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Performance Efficiency Of The Colleges Affiliated With Bharathidasan University:A Fuzzy DEA Approach

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Abstract: The aim of this study is to use Fuzzy Data Envelopment Analysis to assess the efficiency of colleges affiliated with Bharathidasan University. This approach suggests a framework for evaluating the best performing colleges using a combination of input and output variables. The researcher used five uncertain input variables for this study such as the number of students, number of faculty, number of qualified faculty, number of text books in the library and students benefited from scholarships as well as two uncertain output variables such as the number of students placed on campus and the number of students enrolled for Higher Studies.By using a hybrid learningprocedure, the proposed Fuzzy Inference System can construct an input-outputmapping based on the form of fuzzyif-then rules and stipulated input-output data pairs.This analysis compares the colleges and assists us in determining which one is the best.

Keywords:Fuzzy Inference System,Graded Mean Integration Representation,Fuzzy Constant Returns to Scale model, Fuzzy Variable Returns to Scale model.

1. INTRODUCTION

Data Envelopment Analysis (DEA) is a technique that focuses on a specific application of linear programming. It was created for the purpose of evaluating the performance measurement.It's been used to assess the relative performance of a group of companies that produce a variety of identical outputs from a variety of identical inputs. The DEA was introduced by Charnes, Cooper, and Rhodes in 1978 [3]. Farrell [4] developed the DEA principles in 1957. It's a performance evaluation instrument for determining the relative effectiveness of decision-making units [DMUs] in organizations.Many articles have been published on the application of DEA in real-world situations. For multi-input, multi-output production functions, the framework has been used in a variety of industries.The best performing DMU is given a unit or 100 percent efficiency score, and the performance of other DMUs is rated between 0 and 100 percent in comparison to this best performance [7 & 8].

2. LITERATURE REVIEW

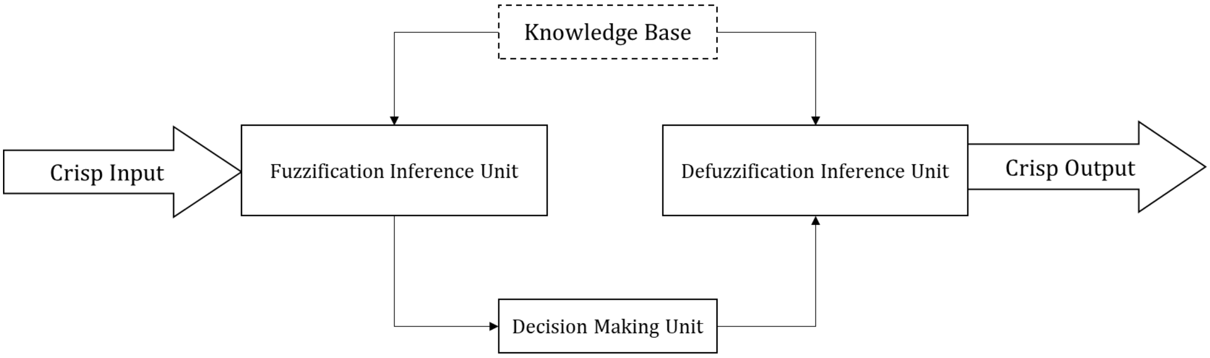
According to a review of previous research literatures, DEA can be used in a variety of fields.
In 2003, Saowanee Lertworasirikul et al.,[9]develops DEA models using imprecise data represented by fuzzy sets. They discussed an approach that transforms fuzzy DEA models into possibilityDEA models by using possibility measures of fuzzy events (fuzzy constraints). A taxonomy and review of the fuzzy DEA methods were discussed by Adel Hatamiet al.,[1]in 2011. Also, they present a classification scheme with fourprimary categories, namely, the tolerance approach, the a-level based approach, the fuzzy rankingapproach and the possibility approach.Loganathan et al.,[6]converted the fractional programming problem into a single objective linear programming problem in parametric form and introduced new fuzzy arithmeticand fuzzy ranking to obtain the optimal solution without converting to its equivalent crisp linear programming problemin 2019.

This analysis differs entirely from the previous studies. In this study, the researcher considered the nature of Fuzzy DEA applications and used them to assess the effectiveness of Bharathidasan University's affiliated colleges from 2015 to 2019.

3. METHODOLOGY

Fuzzy Inference System
A fuzzy inference system is composed of five functionalblocks:
i. A rule base containing a number of fuzzy IF–THEN rules.
ii. A database which defines the membership functions of the fuzzy sets used in thefuzzyrules.
iii. A decision-making unit which performs the inference operations on the rules.
iv. A fuzzification inference which transforms the crisp inputs into degree of match withlinguistic values.
v. A defuzzification interface which transforms the fuzzy results of the inference into acrisp output.

Usually, the rule base and the database are jointly referred to as the knowledge base.
Several types of FIS have been proposed in the literature. It is due to the differences between the specification of the consequent part and the defuzzification schemes [5].



Flowchart: 1 Fuzzy Inference System [5]

4. MATHEMATICAL MODEL

Fractional DEA Program

Let there be N DMUs whose efficiencies have to be compared. Let us take one of the DMUs. Say the m^{th} DMU. And maximize its efficiency, according to the formula given above. Here the m^{th} DMU is the reference DMU [3].

The mathematical problem is,

$$Max\ E_m = \frac{\sum_{j=1}^{J} v_{jm} y_{jm}}{\sum_{i=1}^{I} u_{im} x_{im}}$$

Subject to the Constraints

$$0 \le \frac{\sum_{j=1}^{J} v_{jm} y_{jn}}{\sum_{i=1}^{I} u_{im} x_{in}} \le 1;\ n= 1,2,K,J$$

$$v_{jm}, u_{im} \ge 0;\ \ i = 1,2,K,I;\ \ j = 1,2,K$$

Where,

E_m is the efficiency of the m^{th}DMU,

$Y_{ij}$is the j^{th} output of the m^{th} DMU,

$V_{jm}$is the weight of that output,

X_{im} is i^{th} the input of the m^{th} DMU,

$U_{jm}$is the weight of that input, and

$Y_{jn}$and X_{in} are output j^{th} and i^{th} input, respectively, of the n^{th} DMU, n = 1,2,…,N.

Note that here n includes m.

The Fuzzy DEA principles:

The observed values in real-world problems are often imprecise or vague. Imprecise or vague data may be the result of unquantifiable, incomplete and non-obtainable information.Imprecise or vague data is often expressed with bounded intervals, ordinal (rank order) data orfuzzy numbers. In recent years, many researchers have formulated fuzzy DEA models to deal with situations where some of the input andoutput data are imprecise or vague [1].

Fuzzy Fractional DEA Program

The data in the conventional CCR and BCC models assume the form of specific numerical values. However, the observed value of theinput and output data are sometimes imprecise or vague. Sengupta (1992) [10, 11] was the first to introduce a fuzzy mathematical programmingapproach in which fuzziness was incorporated into the DEA model by defining tolerance levels on both the objective function and constraintviolations.

Let there be N DMUs whose efficiencies have to be compared. Let us take one of the DMUs. Say the m^{th} DMU. And maximize its efficiency, according to the formula given above. Here the m^{th} DMU is the reference DMU [3]. Where tilde represents the fuzzy values.

The mathematical problem is,

$$Max\ \tilde{E}_m = \frac{\sum_{j=1}^{J} v_{jm}\tilde{y}_{jm}}{\sum_{i=1}^{I} u_{im}\tilde{x}_{im}}$$

Subject to the Constraints

$$0 \le \frac{\sum_{j=1}^{J} v_{jm}\tilde{y}_{jm}}{\sum_{i=1}^{I} u_{im}\tilde{x}_{im}} \le 1; \quad \mathrm{n} = 1,2,\dots,\mathrm{k},\mathrm{j}$$

$$v_{jm}, u_{im} \ge 0; \ \ i = 1,2,\dots \mathrm{k}, \mathrm{i}; \ \ j = 1,2,\dots k$$

Where,

$\tilde{E}_m$ is the efficiency of the m^{th}DMU,

$\tilde{v}_{ij}$is the j^{th}fuzzy output of the m^{th} DMU,

$y_{jm}$is the weight of that output,

$\tilde{u}_{im}$ is i^{th} the fuzzy input of the m^{th} DMU,

$x_{jm}$is the weight of that input and

$Y_{jn}$and X_{in} are output j^{th} and i^{th} input, respectively, of the n^{th} DMU, n = 1, 2, …, N.

Note that here n includes m.

Constant Returns to Scale & Variable Returns to Scale Model

The original CRS model was pertinent but to that expertise which is categorized by Constant Returns to Scale. The major promotion was extended by chance, and cooper (VRS) model to facilitate expertise that reveals the variable returns to scale. This study has used input-oriented DEA model, which emphasizes on the minimization of inputs and the maximization of outputs held at their current level and also the VRS model with varying returns to scale is believed.

General Form of F-CRS Model

The general form Output Maximization F-DEA [F-CRS] model can be represented in the form of Fuzzy Fractional Programming Model as follows:Here the general model is built to maximize the efficiency of the output variable:

$\tilde{v}_{jq} - j^{th}$fuzzy output value of theq^{th} DMU

$$y_{jq} - j^{th} \text{ output variable of the } q^{th} \text{ DMU}$$

$$\tilde{u}_{iq} - i^{th} \text{ fuzzy input valueof the } q^{th} \text{ DMU}$$

$$x_{iq} - i^{th} \text{ input valueof the } q^{th} \text{ DMU}$$

$\tilde{E}_q -$ Efficiency of the q^{th} DMU

$$Max\ \ \tilde{E}_q = \frac{\sum_{j=1}^{m} \tilde{v}_{jq}y_{jq}}{\sum_{i=1}^{s} \tilde{u}_{iq}x_{iq}}$$

Subject to the constraints

$$\frac{\sum_{j=1}^{m} \tilde{v}_{jq}y_{jq}}{\sum_{i=1}^{s} \tilde{u}_{iq}x_{iq}} \le 1; q = 1,2,\dots,n$$

$$v_{jq}, y_{jq}, \tilde{u}_{iq}, \tilde{x}_{iq} \ge 0 \ for\ all\ i = 1,2,..s; j = 1,2,..m, q = 1,2,\dots n$$

Solving this Fractional Programming Problem directly is so tedious; hence the Fractional Programming model is changed into regular Linear Programming model as identified infra:

$$\mathrm{Max}\, E_q = \sum_{j=1}^{m} \tilde{v}_{jq}y_{jq}$$

Subject to the constraints

$$\sum_{i=1}^{s} \tilde{u}_{iq} x_{iq} = 1$$
$$\sum_{j=1}^{m} \tilde{v}_{jq} y_{jq} - \sum_{i=1}^{s} \tilde{u}_{iq} x_{iq} \le 0\,; \quad q = 1,2,\ldots n$$
$$\tilde{v}_{jq}, y_{jq}, \tilde{u}_{iq}, x_{iq} \ge 0 \; for\; all\; i = 1,2,\ldots s; j = 1,2,..m, q = 1,2,\ldots n$$

The universal form of Input Minimization F-DEA [F-CRS] Linear Programming model can be interpreted as sticks with:

$$\text{Min}\; \tilde{E}_q = \sum_{i=1}^{s} \tilde{u}_{iq} x_{iq}$$

Subject to the constraints

$$\sum_{j=1}^{m} \tilde{v}_{jq} y_{jq} = 1; \quad \sum_{j=1}^{m} \tilde{v}_{jq} y_{jq} - \sum_{i=1}^{s} \tilde{u}_{iq} x_{iq} \le 0; \qquad q = 1,2,\ldots,n$$
$$\tilde{v}_{jq}, y_{jq}, \tilde{u}_{iq}, x_{iq} \ge 0 \; for\; all\; i = 1,2,..s; j = 1,2,..m, q = 1,2,\ldots n$$

General form of F-VRS Model

The Fuzzy DEA envelopment program for considering variables return to scale is as follows [2]:

$$\text{Min}\; \theta_m$$

Subject to the Constraints

$$\tilde{Y}\lambda \ge \tilde{Y}_m; \quad \tilde{X}\lambda \le \theta \tilde{X}_m$$
$$\sum_{n=1}^{N} \lambda_n = 1;$$
$$\tilde{\lambda} \ge 0; \quad \tilde{\theta}_m \text{free variable}$$

Trapezoidal Fuzzy Number

A Trapezoidal fuzzy number, which represented with four points as follows,$A = (a_1, a_2, a_3, a_4), a_i \in R$, This representation is interpreted as membershipfunction

$$\mu_A = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & a_3 \le x \le a_4 \\ 0 & \text{Otherwise} \end{cases}$$

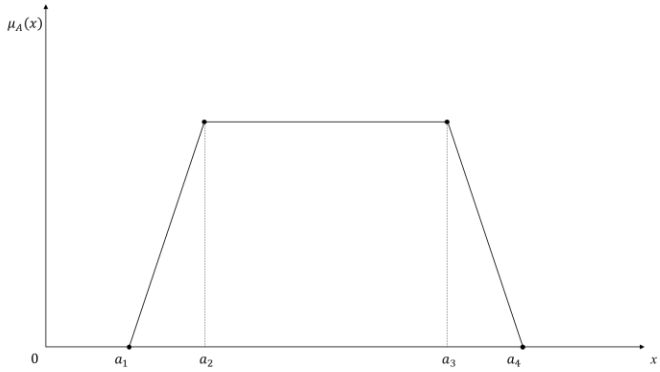


Figure: 1 Membership function of Trapezoidal Fuzzy Number

Defuzzification

Since technical processes require clear control actions, a procedure which generates net values from one or several given fuzzy numbers.

Graded Mean Integration representation

Chen and Hseihpropose graded mean integration representation for representing generalized fuzzynumber [12, 13].

If the generalized fuzzy number $A = (a_1, a_2, a_3, a_4 : w)$, then the graded mean h-level is$\frac{h[L^{-1}(h)+R^{-1}(h)]}{2}$. Where $L^{-1}$and R^{-1} are inverse functions of L and R [12,13].

And, the defuzzified value of the Fuzzy number A by the graded mean integration representation $\Re(A)$is defined as[12,13]

$$\Re(A) = \frac{\int_0^h \left[\frac{L^{-1}(h)+R^{-1}(h)}{2}\right] dh}{\int_0^w h\, dh}$$

Where $h \in (0, w)$, and $0 < w \leq 1$.

If $A = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number. Chen and Hsieh have already found the general formulae ofthe representation of generalized pentagonal fuzzy number as follows[12,13]:

$$\Re(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

For this study, the researcher used Graded Mean Integration representation method for defuzzification[12,13].

Data Collection and Selection of Input and Output Variables

For this research, the required data of selected Colleges under Bharathidasan University based on the availability of reputed data have been carried from the Official Website for the Academic years 2015-2019.

Reviewing the literature on the application of Fuzzy DEA, different studies have used different combination of inputs and yields. The current study considered five input variables and two output variables in order to hold an elaborate study. The variables under the studyare presented in the given figure.

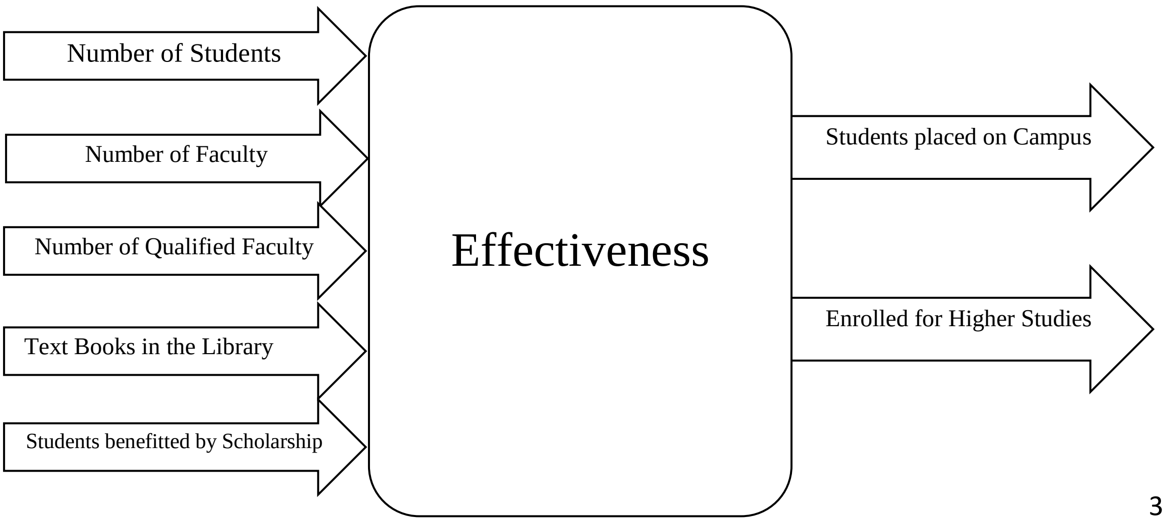


Figure: 2 Selected input and output variables

Problem Formulation: Effectiveness
Fuzzy Constant Returns to Scale [Output Maximization]
A.D.M. College for Women, Nagapattinam (2015 -2016)

$$Max\ E_{ADMC} = \frac{363x_1 + 249x_2}{1674x_3 + 45x_4 + 31x_5 + 33499x_6 + 1497x_7}$$

Subject to the Constraints,

$$\frac{114x_1 + 241x_2}{5415x_3 + 84x_4 + 53x_5 + 78155x_6 + 1210x_7} \le 1$$
$$\frac{304x_1 + 237x_2}{7027x_3 + 149x_4 + 93x_5 + 89354x_6 + 5319x_7} \le 1$$
$$\frac{284x_1 + 49x_2}{3274x_3 + 141x_4 + 32x_5 + 14577x_6 + 1253x_7} \le 1$$
$$\frac{230x_1 + 286x_2}{7710x_3 + 337x_4 + 124x_5 + 99700x_6 + 3302x_7} \le 1$$
$$\frac{356x_1 + 113x_2}{5008x_3 + 202x_4 + 40x_5 + 29944x_6 + 661x_7} \le 1$$
$$\frac{473x_1 + 52x_2}{2446x_3 + 99x_4 + 48x_5 + 41853x_6 + 159x_7} \le 1$$
$$\frac{3x_1 + 71x_2}{3146x_3 + 63x_4 + 37x_5 + 20817x_6 + 3584x_7} \le 1$$
$$\frac{5x_1 + 24x_2}{4182x_3 + 120x_4 + 68x_5 + 57688x_6 + 3585x_7} \le 1$$
$$\frac{20x_1 + 95x_2}{1794x_3 + 87x_4 + 53x_5 + 24026x_6 + 1146x_7} \le 1$$
$$\frac{6x_1 + 56x_2}{4383x_3 + 56x_4 + 33x_5 + 2577x_6 + 4063x_7} \le 1$$
$$\frac{75x_1 + 161x_2}{5012x_3 + 74x_4 + 57x_5 + 75809x_6 + 2167x_7} \le 1$$
$$\frac{12x_1 + 49x_2}{3445x_3 + 144x_4 + 29x_5 + 23980x_6 + 723x_7} \le 1$$
$$\frac{226x_1 + 411x_2}{10150x_3 + 436x_4 + 157x_5 + 177252x_6 + 3184x_7} \le 1$$
$$\frac{7x_1 + 88x_2}{4185x_3 + 104x_4 + 72x_5 + 46211x_6 + 4188x_7} \le 1$$
$$\frac{10x_1 + 125x_2}{2604x_3 + 59x_4 + 48x_5 + 32041x_6 + 1120x_7} \le 1$$
$$\frac{44x_1 + 100x_2}{3434x_3 + 207x_4 + 108x_5 + 97592x_6 + 1458x_7} \le 1$$
$$\frac{198x_1 + 122x_2}{3703x_3 + 175x_4 + 55x_5 + 40694x_6 + 2777x_7} \le 1$$
$$\frac{98x_1 + 154x_2}{1620x_3 + 172x_4 + 135x_5 + 51188x_6 + 2863x_7} \le 1$$

$$\frac{6x_1 + 46x_2}{1333x_3 + 33x_4 + 18x_5 + 23933x_6 + 3872x_7} \leq 1$$
$$\frac{12x_1 + 77x_2}{3516x_3 + 137x_4 + 81x_5 + 65352x_6 + 4122x_7} \leq 1$$
$$\frac{210x_1 + 181x_2}{4272x_3 + 82x_4 + 42x_5 + 47599x_6 + 2091x_7} \leq 1$$
$$\frac{18x_1 + 352x_2}{3860x_3 + 166x_4 + 28x_5 + 29165x_6 + 390x_7} \leq 1$$
$$\frac{246x_1 + 130x_2}{2468x_3 + 171x_4 + 72x_5 + 6980x_6 + 2357x_7} \leq 1$$
$$\frac{189x_1 + 186x_2}{6206x_3 + 302x_4 + 104x_5 + 158909x_6 + 1042x_7} \leq 1$$
$$\frac{41x_1 + 75x_2}{1195x_3 + 138x_4 + 30x_5 + 26505x_6 + 841x_7} \leq 1$$
$$\frac{3x_1 + 28x_2}{3475x_3 + 60x_4 + 53x_5 + 40714x_6 + 3008x_7} \leq 1$$
$$x_1, x_2, x_3, x_4, x_5 x_6, x_7 \geq 0$$

The corresponding LPP structure for the above problem can be written as follows,

$$Max\ E_{ADMC} = 363x_1 + 249x_2$$

Subject to the Constraints,

$$1674\text{x}_3 + 45\text{x}_4 + 31\text{x}_5 + 33499\text{x}_6 + 1497\text{x}_7 = 1$$
$$114x_1 + 241x_2 - 5415x_3 - 84x_4 - 53x_5 - 78155x_6 - 1210x_7 \leq 0$$
$$304\text{x}_1 + 237\text{x}_2 - 7027x_3 - 149x_4 - 93x_5 - 89354x_6 - 5319x_7 \leq 0$$
$$284x_1 + 49x_2 - 3274x_3 - 141x_4 - 32x_5 - 14577x_6 - 1253x_7 \leq 0$$
$$230x_1 + 286x_2 - 7710x_3 - 337x_4 - 124x_5 - 99700x_6 - 3302x_7 \leq 0$$
$$356x_1 + 113x_2 - 5008x_3 - 202x_4 - 40x_5 - 29944x_6 - 661x_7 \leq 0$$
$$473x_1 + 52x_2 - 2446x_3 - 99x_4 - 48x_5 - 41853x_6 - 159x_7 \leq 0$$
$$3x_1 + 71x_2 - 3146x_3 - 63x_4 - 37x_5 - 20817x_6 - 3584x_7 \leq 0$$
$$5x_1 + 24x_2 - 4182x_3 - 120x_4 - 68x_5 - 57688x_6 - 3585x_7 \leq 0$$
$$20x_1 + 95x_2 - 1794x_3 - 87x_4 - 53x_5 - 24026x_6 - 1146x_7 \leq 0$$
$$6x_1 + 56x_2 - 4383x_3 - 56x_4 - 33x_5 - 2577x_6 - 4063x_7 \leq 0$$
$$75x_1 + 161x_2 - 5012x_3 - 74x_4 - 57x_5 - 75809x_6 - 2167x_7 \leq 0$$
$$12x_1 + 49x_2 - 3445x_3 - 144x_4 - 29x_5 - 23980x_6 - 723x_7 \leq 0$$
$$226x_1 + 411x_2 - 10150x_3 - 436x_4 - 157x_5 - 177252x_6 - 3184x_7 \leq 0$$
$$7x_1 + 88x_2 - 4185x_3 - 104x_4 - 72x_5 - 46211x_6 - 4188x_7 \leq 0$$
$$10x_1 + 125x_2 - 2604x_3 - 59x_4 - 48x_5 - 32041x_6 - 1120x_7 \leq 0$$
$$44x_1 + 100x_2 - 3434x_3 - 207x_4 - 108x_5 - 97592x_6 - 1458x_7 \leq 0$$
$$198x_1 + 122x_2 - 3703x_3 - 175x_4 - 55x_5 - 40694x_6 - 2777x_7 \leq 0$$
$$98x_1 + 154x_2 - 1620x_3 - 172x_4 - 135x_5 - 51188x_6 - 2863x_7 \leq 0$$
$$6x_1 + 46x_2 - 1333x_3 - 33x_4 - 18x_5 - 23933x_6 - 3872x_7 \leq 0$$
$$12x_1 + 77x_2 - 3516x_3 - 137x_4 - 81x_5 - 65352x_6 - 4122x_7 \leq 0$$
$$210x_1 + 181x_2 - 4272x_3 - 82x_4 - 42x_5 - 47599x_6 - 2091x_7 \leq 0$$
$$18x_1 + 352x_2 - 3860x_3 - 166x_4 - 28x_5 - 29165x_6 - 390x_7 \leq 0$$
$$246x_1 + 130x_2 - 2468x_3 - 171x_4 - 72x_5 - 6980x_6 - 2357x_7 \leq 0$$
$$189x_1 + 186x_2 - 6206x_3 - 302x_4 - 104x_5 - 158909x_6 - 1042x_7 \leq 0$$
$$41x_1 + 75x_2 - 1195x_3 - 138x_4 - 30x_5 - 26505x_6 - 841x_7 \leq 0$$
$$3x_1 + 28x_2 - 3475x_3 - 60x_4 - 53x_5 - 40714x_6 - 3008x_7 \leq 0$$

$$x_1, x_2, x_3, x_4, x_5 x_6, x_7, x_8 \geq 0$$

Fuzzy Variable Returns to Scale:
A.D.M. College for Women, Nagapattinam (2015 -2016)
Min $x_{28} - x_{29}$

Subject to the constraints

$$387x_1 + 51x_2 + 29x_3 + 128x_4 + 317x_5 + 439x_6 + 705x_7 + 2x_8 + 12x_9 + x_{10} + x_{11} + 272x_{12} + 3x_{13} + 637x_{14} + x_{15} + 21x_{16} + 175x_{17} + 225x_{18} + 98x_{19} + 2x_{20} + 25x_{21} + 209x_{22} + 154x_{23} + 499x_{24} + 280x_{25} + 110x_{26} + x_{27} \geq 387$$

$$59x_1 + 461x_2 + 96x_3 + 49x_4 + 284x_5 + 136x_6 + 64x_7 + 45x_8 + 23x_9 + 45x_{10} + 42x_{11} + 158x_{12} + 33x_{13} + 494x_{14} + 91x_{15} + 107x_{16} + 100x_{17} + 165x_{18} + 154x_{19} + 51x_{20} + 134x_{21} + 204x_{22} + 262x_{23} + 198x_{24} + 186x_{25} + 73x_{26} + 56x_{27} \geq 59$$

$$1928x_1 + 5332x_2 + 7301x_3 + 1957x_4 + 8473x_5 + 5008x_6 + 2434x_7 + 3602x_8 + 4169x_9 + 1999x_{10} + 4374x_{11} + 4868x_{12} + 4008x_{13} + 10133x_{14} + 4145x_{15} + 2847x_{16} + 3330x_{17} + 3550x_{18} + 1620x_{19} + 1415x_{20} + 3673x_{21} + 3964x_{22} + 3860x_{23} + 3711x_{24} + 6063x_{25} + 813x_{26} + 3625x_{27} - 1928x_{28} + 1928x_{29} \leq 0$$

$$42x_1 + 81x_2 + 318x_3 + 141x_4 + 360x_5 + 205x_6 + 124x_7 + 56x_8 + 136x_9 + 86x_{10} + 60x_{11} + 70x_{12} + 164x_{13} + 450x_{14} + 104x_{15} + 55x_{16} + 209x_{17} + 175x_{18} + 172x_{19} + 59x_{20} + 138x_{21} + 112x_{22} + 161x_{23} + 181x_{24} + 273x_{25} + 118x_{26} + 62x_{27} - 42x_{28} + 42x_{29} \leq 0$$

$$29x_1 + 58x_2 + 106x_3 + 32x_4 + 121x_5 + 61x_6 + 52x_7 + 35x_8 + 68x_9 + 51x_{10} + 40x_{11} + 53x_{12} + 32x_{13} + 160x_{14} + 72x_{15} + 46x_{16} + 106x_{17} + 66x_{18} + 135x_{19} + 49x_{20} + 86x_{21} + 60x_{22} + 30x_{23} + 71x_{24} + 117x_{25} + 32x_{26} + 54x_{27} - 29\mathrm{x}_{28} + 29\mathrm{x}_{29} \leq 0$$

$$33780x_1 + 78644x_2 + 89354x_3 + 26518x_4 + 104611x_5 + 31418x_6 + 41853x_7 + 22069x_8 + 58473x_9 + 26078x_{10} + 16387x_{11} + 103556x_{12} + 24272x_{13} + 201673x_{14} + 57026x_{15} + 32203x_{16} + 98015x_{17} + 41096x_{18} + 51188x_{19} + 23971x_{20} + 66038x_{21} + 47749x_{22} + 39922x_{23} + 7981x_{24} + 160250x_{25} + 26535x_{26} + 45513x_{27} - 33780\mathrm{x}_{28} + 33780\mathrm{x}_{29} \leq 0$$

$$1579x_1 + 1347x_2 + 5467x_3 + 1261x_4 + 2803x_5 + 525x_6 + 326x_7 + 3868x_8 + 4347x_9 + 1392x_{10} + 5352x_{11} + 2549x_{12} + 728x_{13} + 2895x_{14} + 4319x_{15} + 1184x_{16} + 1655x_{17} + 2491x_{18} + 2863x_{19} + 3898x_{20} + 3746x_{21} + 2122x_{22} + 3255x_{23} + 2444x_{24} + 1919x_{25} + 717x_{26} + 4462x_{27} - 1579x_{28} + 1579x_{29} \leq 0$$

$$\sum_{i=1}^{27} x_i = 1$$

$$x_i \geq 0, i = 1,2, \ldots ,27$$

All Such 216 problems were generated from the collected dataand solved using the software TORA.

5. EMPIRICAL RESULTS

Fuzzy Constant Return to Scale [F-CCR Model]

The Fuzzy DEA performance efficiency score based on Technical Efficiency [Fuzzy Constant Returns to Scale] under the CCR Model is shown in Table 1. The Analysis reveals that among the selected 27 Colleges taken for the study two colleges attained the maximum efficiency score as 1.

Table 1: F-DEA Efficiency Score – F-CCR Model

Name of the Institution	Efficiency Score	De-fuzzified Score
A.D.M. College for Women, Nagapattinam	1, 1, 1, 0.48	0.91
A.V.C. College, Mayiladuthurai	0.831, 1, 0.626, 0.354	0.74
A.V.V.M. Sri Pushpam College, Tanjore	0.341, 0.165, 0.62, 0.622	0.42
Annai College of Arts & Science, Kumbakonam	1, 0.449, 0.646, 0.321	0.59
Bishop Heber College, Trichy	0.323, 0.477, 1, 0.794	0.68
Cauvery College for Women, Trichy	1, 1, 1, 0.879	0.98
Dhanalakshmi Srinivasan College, Perambalur	1, 1, 1, 1	1.00
Govt. Arts College for Women, Pudukkottai	0.37, 0.284, 0.945, 0.927	0.63
Government Arts College, Karur	0.051, 0.068, 0.286, 0.24	0.17
Government Arts College, Trichy	0.448, 0.275, 0.589, 0.44	0.44
Government College Women, Kumbakonam	1, 0.314, 0.633, 0.164	0.51
Holy Cross College, Trichy	0.427, 0.699, 0.646, 0.766	0.65
J.J. College of Arts & Science, Pudukkottai	0.175, 0.203, 0.71, 0.093	0.35
Jamal Mohamed College, Trichy	0.38, 0.685, 0.89, 0.509	0.67
K. N. Govt. College for Women, Thanjavur	0.227, 0.262, 0.493, 0.5	0.37
Khadir Mohideen College, Adirampattinam	0.535, 0.515, 1, 0.699	0.71
National College, Trichy	0.254, 0.425, 0.614, 0.589	0.49
Nehru Memorial College, Puthanampatti	0.396, 0.671, 0.472, 0.382	0.51
Periyar E.V.R. College, Trichy	0.639,1,0.2,1	0.67
Poompuhar College, Melaiyur	0.304,0.412,0.493,0.288	0.40
Rajah Serfoji Government College, Thanjavur	0.154,0.415,0.398,0.343	0.35
Seethalakshmi Ramaswami College, Trichy	0.505,0.721,0.919,0.678	0.74
S. T. Educational Trust College, Mannargudi	1,1,1,0.978	1.00

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Name of the Institution	Efficiency Score	De-fuzzified Score
Srimad Andavan Arts & Science College, Trichy	1,1,1,1	1.00
St. Joseph's College, Trichy	0.389,0.436,0.741,0.509	0.54
Thanthai Hans Roever College, Perambalur	0.465,1,1,1	0.91
Thiru. Vi. Ka. Govt. Arts College, Thiruvarur	0.091,0.2,0.239,0.536	0.25

Fuzzy Variable Return to Scale [F-BCC Model]

The Fuzzy DEA efficiency score based on Technical Efficiency [Fuzzy Variable Returns to Scale] under the F-BCC Model is shown in Table 2. In F-BCC Model there is an increment in the number of efficient colleges. The Analysis reveals that among the 27 colleges taken for the study, thirteen Colleges attained the maximum efficiency score as 1.

Table 2: F-DEA Efficiency Score – F-BCC Model

Name of the Institution	Efficiency Score	De-fuzzified Score
A.D.M. College for Women, Nagapattinam	1,1,1,1	1
A.V.C. College, Mayiladuthurai	0.904,1,0.63,0.464	0.77
A.V.V.M. Sri Pushpam College, Poondi	0.343,0.317,0.714,0.718	0.52
Annai College of Arts & Science, Kumbakonam	1,1,1,1	1.00
Bishop Heber College, Trichy	0.324,0.481,1,0.895	0.70
Cauvery College for Women, Trichy	1,1,1,0.922	0.99
Dhanalakshmi Srinivasan College, Perambalur	1,1,1,1	1
Govt. Arts College for Women, Pudukkottai	0.847,1,1,1	0.97
Government Arts College, Karur	0.485,0.484,0.485,0.575	0.50
Government Arts College, Trichy	1,1,1,0.861	0.98
Government College Women, Kumbakonam	1,1,1,0.744	0.96
Holy Cross College, Trichy	0.651,0.737,0.648,0.797	0.70
J.J. College of Arts & Science, Pudukkottai	1,1,1,1	1
Jamal Mohamed College, Trichy	1,1,1,0.527	0.92
K. N. Govt. College for Women, Thanjavur	0.526,0.554,0.541,0.694	0.57
Khadir Mohideen College, Adirampattinam	1,1,1,1	1
National College, Trichy	0.506,0.494,0.643,0.71	0.58
Nehru Memorial College, Puthanampatti	0.574,0.707,0.613,0.568	0.63
Periyar E.V.R. College, Trichy	0.872,1,0.63,1	0.86
Poompuhar College, Melaiyur	1,1,1,1	1
Rajah Serfoji Government College, Thanjavur	0.446,0.558,0.481,0.537	0.51
Seethalakshmi Ramaswami College, Trichy	0.714,0.816,0.942,0.918	0.86
S. T. Educational Trust College, Mannargudi	1,1,1,1	1
Srimad Andavan Arts & Science College, Trichy	1,1,1,1	1
St. Joseph's College, Trichy	0.437,0.44,0.792,0.551	0.58
Thanthai Hans Roever College, Perambalur	1,1,1,1	1
Thiru. Vi. Ka. Govt. Arts College, Thiruvarur	0.721,0.705,0.64,1	0.74

Table 3 shows that nine of the 27 colleges considered for this analysis are extremely standardized, with an efficiency score of 1.

Table 3: Efficient Countries Identified by F-CCR and F-BCC Models

Name of the Institution	CRS	VRS	Mean Score
A.D.M. College for Women, Nagapattinam	0.91	1.00	0.96
A.V.C. College, Mayiladuthurai	0.74	0.77	0.76
A.V.V.M. Sri Pushpam College, Poondi	0.42	0.52	0.47
Annai College of Arts & Science, Kumbakonam	0.59	1.00	0.79
Bishop Heber College, Trichy	0.68	0.70	0.69
Cauvery College for Women, Trichy	0.98	0.99	0.98
Dhanalakshmi Srinivasan College, Perambalur	1.00	1.00	1
Govt. Arts College for Women, Pudukkottai	0.63	0.97	0.80
Government Arts College, Karur	0.17	0.50	0.33
Government Arts College, Trichy	0.44	0.98	0.71
Government College Women, Kumbakonam	0.51	0.96	0.73
Holy Cross College, Trichy	0.65	0.70	0.68
J.J. College of Arts & Science, Pudukkottai	0.35	1.00	0.67
Jamal Mohamed College, Trichy	0.67	0.92	0.80
K. N. Govt. College for Women, Thanjavur	0.37	0.57	0.47
Khadir Mohideen College, Adirampattinam	0.71	1.00	0.86
National College, Trichy	0.49	0.58	0.53
Nehru Memorial College, Puthanampatti	0.51	0.63	0.57
Periyar E.V.R. College, Trichy	0.67	0.86	0.76
Poompuhar College, Melaiyur	0.40	1.00	0.70
Rajah Serfoji Government College, Thanjavur	0.35	0.51	0.43
Seethalakshmi Ramaswami College, Trichy	0.74	0.86	0.80
S.T Educational Trust College, Mannargudi	1.00	1.00	1
Srimad Andavan Arts & Science College, Trichy	1.00	1.00	1
St. Joseph's College, Trichy	0.54	0.58	0.56
Thanthai Hans Roever College, Perambalur	0.91	1.00	0.96
Thiru. Vi. Ka. Govt. Arts College, Thiruvarur	0.25	0.74	0.49

6. CONCLUSION

The Efficiency Analysis based on Fuzzy Constant Returns to Scale reveals that two colleges such as: Dhanalakshmi Srinivasan College, Srimad Andavan Arts & Science College, stand first and the analysis based on the Fuzzy Variable Returns to Scale Communicates that nine Colleges such as A.D.M. College for Women, Annai College of Arts & Science, Cauvery College for Women,Dhanalakshmi Srinivasan College of Arts & Science, J.J. College of Arts & Science, Khadir Mohideen College, Rajah Serfoji Government College, S. T. Educational Trust College, Srimad Andavan Arts & Science College, Thanthai Hans Roever College take the first place.

Comparing both the analysis one can conclude that Dhanalakshmi Srinivasan College of Arts & Science,Srimad Andavan Arts & Science Collegeand S.T Educational Trust Collegeare the most efficient colleges.

7. REFERENCES

[1] Adel Hatami-Marbini, Ali Emrouznejad, Madjid Tavana, “A taxonomy and review of the fuzzy data envelopment analysis literature: Two decades in the making”, European Journal of Operational Research, doi:10.1016/j.ejor.2011.02.001, 214 (2011) 457–472.

[2] Banker, R.D., Charnes, A., and Cooper, W. W., “Some models for the estimation of technical and scale inefficiencies in data envelopment analysis”, Management Science, 1078 – 1092, Vol. 30, No.9, pp. 1078-1092, 1984.

[3] Charnes, A., Cooper, W.W., and Rhodes, E. “Measuring the Efficiency of Decision-Making Units,” European Journal of Operation Research, Vol. 2, Issue 6, 1978, pp. 429-444.

[4] Farrel, M.J. “The measurement of Productivity efficiency,” Journal of Royal Statistical Society, Vol. 120, Part III, 1957, pp. 253 – 281.

[5] Jyh-Shing Roger Jang, “ANFIS : Adap tive-Ne twork-Based Fuzzy Inference System”, IEEE Transactions On Systems, Man, And Cybernetics, Vol. 23, No. 3,1993.

[6] Loganathan., Ganesan, “A solution approach to fully fuzzy linear fractional programming problems”, Journal of Physics: Conference Series, 1377 (2019) 012040, doi:10.1088/1742-6596/1377/1/012040

[7] Perumal Mariappan “Introduction to Data Envelopment Analysis [DEA]”, LAP LAMBERT Academic Publishing, (2016).

[8] Ramanathan R., “An Introduction to data envelopment analysis-A tool for performance measurement”, Sage publications, ISBN: 81-7829-260-2, 2003.

[9] Saowanee Lertworasirikula, Shu-Cherng Fanga, Jerey A. Joinesb, Henry L.W. Nuttlea, “Fuzzy data envelopment analysis (DEA): a possibilityapproach”Fuzzy Sets and Systems 139 (2003) 379–394,

[10] Sengupta, J.K., 1992. A fuzzy systems approach in data envelopment analysis. Computers and Mathematics with Applications 24 (8-9), 259–266.

[11] Sengupta, J.K., 1992. Measuring efficiency by a fuzzy statistical approach. Fuzzy Sets and Systems 46 (1), 73–80.

[12] Shan-Huo Chen and Chin Hsun Hseih, (2000), Graded Mean Integration Representation of Generalized Fuzzy Number journal of the chunese Fuzzy System Association, Taiwan, 5(2): 1-7.

[13] Shan-Huo chen, and Chin Hsun Hseih, (2000), Representation, Ranking, Distance and Similarity of L-R Type Fuzzy Number and Application, Australia Journal of Intelligent Information Processing Systems, Australia, 6(4): 217 – 229.

[14] Zadeh, L. A., Fuzzy Sets. Information and Control, 8, 338 – 353. 1965.