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Dual Hesitant Multi Objective Fractional Transportation Problem With Non-Linear Discount Cost

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Abstract: This paper focuses a multi-objective fractional transportation problem based on dual hesitant fuzzy numbers. The traditional fuzzy set deals with the single membership value to express vagueness of an element. It is not applicable to give several possible membership values at the same time. The hesitant fuzzy set provides the better way to overcome these uncertainties by giving the several possible membership degrees to single element. The mathematical model of dual hesitant multi objective fractional transportation problem is formulated. A new method is proposed to solve dual hesitant multi objective fractional transportation problem with non-linear discount cost. The aim of this proposed method to optimize the ratios of the objective functions with non-linear discount cost. A numerical example is shown to check the effectiveness of the proposed method.

Key Words: Multi Objective Transportation Problem, Fractional Transportation Problem, Dual Hesitant Fuzzy Set

1. INTRODUCTION

The transportation problem (TP) is very famous due to its versatile applications in the real world. The traditional TP is the special case of linear programming problem that finds the best transportation plan for a homogenous commodity from a number of sources to a number of destinations at the minimum total transportation cost.

In reality more transportation problems are described as multi objectives which can be measured in different scales at the same time. This situation leads to concept of multi objective transportation problems (MOTP). In MOTP, objectives may be of minimizing the total time of transportation, total cost of transportation and total deterioration of products during transportation etc. Diaz(1973) have developed optimal solutions of the MOTP with two objectives. Ringuest (1987), Aneja(2004) have studied MOTP. Lakhveer(2018) formulated the algorithm for finding the optimal solution to MOTP in tabular method which gives the better solution than solution by software.

The fractional programming problem (FPP) is the generalization of linear programming problem in which the objectives are ratio of two functions. The objective of this FPP is to optimize the ratio of the cost functions. It is very useful in many real life situations. For examples ratio between the profit and time, profit and cost , minimizing the inventory and sales etc. Several methods have been developed by many authors, Charnes and Cooper(1962), Birtan(1973) ,Cravan(1988), Schaible (1995).

There are several ways to solve multi objective linear transportation problem and fractional transportation problem. Changkong(1983),Borza(2012), Chakraborty(2002) ,Abouzar (2018) solved the fractional transportation problem with multi objectives. Dangwal (2012) have developed the algorithm for MOFTP by using Taylor series.

In many situations, all the parameters of the transportation problem are imprecise. Due to this the parameters of TP become fuzzy in nature. The traditional fuzzy set deals with the single membership value to express vagueness of an element. It is not applicable to give several possible membership values at the same time. The hesitant fuzzy set provides the better way to overcome these uncertainties by giving the several possible membership degree to single element.

Torra (2009) was introduced the concept of a Hesitant Fuzzy Set (HFS) and the proper definition of HFS has developed by Torra (2010).Dual-Hesitant Fuzzy Sets (DHFS) is the extension of HFS which was initiated by Zhu (2012). This DHFSincludes the concepts of fuzzy sets, intuitionstic fuzzy set , hesitant fuzzy set and multi fuzzy sets.Torra(2010) and Zhu(2012)have introduced the basic properties and operations of DHFSs. Thereafter, they presented the concept of DHFSs in a group forecasting problem.

Discounts are sometimes available for large transports so that the marginal costs of unit shipping cost might follow a particular pattern. In some real problems this discount cost can be expressed as non linear functions because of its sustainability. Aim of this problem is to develop the mathematical model of TP using optimization techniques so as to achieve the best demand and supply by discounting at the minimum total transportation cost.

In thispaper the mathematical model of dual hesitant multi objective fractional transportation problem (DHMOFTP) is described with non-linear discount cost. A new method is proposed to solve the proposed model (DHMOFTP). The aim of this proposed method to optimize the ratio of the two objective functions with non-linear discount cost. A numerical example is shown to check the effectiveness of the proposed method.
In section 2, basic definitions of hesitant fuzzy set and dual hesitant fuzzy set are discussed which are useful to solve DHFMOTP. In section 3, the mathematical formulation of DHFMOFTP is described and the effective algorithm is proposed and numerical example is solved to show the feasibility of proposed method in section 4.

2. PRELIMINARIES

2.1 Hesitant Fuzzy Set : Torra (2009)
A hesitant fuzzy set HF on Y is defined in terms of a function h(y) from Y to the subset of values in the interval [0, 1].
If $\rho([0,1])$is the power set of [0,1] then h is the function from Y to $\rho([0,1])$
h: Y $\rightarrow \rho([0,1])$

Mathematically it can be stated that $HF = \{ (y_i , h(y_i)) : y_i \in Y \}$ where $h(y_i)$ is a set of several values in [0,1].In general each member of $h(y_i)$ is called a hesitant fuzzy element denoted by h_i .

2.2 Dual hesitant fuzzy set:

A dual hesitant fuzzy set DHF on Y is defined in terms of a functions h(y) and g(y) that returns a subset of values in the interval [0,1] once it is applied to Y

$h : Y \rightarrow \rho([0,1])$ and $g : Y \rightarrow \rho([0,1])$

where h(y) and g(y) are mappings that takes set of values in [0,1]; they are defined as the possible membership degree and non-membership degree of any element $y \in Y$ provided $0 \leq h(y_i) + g(y_i) \leq 1$.

Mathematically, DH = $\{ (y_i , h(y_i), g(y_i)): y_i \in Y \}$ where $h(y_i)$ is a set of several membership values in [0,1] and $g(y_i)$is a set of several possible non-membership values in [0,1] with $0 \leq h(y_i) + g(y_i) \leq 1$.

2.3 Ranking function of dual hesitant fuzzy sets:

Let DHF= $\{ (y_i , h(y_i), g(y_i)): y_i \in Y \}$ be a dual hesitant fuzzy set where $\{y_1, y_2, \dots \dots y_n\}$ and $d = (h_d, g_d)$ be dual hesitant fuzzy element. The score function $sf_d$of dual hesitant fuzzy set is defined as

$$sf_d = \left| \frac{1}{k} \sum_{i=1}^{k} h_d(y_i) - \frac{1}{k} \sum_{i=1}^{k} g_d(y_i) \right|$$

Let $d_1 and d_2$ be any two dual hesitant fuzzy elements. Then using the score function the order relations are defined as follows:

1. If $s_{d_1} > s_{d_2}$, then d_1 is said to be superior to d_2 and it is denoted by $d_1 > d_2$
2. If $s_{d_1} < s_{d_2}$, then d_1 is said to be inferior to d_2 and it is denoted by $d_1 < d_2$
3. If $s_{d_1} = s_{d_2}$, then d_1 is said to be equivalent to d_2 and it is denoted by $d_1 = d_2$

Mathematical formulation dual hesitant multi-objective fractional transportation problem (DFMOFTP) **:**

Mathematical formulation of dual hesitant multi-objective fractional transportation problem is defined as follows:

$$\min(\max)p^1 = \frac{\sum_{a=1}^{c} \sum_{b=1}^{d} \widetilde{u_{ab1}} y_{ab}}{\sum_{a=1}^{c} \sum_{b=1}^{d} \widetilde{v_{ab1}} y_{ab}}$$

$$\min(\max)p^2 = \frac{\sum_{a=1}^{c} \sum_{b=1}^{d} \widetilde{u_{ab2}} y_{ab}}{\sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{v_{ab2}} y_{ab}}$$

….

$$\min(\max)p^l = \frac{\sum_{a=1}^{c} \sum_{b=1}^{d} \widetilde{u_{abl}} y_{ab}}{\sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{v_{abl}} y_{ab}}$$

Subject to $\sum_{b=1}^{d} y_{ab} = s_a$

$$\sum_{a=1}^{c} y_{ab} = d_b$$

$y_{ab} \geq 0$ for every $a = 1\ toc , b = 1\ todandk = 1\ ol$

where$\widetilde{u_{abk}}$ and $\widetilde{v_{abk}}$ are dual hesitant cost elements of the k^{th} objective function.

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The objective function of the above model is based on dual hesitant fuzzy environment, aim of the problem is to maximize the member values and minimize the non-membership values of the cost value of $\widetilde{u_{abk}}$ and $\widetilde{v_{abk}}$. Hence it can be formulated as given below:

$$\min(\max)p^1 = \frac{\sum_{a=1}^{c}\sum_{b=1}^{d}\left(h_{ab1}^{r}, g_{ab1}^{r}\ \right)y_{ab}}{\sum_{a=1}^{c}\sum_{b=1}^{d}\ (h_{ab1}^{s}, g_{ab1}^{s})\ y_{ab}}$$

$$\min(\max)p^2 = \frac{\sum_{a=1}^{c}\sum_{b=1}^{d}\left(h_{ab2}^{r}, g_{ab2}^{r}\ \right)y_{ab}}{\sum_{a=1}^{c}\sum_{b=1}^{d}\ (h_{ab1}^{s}, g_{ab1}^{s})\ y_{ab}}$$

….

$$\min(\max)p^l = \frac{\sum_{a=1}^{c}\sum_{b=1}^{d}\left(h_{abl}^{r}, g_{abl}^{r}\ \right)y_{ab}}{\sum_{a=1}^{c}\sum_{b=1}^{d}\ (h_{abl}^{s}, g_{abl}^{s})\ y_{ab}}$$

Subject to

$$\sum_{b=1}^{d} y_{ab} = s_a$$

$$\sum_{a=1}^{c} y_{ab} = d_b$$

$$y_{ab} \geq 0 \text{ for every } a = 1\ toc\ , b = 1\ todandk = 1\ ol$$

Where $\left(h_{ijk}^{s}\ , g_{ijk}^{s}\right)$is dual hesitant cost values of the kth objective in the numerator and $\left(h_{ijl}^{s}\ , g_{ijl}^{s}\right)$ is dual hesitant cost values of the kth objective in the denominator.

Procedure to solve multi objective fractional transportation problem:

Step 1 **:**Find the crisp value for every fuzzy number using any ranking number.

Step 2 : Find the fractional values of the each cell as shown below:

	W₁	**W₂**	**….**	**W_y**	**Supply**
U₁	$\frac{s_{111}}{u_{111}}$ $\frac{s_{112}}{u_{112}}$ …. $\frac{s_{11r}}{u_{11r}}$	$\frac{s_{121}}{u_{121}}$ $\frac{s_{122}}{u_{122}}$ …. $\frac{s_{12r}}{u_{12r}}$	….	$\frac{s_{1y1}}{u_{1y1}}$ $\frac{s_{1y2}}{u_{1y2}}$ …. $\frac{s_{1yr}}{u_{1yr}}$	a_1
U₂	$\frac{s_{211}}{u_{211}}$ $\frac{s_{212}}{u_{212}}$ …. $\frac{s_{21r}}{u_{21r}}$	$\frac{s_{221}}{u_{221}}$ $\frac{s_{222}}{u_{222}}$ …. $\frac{s_{22r}}{u_{22r}}$	….	$\frac{s_{2y1}}{u_{2y1}}$ $\frac{s_{2y2}}{u_{2y2}}$ …. $\frac{s_{2yr}}{u_{2yr}}$	a_2
….					
U_x	$\frac{s_{x11}}{u_{x11}}$	$\frac{s_{x21}}{u_{x21}}$		$\frac{s_{xy1}}{u_{xy1}}$	

	$\frac{s_{x12}}{u_{x12}}$ …. $\frac{s_{x1r}}{u_{x1r}}$	$\frac{s_{x22}}{u_{x22}}$ …. $\frac{s_{x2r}}{u_{x2r}}$	….	$\frac{s_{xy2}}{u_{xy2}}$ …. $\frac{s_{xyr}}{u_{xyr}}$	a_x
Demand	b_1	b_2	…..	b_y	

Step 3 : If all the objectives are maximization then it can be converted into minimization type by subtracting the greatest element from all the fractional values.

Step 4 : Find the maximum ratio of the each row $\gamma_{lk}$and each column $\delta_{pk}$and fix as given below:

	W1	**W2**	**….**	**Wy**	**Supply**	
U1	$\frac{s_{111}}{u_{111}}$ $\frac{s_{112}}{u_{112}}$ …. $\frac{s_{11r}}{u_{11r}}$	$\frac{s_{121}}{u_{121}}$ $\frac{s_{122}}{u_{122}}$ …. $\frac{s_{12r}}{u_{12r}}$	….	$\frac{s_{1y1}}{u_{1y1}}$ $\frac{s_{1y2}}{u_{1y2}}$ …. $\frac{s_{1yr}}{u_{1yr}}$	a_1	γ_{11} γ_{12} …. γ_{1r}
U2	$\frac{s_{211}}{u_{211}}$ $\frac{s_{212}}{u_{212}}$ …. $\frac{s_{21r}}{u_{21r}}$	$\frac{s_{221}}{u_{221}}$ $\frac{s_{222}}{u_{222}}$ …. $\frac{s_{22r}}{u_{22r}}$	….	$\frac{s_{2y1}}{u_{2y1}}$ $\frac{s_{2y2}}{u_{2y2}}$ …. $\frac{s_{2yr}}{u_{2yr}}$	a_2	γ_{21} γ_{22} … γ_{2r}
….	….	….	….	…	…	…

Ux	$\frac{s_{x11}}{u_{x11}}$ $\frac{s_{x12}}{u_{x12}}$ …. $\frac{s_{x1r}}{u_{x1r}}$	$\frac{s_{x21}}{u_{x21}}$ $\frac{s_{x22}}{u_{x22}}$ …. $\frac{s_{x2r}}{u_{x2r}}$	….	$\frac{s_{xy1}}{u_{xy1}}$ $\frac{s_{xy2}}{u_{xy2}}$ …. $\frac{s_{xyr}}{u_{xyr}}$	a_x	γ_{2r} γ_{2r} … γ_{xr}
Demand	b_1	b_2	…..	b_y		
	δ_{11} δ_{12} … δ_{1r}	δ_{21} δ_{22} … δ_{2r}	… …. …	δ_{y1} δ_{y2} … δ_{yr}		

Step 5 : Choose L = max { γ_{lr}, δ_{pr} } for every $l = 1\ tox\ , p = 1\ toyandk = 1\ to\ r$ **.**
Step 6: Select the call having L as one of its ratio. Suppose there are more than one cell choose the cell which has maximum ration for other fractional objectives.
Step 7: Choose the cell containing min $\left\{\sum_{l=1}^{x} \frac{s_{lpk}}{u_{lpk}} forfixedp\right\}$. If there is a tie then select one to which maximum allocation.
Step 8: Do the procedure of step 4 to step 6 until supply and demand requirement are not met.

Procedure to solve dual hesitant multi objective fractional transportation problem:
The methodology for solving DFMOFTP with non-linear discount cost is described as follows:

- Find the score value for each objective functions of given DFMOFTP.
- Covert the given DFMOFTP to the fractional form
- Obtain the initial basic feasible solution for DFMOFTP by using the steps described in section 4.
- Determine the ratio of each objective at optimal value.

Test for optimality :
After finding the initial basic feasible solution , test the optimality of IBFS of DFMOFTP as follows:

- Add w_{abk} and z_{ab} to every sources and destinations and find the values of each w_{ak} and z_{bk} by using $\frac{\partial p_k}{\partial y_{ab}} = w_{ak} + z_{bk}$ and taking any one w_{abk} and z_{ab}as zero.
- calculate $Q_{abk} = \frac{\partial p_k}{\partial y_{ab}} - w_{ak} - z_{bk}$ for all non-basic y_{ij} variables

If $Q_{abk} = \frac{\partial p_k}{\partial y_{ab}} - w_{ak} - z_{bk} \geq 0$ then obtained solution is optimal solution. Otherwise go to the next step.
If $Q_{abk} = \frac{\partial p_k}{\partial y_{ab}} - w_{ak} - z_{bk} < 0$ then obtained solution is non-optimal solution.

- If the solution is non optimal then find next set of feasible solution and check the optimality.

Numerical Example :

Consider the following transportation problem with two objectives with discount costs which are transported from three sources to four destinations. The first objective will be the ratio of profit and transportation cost and second objective function will be the ratio of speed of transportation and wastage cost. Aim of these objectives is to maximize the ratio so that cost is to be minimized.

Table: 1 Profit per unit when an item transported

	D1	D2	D3	D4	SUPPLY
S1	12 (0.5,0.4,0.1: 0.5,0.5, 0.9)	18 (0.8,0.6,0.4,0.2: 0.1, 0.2,0.6,0.8)	10 (0.5,0.4: 0.4, 0.6)	14 (0.6,0.7,0.9: 0.3,0.2,0.1)	150
S2	10 (0.3,0.1: 0.7, 0.8)	14 (0.7,0.6,0.3: 0.1, 0.3,0.6)	16 (0.6,0.5,0.4: 0.2,0.4,0.7)	13 (0.6,0.5,0.3: 0.3, 0.3,0.7)	250
S3	15 (0.3,0.2,0.1: 0.2,0.6, 0.7)	8 (0.2,0.1: 0.6,0.8)	19 (0.6,0.5: 0.3,0.2)	15 (0.5,0.4,0.3, 0.2: 0.4,0.6,0.7,0.8)	200
DEMAND	100	250	100	150	600

Table: 2 transportation cost per unit when an item transported

	D1	D2	D3	D4	SUPPLY
S1	17 (0.6,0.5,0.4: 0.3, 0.4, 0.5)	14 (0.7,0.3: 0.2, 0.5)	18 (0.7,0.5,0.4: 0.2,0.4, 0.5)	10 (0.7,0.5: 0.1,0.4)	150
S2	12 (0.5, 0.4,0.1: 0.3, 0.6,0.8)	8 (0.8,0.5: 0.1, 0.5)	15 (0.5,0.4,0.2: 0.4,0.5,0.6)	14 (0.4,0.3: 0.5,0.6)	250
S3	15 (0.7,0.6,0.4 : 0.2, 0.3, 0.4)	17 (0.5,0.3,0.1: 0.4,0.7,0.8)	14 (0.4,0.3: 0.5,0.4)	12 (0.5, 0.4, 0.3: 0.4,0.5,0.6)	200
DEMAND	100	250	100	150	600

Table: 3 Delivery speed of transportation

	D1	D2	D3	D4	SUPPLY
S1	4 (0.6,0.5: 0.4, 0.5)	6 (0.8,0.5: 0.1,0.4)	5 (0.6,0.5,0.4 : 0.3,0.4, 0.5)	2 (0.6,0.5: 0.3,0.4)	150
S2	2 (0.8,0.6,0.5: 0.2,0.3,0.4)	5 (0.7,0.5,0.4: 0.3, 0.4,0.5)	1 (0.6,0.4,0.3: 0.3,0.5,0.6)	4 (0.7,0.6: 0.3,0.4)	250
S3	2 (0.8,0.7:	1 (0.7,0.6,0.5: 0.2,0.3,0.4)	4 (0.9,0.8:	3 (0.7,0.6:	200

	0.2, 0.3)		0.1,0.2)	0.2,0.4)	
DEMAND	100	250	100	150	600

Table 4 : Wastage cost per unit

	D1	D2	D3	D4	SUPPLY
S1	1 (0.7,0.6: 0.2, 0.3)	3 (0.7,0.6,0.4: 0.2,0.3,0.5)	4 (0.7,0.6 : 0.2,0.3)	6 (0.6, 0.5: 0.4,0.5)	150
S2	4 (0.6,0.5,0.4: 0.3,0.4,0.5)	3 (0.8,0.6,0.4: 0.2, 0.3,0.5)	6 (0.9,0.7: 0.1,0.2)	2 (0.8,0.7: 0.1,0.2)	250
S3	5 (0.7,0.6: 0.2, 0.3)	3 (0.6,0.5: 0.3,0.4)	3 (0.9,0.8: 0.1,0.2)	2 (0.6,0.5: 0.3,0.4)	200
DEMAND	100	250	100	150	600

Table 5: Discount cost per unit

	D1	D2	D3	D4	SUPPLY
S1	0.04	0.01	0.1	0.04	150
S2	0.02	0.02	0.01	0.02	250
S3	0.028	0.08	0.01	0.04	200
DEMAND	100	250	100	150	600

Finding the score value for each dual hesitant elements then the above tables become as follows :

Table: 6 Profit per unit when an item transported

	D1	D2	D3	D4	SUPPLY
S1	(12,0.3)	(18.75,0.075)	(10.5,0.05)	(13.67,0.53)	150
S2	(11,0.55)	(15,0.2)	(16.67,0.2)	(13,0.04)	250
S3	(12.67,0.3)	(10.5,0.55)	(18,0.3)	(12.25,0.25)	200
DEMAND	100	250	100	150	600

Table: 7 transportation cost per unit when an item transported

	D1	D2	D3	D4	SUPPLY
S1	(17.33,0.1)	(13.5,0.15)	(17,0.16)	(11.5,0.35)	150
S2	(11,0.24)	(9.5,0.35)	(15,0.13)	(15.5,0.2)	250
S3	(16.67,0.3)	(17,0.33)	(15.5,0.2)	(12,0.1)	200
DEMAND	100	250	100	150	600

Table: 8 Delivery speed of transportation

	D1	D2	D3	D4	SUPPLY
S1	(3.5,0.1)	(6.5,0.4)	(4,0.1)	(3,0.2)	150
S2	(2.33,0.33)	(4.67,0.13)	(2.67,0.04)	(5,0.3)	250
S3	(2.5,0.5)	(2.67,0.3)	(3.5,0.7)	(2.5,0.35)	200
DEMAND	100	250	100	150	600

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Table : 9 Wastage cost per unit

	D1	D2	D3	D4	SUPPLY
S1	(1.5,0.4)	(3,0.14)	(4.5,0.4)	(6.5,0.2)	150
S2	(4.33,0.1)	(3.33,0.27)	(6.5,0.65)	(2.5,0.6)	250
S3	(5.5,0.4)	(4,0.2)	(3.5,0.7)	(1.5,0.2)	200
DEMAND	100	250	100	150	600

Mathematical formulation of given multi objective transportation problem

$$\max p^1 = \frac{\begin{array}{c}0.3y_{11} + 0.075y_{12} + 0.05y_{13} + 0.5y_{14} + 0.55y_{21} + 0.2y_{22} + 0.2y_{23} \\ +0.04y_{24} + 0.3y_{31} + 0.55y_{32} + 0.3y_{33} + 0.25y_{34}\end{array}}{\begin{array}{c}0.1y_{11} + 0.15y_{12} + 0.16y_{13} + 0.35y_{14} + 0.24y_{21} + 0.35y_{22} + 0.13y_{23} \\ +0.2x_{24} + 0.3x_{31} + 0.33x_{32} + 0.2x_{33} + 0.1x_{34}\end{array}}$$

$$\max z^2 = \frac{\begin{array}{c}0.1y_{11} + 0.4y_{12} + 0.1y_{13} + 0.2y_{14} + 0.33y_{21} + 0.13y_{22} + 0.04y_{23} \\ +0.13y_{24} + 0.5y_{31} + 0.3y_{32} + 0.7y_{33} + 0.35y_{34}\end{array}}{\begin{array}{c}0.4y_{11} + 0.24y_{12} + 0.4y_{13} + 0.2y_{14} + 0.1y_{21} + 0.27y_{22} + 0.65y_{23} \\ +0.6y_{24} + 0.4y_{31} + 0.2y_{32} + 0.7y_{33} + 0.2y_{34}\end{array}}$$

Subject to $y_{11} + y_{12} + y_{13} + y_{14} \leq 150$

$y_{21} + y_{22} + y_{23} + y_{24} \leq 250$

$y_{31} + y_{32} + y_{33} + y_{34} \leq 200$

$y_{11} + y_{21} + y_{31} \leq 100$

$y_{12} + y_{22} + y_{32} \leq 250$

$y_{13} + y_{23} + y_{33} \leq 100$

$y_{14} + y_{24} + y_{34} \leq 150$

and $y_{ab} \geq 0$

Adding the following discount cost function to the above model

$\frac{u_{111}}{v_{111}} = 3y_{11} - 0.04y_{11}^2$ $\frac{u_{112}}{v_{112}} = 0.25y_{11} - 0.04y_{11}^2$

$\frac{u_{121}}{v_{121}} = 0.5y_{12} - 0.01y_{12}^2$ $\frac{u_{122}}{v_{122}} = 1.67y_{12} - 0.01y_{12}^2$

$\frac{u_{131}}{v_{131}} = 0.31y_{13} - 0.1y_{13}^2$ $\frac{u_{132}}{v_{132}} = 0.25y_{13} - 0.1y_{13}^2$

$\frac{u_{141}}{v_{141}} = 1.51y_{14} - 0.04y_{14}^2$ $\frac{u_{142}}{v_{142}} = 1y_{14} - 0.04y_{14}^2$

$\frac{u_{211}}{v_{211}} = 2.29y_{21} - 0.02y_{21}^2$ $\frac{u_{212}}{v_{212}} = 3.31y_{21} - 0.02y_{21}^2$

$\frac{u_{221}}{v_{221}} = 0.57y_{22} - 0.02y_{22}^2$ $\frac{u_{222}}{v_{223}} = 0.48y_{22} - 0.02y_{22}^2$

$\frac{u_{231}}{v_{231}} = 1.53y_{23} - 0.01y_{23}^2$ $\frac{u_{232}}{v_{232}} = 0.06y_{23} - 0.01y_{23}^2$

$\frac{u_{241}}{v_{241}} = 0.2y_{24} - 0.02y_{24}^2$ $\frac{u_{242}}{v_{242}} = 0.5y_{24} - 0.02y_{24}^2$

$\frac{u_{311}}{v_{311}} = 1y_{31} - 0.028y_{31}^2$ $\frac{u_{312}}{v_{312}} = 1.231y_{31} - 0.028y_{31}^2$

$\frac{u_{321}}{v_{321}} = 1.67y_{32} - 0.08y_{32}^2$ $\frac{u_{322}}{v_{322}} = 1.5y_{32} - 0.08y_{32}^2$

$\frac{u_{331}}{v_{331}} = 1.5y_{33} - 0.01y_{33}^2$ $\frac{u_{332}}{v_{332}} = 1y_{33} - 0.01y_{33}^2$

$$\frac{u_{341}}{v_{341}} = 2.5y_{34} - 0.04y_{34}^2 \quad \frac{u_{342}}{v_{342}} = 2.5y_{34} - 1.75y_{34}^2$$

Using the algorithm described in section 4 for solving MOTP and the solution table given below:

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Table 9: Solution Table

	D1	D2	D3	D4	SUPPLY
S1	3 0.25	0.5 **100** 1.67	0.31 0.25	1.57 **50** 1	150
S2	2.29 **100** 3.3	0.57 **150** 0.48	1.53 0.06	0.2 0.5	250
S3	1 1.25	1.67 1.5	1.5 **100** 1	2.5 **100** 1.75	200
DEMAND	100	250	100	150	600

From the above table the objective values are

$$p^1 = \frac{18(100) + 14(50) + 10(100) + 14(150) + 17(100) + 11(100)}{14(100) + 10(50) + 12(100) + 8(150) + 14(100) + 12(100)} = 1.217$$

$$p^2 = \frac{6(100) + 2(50) + 2(100) + 5(150) + 4(100) + 3(100)}{3(100) + 6(50) + 4(100) + 3(150) + 3(100) + 2(100)} = 1.205$$

To check the optimality condition finds the partial derivatives of each variables as follow:

$\frac{\partial p^1}{\partial y_{11}} = 3 - 0.04y_{11} = 3$ $\quad$ $\frac{\partial p^2}{\partial y_{11}} = 0.25 - 0.04y_{11} = 0.25$

$\frac{\partial p^1}{\partial y_{12}} = 0.5 - 0.01y_{12} = -0.5$ $\quad$ $\frac{\partial p^2}{\partial y_{12}} = 1.67 - 0.01y_{12} = 0.67$

$\frac{\partial p^1}{\partial y_{13}} = 0.31 - 0.1y_{13} = 0.31$ $\quad$ $\frac{\partial p^2}{\partial y_{13}} = 0.25 - 0.1y_{13} = 0.25$

$\frac{\partial p^1}{\partial y_{14}} = 1.51 - 0.04y_{14} = -0.49$ $\quad$ $\frac{\partial p^2}{\partial y_{14}} = 1 - 0.04y_{14} = 1$

$\frac{\partial p^1}{\partial y_{21}} = 2.29 - 0.02y_{21} = 0.29$ $\quad$ $\frac{\partial p^2}{\partial y_{21}} = 3.3 - 0.02y_{21} = 1.3$

$\frac{\partial p^1}{\partial y_{22}} = 0.57 - 0.02y_{22} = -2.43$ $\quad$ $\frac{\partial p^2}{\partial y_{22}} = 0.48 - 0.02y_{22} = -2.52$

$\frac{\partial p^1}{\partial y_{23}} = 1.53 - 0.01y_{23} = 1.54$ $\quad$ $\frac{\partial p^2}{\partial y_{23}} = 0.06 - 0.01y_{23} = 0.06$

$\frac{\partial p^1}{\partial y_{24}} = 0.2 - 0.02y_{24} = 0.2$ $\quad$ $\frac{\partial p^1}{\partial y_{24}} = 0.5 - 0.02y_{24} = 0.5$

$\frac{\partial p^1}{\partial y_{31}} = 1 - 0.028y_{31} = 1$ $\quad$ $\frac{\partial p^1}{\partial y_{31}} = 1.25 - 0.028y_{31} = 1.25$

$\frac{\partial p^1}{\partial y_{32}} = 1.67 - 0.08y_{32} = 1.67$ $\quad$ $\frac{\partial p^2}{\partial y_{32}} = 1.5 - 0.08y_{32} = 1.5$

$\frac{\partial p^1}{\partial y_{33}} = 1.5 - 0.01y_{33} = 0.5$ $\quad$ $\frac{\partial p^2}{\partial y_{33}} = 1 - 0.01y_{33} = 0$

$\frac{\partial p^1}{\partial y_{34}} = 2.5 - 0.04y_{11} = -1.5$ $\quad$ $\frac{\partial p^1}{\partial y_{34}} = 1.75 - 0.04y_{11} = -2.25$

Find the cost function of the occupied equation using

$$\frac{\partial p_k}{\partial y_{ab}} = w_{ak} + z_{bk}$$

$w_1^1 + z_2^1 = -0.5$ $\quad$ $w_2^2 + z_2^2 = -2.52$

$w_1^1 + z_4^1 = -0.49$ $\quad$ $w_3^2 + z_4^2 = -2.25$

$w_2^1 + z_2^1 = -2.43$ $\quad$ $w_3^2 + z_3^2 = 0$

$$w_3^1 + z_4^1 = -1.5$$

Let $w_1^1 = 0$ and $w_2^2 = 0$; $w_3^2 = 0$ other values are $z_2^1 = -0.5$; $z_4^1 = -0.49$; $w_2^1 = -1.93; w_3^1 = -1.01$; $z_2^2 = -2.52$; $z_4^2 = -6.25$

Calculate the net evaluation using the following equation :

$$Q_{abk} = \frac{\partial p_k}{\partial y_{ab}} - w_{ak} - z_{bk}$$

$$Q_{121} = \frac{\partial p^1}{\partial y_{12}} - w_1^1 - z_2^1 = -0.5 - 0 + 0.5 = 0$$

$$Q_{141} = \frac{\partial p^1}{\partial y_{14}} - w_1^1 - z_4^1 = -0.49 - 0 + 0.49 = 0$$

$$Q_{221} = \frac{\partial p^1}{\partial y_{22}} - w_2^1 - z_2^1 = -0.57 + 1.93 + 0.5 = 1.86$$

$$Q_{341} = \frac{\partial p^1}{\partial y_{34}} - w_3^1 - z_4^1 = -1.5 + 1.01 + 0.49 = 0$$

$$Q_{222} = \frac{\partial p^2}{\partial y_{22}} - w_2^2 - z_2^2 = -2.52 - 0 + 2.52 = 0$$

$$Q_{342} = \frac{\partial p^2}{\partial y_{34}} - w_3^2 - z_4^2 = -2.25 - 0 + 2.25 = 0$$

Now all $Q_{abk} \geq 0$ therefore obtained solution is the optimal solution which optimizes the ratio.

The optimal value of the ratio is shown below

$$p^1 = \frac{18(100) + 14(50) + 10(100) + 14(150) + 17(100) + 11(100)}{14(100) + 10(50) + 12(100) + 8(150) + 14(100) + 12(100)} = 1.217$$

$$p^2 = \frac{6(100) + 2(50) + 2(100) + 5(150) + 4(100) + 3(100)}{3(100) + 6(50) + 4(100) + 3(150) + 3(100) + 2(100)} = 1.205$$

Therefore it is noticed that distribution of 100 units from S1 to D2 at 1% discount , 50 units from S1 to D4 at 4% discount , 100 units from S2 to D1 at 2% discount , 150 units from S2 to D2 at 2% discount 100 units from S3 to D3 and D4 with 1% and 4% discount respectively so as to obtain the highest ratio of profit to cost and speed to cost of wastage.

3. CONCLUSION :

In thispaper a new algorithm is proposed to solve dual hesitant multi objective fractional transportation problem (DHMOFTP) with non-linear discount cost. The traditional fuzzy set deals with the single membership value to express vagueness of an element. It is not applicable to give several possible membership values at the same time. The hesitant fuzzy set provides the better way to overcome these uncertainties by giving the several possible membership degrees to single element. The aim of this proposed method to optimize the ratio of the two objective functions with non-linear discount cost. KKT optimality condition is used to check the optimal level of the objective functions. In example our aim is to maximize the profit with discount cost that results the best solution with minimum transportation cost. Similarly in second objective is to maximize the ratio of speed and wastage cost that will reflect the best solution in transporting the food items.

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