 + 7.95x_2 - 78937x_3 - 0.90x_4 - 17061469x_5 \leq 0$$

$$59.89x_1 + 6.29x_2 - 1837x_3 - 1.80x_4 - 675471x_5 \leq 0$$

$$49.10x_1 + 5.85x_2 - 1775x_3 - 2.60x_4 - 559237x_5 \leq 0$$

$$82.26x_1 + 7.44x_2 - 690x_3 - 0.80x_4 - 425093x_5 \leq 0$$

$$61.65x_1 + 6.86x_2 - 1210x_3 - 2.50x_4 - 470325x_5 \leq 0$$

$$49.09x_1 + 5.16x_2 - 31660x_3 - 1.00x_4 - 4445480x_5 \leq 0$$

$$74.74x_1 + 12.65x_2 - 808x_3 - 5.50x_4 - 342782x_5 \leq 0$$

$$58.51x_1 + 7.28x_2 - 20282x_3 - 8.30x_4 - 4986902x_5 \leq 0$$

$$38.55x_1 + 4.49x_2 - 44006x_3 - 1.20x_4 - 7970482x_5 \leq 0$$

$$56.94x_1 + 4.82x_2 - 406x_3 - 3.10x_4 - 238440x_5 \leq 0$$

$$62.66x_1 + 9.80x_2 - 55859x_3 - 2.60x_4 - 12619123x_5 \leq 0$$

$$60.44x_1 + 8.07x_2 - 2757x_3 - 0.60x_4 - 1033665x_5 \leq 0$$

$$40.71x_1 + 5.16x_2 - 132062x_3 - 2.70x_4 - 20798452x_5 \leq 0$$

$$57.75x_1 + 9.20x_2 - 7051x_3 - 0.01x_4 - 3813950x_5 \leq 0$$

$$57.70x_1 + 6.49x_2 - 68078x_3 - 3.50x_4 - 13553003x_5 \leq 0$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Fuzzy Variable Returns to Scale:

Andaman and Nicobar Islands (1991)

Min $x_{36} - x_{37}$

Subject to the constraints

$$73.02x_1 + 44.08x_2 + 41.59x_3 + 52.89x_4 + 37.49x_5 + 77.81x_6 + 42.91x_7 + 40.71x_8 + 71.20x_9 + 75.29x_{10} + 75.51x_{11} + 61.29x_{12} + 55.85x_{13} + 63.86x_{14} + 51.50x_{15} + 41.39x_{16} + 56.04x_{17} + 89.81x_{18} + 81.78x_{19} + 44.67x_{20} + 64.87x_{21} + 59.89x_{22} + 49.10x_{23} + 82.26x_{24} + 61.65x_{25} + 49.09x_{26} + 74.74x_{27} + 58.51x_{28} + 38.55x_{29} + 56.94x_{30} + 62.66x_{31} + 60.44x_{32} + 40.71x_{33} + 57.75x_{34} + 57.70x_{35} \geq 73.02$$

$$8.20x_1 + 5.72x_2 + 4.70x_3 + 6.23x_4 + 3.70x_5 + 5.72x_6 + 4.30x_7 + 4.59x_8 + 2.30x_9 + 8.17x_{10} + 10.42x_{11} + 7.19x_{12} + 6.36x_{13} + 11.05x_{14} + 6.17x_{15} + 7.40x_{16} + 5.65x_{17} + 10.53x_{18} + 10.78x_{19} + 5.55x_{20} + 7.95x_{21} + 6.29x_{22} + 5.85x_{23} + 7.44x_{24} + 6.86x_{25} + 5.16x_{26} + 12.65x_{27} + 7.28x_{28} + 4.49x_{29} + 4.82x_{30} + 9.80x_{31} + 8.07x_{32} + 5.16x_{33} + 9.20x_{34} + 6.49x_{35} \geq 8.20$$

$$281x_1 + 66508x_2 + 865x_3 + 22414x_4 + 64531x_5 + 642x_6 + 17615x_7 + 138x_8 + 102x_9 + 9421x_{10} + 1170x_{11} + 41310x_{12} + 16464x_{13} + 5171x_{14} + 7837x_{15} + 21844x_{16} + 44977x_{17} + 29099x_{18} + 52x_{19} + 48566x_{20} + 78937x_{21} + 1837x_{22} + 1775x_{23} + 690x_{24} + 1210x_{25} + 31660x_{26} + 808x_{27} + 20282x_{28} + 44006x_{29} + 406x_{30} + 55859x_{31} + 2757x_{32} + 132062x_{33} + 7051x_{34} + 68078x_{35} + 281x_{36} - 281x_{37} \leq 0$$

$$5.00x_1 + 1.40x_2 + 12.80x_3 + 1.10x_4 + 2.80x_5 + 5.20x_6 + 0.01x_7 + 7.00x_8 + 9.90x_9 + 6.60x_{10} + 3.70x_{11} + 0.01x_{12} + 10.40x_{13} + 2.10x_{14} + 4.50x_{15} + 0.01x_{16} + 0.70x_{17} + 6.80x_{18} + 4.00x_{19} + 4.50x_{20} + 0.90x_{21} + 1.80x_{22} + 2.60x_{23} + 0.80x_{24} + 2.50x_{25} + 1.00x_{26} + 5.50x_{27} + 8.30x_{28} + 1.20x_{29} + 3.10x_{30} + 2.60x_{31} + 0.60x_{32} + 2.70x_{33} + 0.01x_{34} + 3.50x_{35} + 5.00x_{36} - 5.00x_{37} \leq 0$$

$$180539x_1 + 9884163x_2 + 368690x_3 + 4403883x_4 + 11981670x_5 + 396152x_6 + 4281165x_7 + 29850x_8 + 38604x_9 + 2853507x_{10} + 660982x_{11} + 8880771x_{12} + 3169995x_{13} + 1827795x_{14} + 1429159x_{15} + 3717689x_{16} + 7808732x_{17} + 7613447x_{18} + 46045x_{19} + 8750554x_{20} + 17061469x_{21} + 675471x_{22} + 559237x_{23} + 425093x_{24} + 470325x_{25} + 4445480x_{26} + 342782x_{27} + 4986902x_{28} + 7970482x_{29} + 238440x_{30} + 12619123x_{31} + 1033665x_{32} + 20798452x_{33} + 3813950x_{34} + 13553003x_{35} + 180539x_{36} - 180539x_{37} \leq 0$$

$$\sum_{i=1}^{37} x_i = 1$$

$$x_i \geq 0, i = 1,2, \ldots, 37$$

All Such 350 problems were generated from the collected dataand solved using the software TORA.

5. EMPIRICAL RESULTS

Fuzzy Constant Return to Scale [F-CCR Model]

The F-DEA performance score based on Technical Efficiency [Fuzzy Constant Returns to Scale] under the F-CCR Model is shown in Table 1. The Analysis reveals that among the selected Indian States&Union Territoriestaken for the study only 2 union territoriesattained the maximum efficiency score as 1.

Table 1: F-DEA Efficiency Score – F-CCR Model

State/Union Territory	Efficiency Score	De-fuzzified Score
Andaman and Nicobar Islands	0.6,0.687,1,1	0.83
Andhra Pradesh	0.162,0.083,0.351,1	0.34
Arunachal Pradesh	0.14,0.734,0.782,0.596	0.63
Assam	0.318,0.225,1,0.979	0.62
Bihar	0.084,0.328,0.123,0.245	0.21
Chandigarh	0.488,1,1,1	0.91
Chhattisgarh	0.739,0.903,0.069,0.284	0.49

State/Union Territory	Efficiency Score	De-fuzzified Score
Dadra and Nagar Haveli	0.762,0.797,0.526,0.589	0.67
Daman and Diu	1,1,1,0.873	0.98
Delhi	0.154,0.096,0.258,0.15	0.17
Goa	0.584,0.507,0.654,0.639	0.59
Gujarat	1,0.15,0.102,0.488	0.33
Haryana	0.095,0.094,0.209,0.145	0.14
Himachal Pradesh	0.502,0.178,0.168,0.231	0.24
Jammu and Kashmir	0.208,0.843,0.122,0.179	0.39
Jharkhand	0.825,0.765,0.189,0.885	0.60
Karnataka	0.307,0.114,0.159,0.513	0.23
Kerala	0.136,0.083,0.329,0.166	0.19
Lakshadweep	1,1,1,1	1
Madhya Pradesh	0.091,0.092,0.214,0.189	0.15
Maharashtra	0.191,0.079,0.336,0.877	0.32
Manipur	0.478,1,1,0.594	0.85
Meghalaya	0.426,1,1,0.634	0.84
Mizoram	1,1,1,1	1
Nagaland	0.556,0.544,0.671,0.595	0.60
Odisha	0.305,0.105,0.293,0.466	0.26
Puducherry	0.688,0.572,0.859,0.619	0.69
Punjab	0.092,0.972,0.109,0.085	0.39
Rajasthan	0.166,0.057,0.329,0.411	0.22
Sikkim	0.596,0.924,0.766,0.578	0.76
Tamil Nadu	0.18,0.031,0.384,0.22	0.21
Tripura	1,0.562,0.384,0.346	0.54
Uttar Pradesh	0.072,0.041,0.1,0.188	0.09
Uttarakhand	1,1,0.205,0.411	0.64
West Bengal	0.108,0.065,0.151,0.211	0.13

Fuzzy Variable Return to Scale [F-BCC Model]

The F-DEA efficiency score based on Technical Efficiency [Fuzzy Variable Returns to Scale] under the F-BCC Model is shown in Table 2. In F-BCC Model there is an increment in the number of efficient DMUs. The Analysis reveals that among the selected States & Union Territoriestaken for the study, Kerala, Lakshadweep& Mizoram attained the maximum efficiency score as 1.

Table:2F-DEA Efficiency Score – F-BCC Model

State/Union Territory	Efficiency Score	De-fuzzified Score
Andaman and Nicobar Islands	0.673,0.688,1,1	0.84
Andhra Pradesh	0.232,0.085,0.857,1	0.52
Arunachal Pradesh	0.276,1,0.977,0.708	0.82
Assam	0.4,0.245,1,1	0.65
Bihar	0.152,0.5,0.147,0.268	0.29
Chandigarh	0.514,1,1,1	0.92
Chhattisgarh	1,1,0.09,0.31	0.58

State/Union Territory	Efficiency Score	De-fuzzified Score
Dadra and Nagar Haveli	1,1,0.617,0.627	0.81
Daman and Diu	1,1,1,0.886	0.98
Delhi	0.191,0.099,1,0.151	0.42
Goa	0.885,0.529,0.659,0.641	0.65
Gujarat	1,0.152,0.106,0.52	0.34
Haryana	0.118,0.121,0.318,0.147	0.19
Himachal Pradesh	1,0.198,0.179,0.238	0.33
Jammu and Kashmir	0.265,1,0.145,0.224	0.46
Jharkhand	1,1,0.211,1	0.74
Karnataka	0.346,0.117,0.238,0.534	0.27
Kerala	1,1,1,1	1
Lakshadweep	1,1,1,1	1
Madhya Pradesh	0.129,0.093,0.217,0.22	0.16
Maharashtra	0.238,1,1,0.887	0.85
Manipur	0.585,1,1,0.654	0.87
Meghalaya	0.599,1,1,0.764	0.89
Mizoram	1,1,1,1	1
Nagaland	0.678,0.665,0.753,0.703	0.70
Odisha	0.418,0.115,0.297,0.543	0.30
Puducherry	1,0.588,0.861,0.668	0.76
Punjab	0.095,1,0.118,0.1	0.41
Rajasthan	0.28,0.061,0.367,0.487	0.27
Sikkim	0.858,1,0.77,0.665	0.84
Tamil Nadu	0.368,0.036,1,0.253	0.45
Tripura	1,0.562,0.388,0.378	0.55
Uttar Pradesh	0.114,0.05,0.126,0.231	0.12
Uttarakhand	1,1,0.226,0.489	0.66
West Bengal	0.128,0.068,0.155,0.24	0.14

Table 3 shows that only two Union Territories are extremely standardized, with an efficiency score of 1.

Table 3: Efficient Countries Identified by F-CCR and F-BCC Models

State/Union Territory	CRS Score	VRS Score	Mean Score
Andaman and Nicobar Islands	0.83	0.84	0.84
Andhra Pradesh	0.34	0.52	0.43
Arunachal Pradesh	0.63	0.82	0.73
Assam	0.62	0.65	0.64
Bihar	0.21	0.29	0.25
Chandigarh	0.91	0.92	0.92
Chhattisgarh	0.49	0.58	0.54
Dadra and Nagar Haveli	0.67	0.81	0.74
Daman and Diu	0.98	0.98	0.98
Delhi	0.17	0.42	0.30

State/Union Territory	CRS Score	VRS Score	Mean Score
Goa	0.59	0.65	0.62
Gujarat	0.33	0.34	0.34
Haryana	0.14	0.19	0.17
Himachal Pradesh	0.24	0.33	0.28
Jammu and Kashmir	0.39	0.46	0.42
Jharkhand	0.60	0.74	0.67
Karnataka	0.23	0.27	0.25
Kerala	0.19	1	0.59
Lakshadweep	1	1	1
Madhya Pradesh	0.15	0.16	0.16
Maharashtra	0.32	0.85	0.59
Manipur	0.85	0.87	0.86
Meghalaya	0.84	0.89	0.87
Mizoram	1	1	1
Nagaland	0.60	0.70	0.65
Odisha	0.26	0.30	0.28
Puducherry	0.69	0.76	0.73
Punjab	0.39	0.41	0.40
Rajasthan	0.22	0.27	0.25
Sikkim	0.76	0.84	0.80
Tamil Nadu	0.21	0.45	0.33
Tripura	0.54	0.55	0.54
Uttar Pradesh	0.09	0.12	0.10
Uttarakhand	0.64	0.66	0.65
West Bengal	0.13	0.14	0.13

6. CONCLUSION

The Efficiency Analysis based on Fuzzy Constant Returns to Scale demonstrates that two Indian Union Territoriessuch as: Lakshadweep and Mizoramcome out on top in Demographic & Literacy and the analysis based on the Fuzzy Variable Returns to Scale Communicates that Kerala, Lakshadweep and Mizoramare the top state & union territories in the efficiency level of Demographic & Literacy. When these analyses are compared, it is clear that Lakshadweep and Mizoram are the most efficient places in terms of demographics and literacy.

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