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F-Centroidal Mean Labelling of Trees and Cycle Related Graphs

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Abstract: In this paper, we have discussed the F–centroidal Mean Labelling of tree and cycle related graphs. A function f is called an F–centroidal mean labelling of a graph G(V,E) with p vertices and q edges if $f : V(G) \rightarrow \{1,2,3,..........,q+1\}$ ***is injective and the induced function*** $f^* : E(G) \rightarrow \{1,2,3,..........,q\}$ ***defined as***

$$f^*(uv) = \left\lfloor \frac{2\left[f(u)^2 + f(u)f(v) + f(v)^2\right]}{3\left[f(u)+f(v)\right]} \right\rfloor,$$

for all $uv \in E(G)$***, is bijective. A graph that admits an F–centroidal mean labelling is called an F–centroidal mean graph.***

Key Words: F–centroidal mean labelling, F–centroidal mean graph.

1. INTRODUCTION:

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of [10]. Let *G(V, E)* be a graph with *p* vertices and *q* edges. For a detailed survey on graph labelling, we refer [9].

DuraiBaskar and Arockiaraj defined the *F*-harmonic mean labelling [8] and discussed its meanness of some standard graphs. The concept of *F*-geometric mean labelling was introduced by DuraiBaskar and Arockiaraj [7] and it was developed [6]. The concept of *F*-root square mean labelling was introduced by Arockiaraj, [3] and they studied the *F*-root square mean labelling of some standard graphs [4]. DuraiBaskar and Manivannan were introduced *F*-heronian mean labelling [5]. Motivated by the works of so many authors in the area of graph labelling we introduced a new type of labeling called an *F*-centroidal mean labelling.

A function f is called an F–centroidal mean labelling of a graph *G(V,E)* with *p* vertices and *q* edges if $f : V(G) \rightarrow \{1,2,3,..........,q+1\}$ is injective and the induced function $f^* : E(G) \rightarrow \{1,2,3,..........,q\}$ defined as

$$f^*(uv) = \left\lfloor \frac{2\left[f(u)^2 + f(u)f(v) + f(v)^2\right]}{3\left[f(u)+f(v)\right]} \right\rfloor,$$

for all $uv \in E(G)$, is bijective. A graph that admits an ***F*–centroidal mean labeling** is called an ***F*–centroidal mean graph.**

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In this paper we have discussed the *F*–centroidal mean labelling of the graph M(P_n), the graph Spl(P_n), the graph P_n^2, the graph VD(P_n), the graph T_n and the graph PC_n .

2. MAIN RESULTS:

Definition: 2.1

Let G be a graph. Let G' be a copy of G. The mirror graph M(G) of G is defined as the disjoint union of G and G' with additional edges joining each vertex of G to its corresponding vertex in G'.

Theorem: 2.2

The graph M(P_n) ($n \geq 3$) is a *F*-centroidal mean graph.

Proof:

Let $\{v_i, v_i', 1 \leq i \leq n\}$ be the vertices and $\{a_i, b_i, 1 \leq i \leq n-1, c_i, 1 \leq i \leq n\}$ be the edges which are denoted as in Figure 1.1.

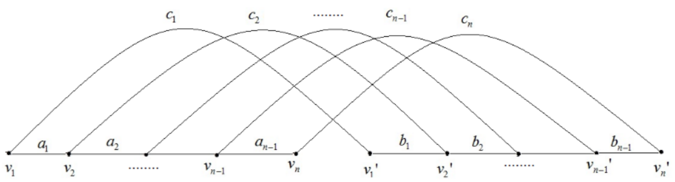

Figure 1.1: Ordinary labelling of M(P_n)

First we label the vertices as follows:

For $1 \leq i \leq n$, $f(v_i) = 3i - 2$; $f(v_i') = 3i - 1$

Then the induced edge labels are:

For $1 \leq i \leq n-1$, $f^*(a_i) = 3i - 1$, $f^*(b_i) = 3i$

For $1 \leq i \leq n$, $f^*(c_i) = 3i - 2$

Hence, f is an *F*-Centroidal mean labelling of M(P_n) ($n \geq 3$).

F-Centroidal mean labelling of M(P_5) is shown in Figure 1.2

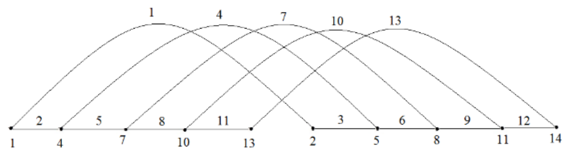

Figure 1.2: *F*-centroidal mean labeling of M(P_5)

Definition: 2.3

For a graph G, the split graph is obtained by adding to each vertex v, a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G. The resultant graph is denoted as Spl(P_n)

Theorem: 2.4

The graph Spl (P_n) ($n \geq 3$) is an *F*-centroidal mean graph.

Proof:

Let $\{u_i, v_i, 1 \leq i \leq n\}$ be the vertices and $\{a_i, b_i, c_i, 1 \leq i \leq n-1\}$ be the edges which are denoted as in Figure 1.3.

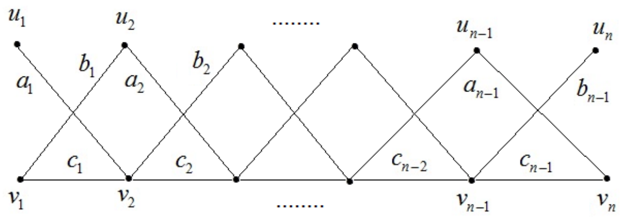

Figure 1.3: Ordinary labelling of Spl(P_n)

First we label the vertices as follows:

$$f(u_1) = 1$$

For $2 \leq i \leq n$, $f(u_i) = \begin{cases} 3i-2 & i \text{ is even} \\ 3i-3 & i \text{ is odd} \end{cases}$

$$f(v_1) = 3 \;;\; f(v_2) = 2$$

For $3 \leq i \leq n$, $f(v_i) = \begin{cases} 3i-1 & i \text{ is odd} \\ 3i-3 & i \text{ is even} \end{cases}$

Then the induced edge labels are:

For $1 \leq i \leq n-1$, $f^*(a_i) = \begin{cases} 3i-2 & i \text{ is odd} \\ 3i & i \text{ is even} \end{cases}$

For $1 \leq i \leq n-1$, $f^*(b_i) = \begin{cases} 3i & i \text{ is odd} \\ 3i-2 & i \text{ is even} \end{cases}$

$$f^*(c_1) = 2$$

For $2 \leq i \leq n-1$, $f^*(c_i) = 3i-1$

Hence, f is an *F*-Centroidal mean labeling of Spl(P_n) ($n \geq 3$).

F-Centroidal mean labeling of Spl(P_6) is shown in Figure 1.4.

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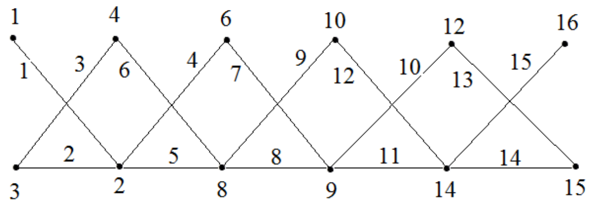

Figure 1.4: *F*-centroidal mean labelling of Spl(P_6)

Definition: 2.5

The square G^2 of a graph G has $V(G^2) = V(G)$, with u, v adjacent in G^2 whenever $d(u,v) \leq 2$ in G. The powers G^3, G^4, … of G are similarly defined.

Theorem: 2.6

The graph P_n^2 is an *F*-centroidal mean graph.

Proof:

Let $\{v_i, 1 \leq i \leq n\}$ be the vertices and $\{a_i, 1 \leq i \leq n-2; b_i, 1 \leq i \leq n-1\}$ be the edges which are denoted as in Figure 1.5.

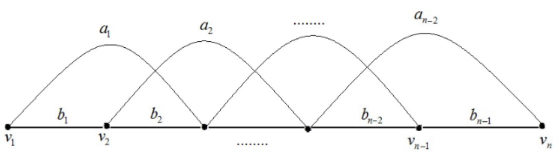

Figure 1.5: Ordinary labelling of P_n^2

First we label the vertices as follows:

$f(v_1) = 1$

For $2 \leq i \leq n$, $f(v_i) = 2i - 2$

Then the induced edge labels are:

For $1 \leq i \leq n-2$, $f^*(a_i) = 2i$.

For $1 \leq i \leq n-1$, $f^*(b_i) = 2i - 1$.

Hence, *f* is an *F*-Centroidal mean labelling of P_n^2

F-Centroidal mean labelling of P_6^2 is shown in Figure 1.6.

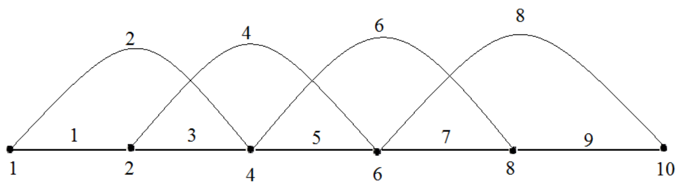

Figure 1.6: *F*-centroidal mean labelling of P_6^2

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Definition: 2.7

Let *G* be a graph and *v* be any vertex of *G*. A new vertex v' is said to be duplication of *v* if all the vertices which are adjacent to *v* are adjacent to v'. The graph obtained by duplication *v* is denoted by *VD(G)*.

Theorem: 2.8

The graph *VD(G)* is an *F*-centroidal mean graph.

Proof:

Let $\{v_i, 1 \le i \le n, v_1'\}$ be the vertices and $\{e_1', e_2', e_i, 1 \le i \le n-1\}$ be the edges which are denoted as in Figure 1.7.

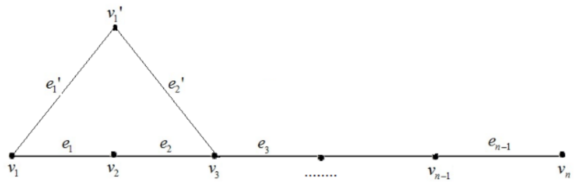

Figure 1.7: Ordinary labelling of P_n^2

First we label the vertices as follows:

$f(v_1) = 1$; $f(v_2) = 2$; $f(v_1') = 3$

For $3 \le i \le n$, $f(v_i) = i + 2$

Then the induced edge labels are:

$f^*(e_1) = 1$; $f^*(e_2) = 3$

$f^*(e_1') = 2$; $f^*(e_2') = 4$

For $3 \le i \le n-1$, $f^*(e_i) = i + 2$.

Hence, *f* is an *F*-Centroidal mean labelling of P_n^2

F-Centroidal mean labelling of P_6^2 is shown in Figure 1.8.

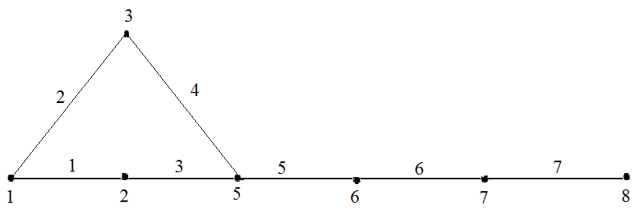

Figure 1.8: *F*-centroidal mean labelling of P_6^2

Definition: 2.9

A tortoise T_n (n ≥ 4) is obtained a path $v_1, v_2, \ldots\ldots, v_n$ by attaching an edge between v_i and v_{n-i+1} for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$.

Theorem: 2.10

The graph T_n (n ≥ 4), n ≡ 1, 3 (mod 4) is a *F*-centroidal mean graph.

Proof:

Let $\{v_i, 1 \le i \le n\}$ be the vertices and $\left\{ e_i, 1 \le i \le \frac{3(n-1)}{2} \right\}$ be the edges which are denoted as in Figure 1.9.

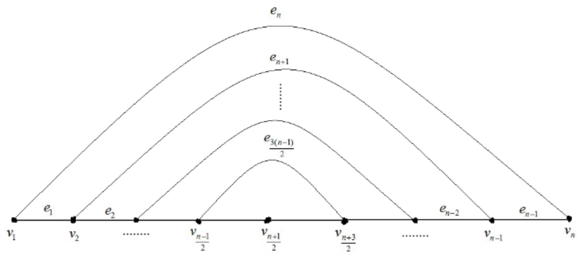

Figure 1.9: Ordinary labelling of T_n

First we label the vertices as follows:

For $1 \le i \le \frac{n-1}{2}$, $f(v_i) = \begin{cases} \frac{3(n+1)}{2} - 3i & i \text{ is odd} \\ \frac{3n+1}{2} - 3i & i \text{ is even} \end{cases}$

For $\frac{n+1}{2} \le i \le n$, $f(v_i) = 3i - \frac{3n+1}{2}$

Then the induced edge labels are:

For $1 \le i \le \frac{n-1}{2}$, $f^*(e_i) = 3(1-i) + \frac{3n-7}{2}$

For $\frac{n+1}{2} \le i \le n-1$, $f^*(e_i) = 3i + 2 - \frac{3(n+1)}{2}$

For $n \le i \le \frac{3(n-1)}{2}$, $f^*(e_i) = 3n + \frac{3(n-1)}{2} - 3i$

Hence, f is an *F*-Centroidal mean labelling of T_n

F-Centroidal mean labelling of T_9 is shown in Figure 1.10.

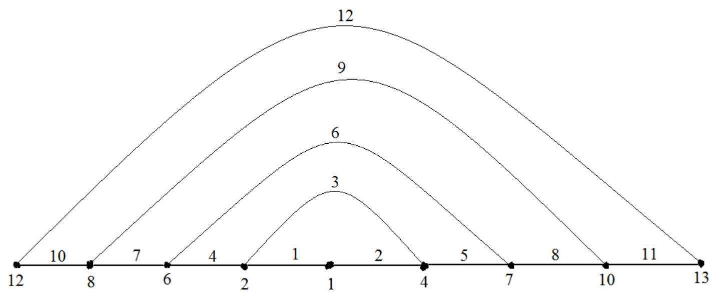

Figure 1.10: *F*-centroidal mean labelling of T_9

Definition: 2.11

A graph PC_n $(n \geq 5)$ is obtained from Cn = $v_1 v_2, \ldots\ldots, v_n v_1$ by adding chords v_i and v_{n-i+2} for $2 \leq i \leq l$ where $l = \frac{n}{2}$ or $\frac{n-1}{2}$ when n is even or odd.

Theorem: 2.12

The graph PC_n $(n \geq 5)$ is a *F*-centroidal mean graph.

Proof:

Case(i): n is even.

Let $\{v_i, 1 \leq i \leq n\}$ be the vertices and $\left\{e_i, 1 \leq i \leq n; e_i', 1 \leq i \leq \frac{n-2}{2}\right\}$ be the edges which are denoted as in Figure 1.11.

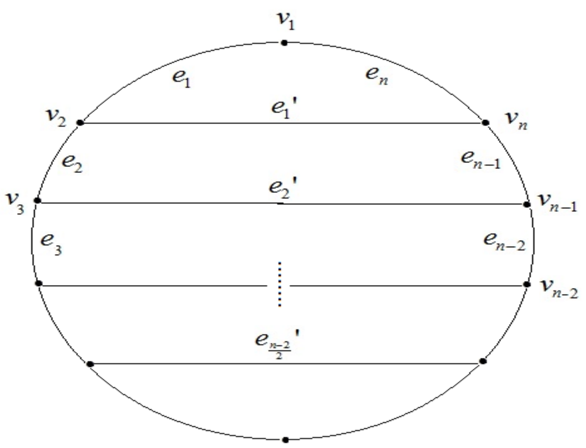

Figure 1.11: Ordinary labelling of PC_n

First we label the vertices as follows:

$$f(v_1) = 1$$

For $2 \le i \le \frac{n+1}{2}$, $f(v_i) = \begin{cases} 3i-4 & i \text{ is even} \\ 3(i-1) & i \text{ is odd} \end{cases}$

For $\frac{n+4}{2} \le i \le n$, $f(v_i) = 3n + 4 - 3i$

Then the induced edge labels are:

For $1 \le i \le \frac{n}{2}$, $f^*(e_i) = 3i - 2$

For $\frac{n+2}{2} \le i \le n$, $f^*(e_i) = 3n + 2 - 3i$

For $1 \le i \le \frac{n-2}{2}$, $f^*(e_i) = 3i$

Hence, f is an F-Centroidal mean labeling of PC_n

F-Centroidal mean labeling of PC_n is shown in Figure 1.12.

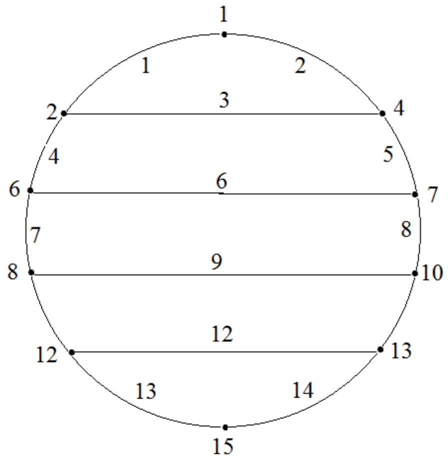

Figure 1.12: F-centroidal mean labelling of PC_n

Case(ii): n is odd

Let $\{v_i, 1 \le i \le n\}$ be the vertices and $\left\{e_i, 1 \le i \le n; e_i', 1 \le i \le \frac{n-3}{2}\right\}$ be the edges which are denoted as in Figure 1.13.

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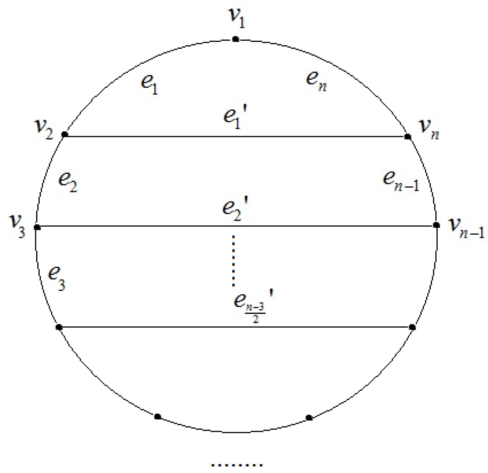

Figure 1.13: Ordinary labelling of PC_n

First we label the vertices as follows:

$$f(v_1) = 1$$

For $2 \le i \le \frac{n+1}{2}$, $f(v_i) = \begin{cases} 3i-4 & i \text{ is even} \\ 3(i-1) & i \text{ is odd} \end{cases}$

For $\frac{n+3}{2} \le i \le n$, $f(v_i) = 3n + 4 - 3i$

Then the induced edge labels are:

For $1 \le i \le \frac{n-1}{2}$, $f^*(e_i) = 3i - 2$, $f^*\left(e_{\frac{n+1}{2}}\right) = \frac{3(n-1)}{2}$

For $\frac{n+3}{2} \le i \le n$, $f^*(e_i) = 3n + 2 - 3i$

For $1 \le i \le \frac{n-3}{2}$, $f^*(e_i) = 3i$

Hence, f is an F-Centroidal mean labelling of PC_n

F-Centroidal mean labelling of PC_9 is shown in Figure 1.14.

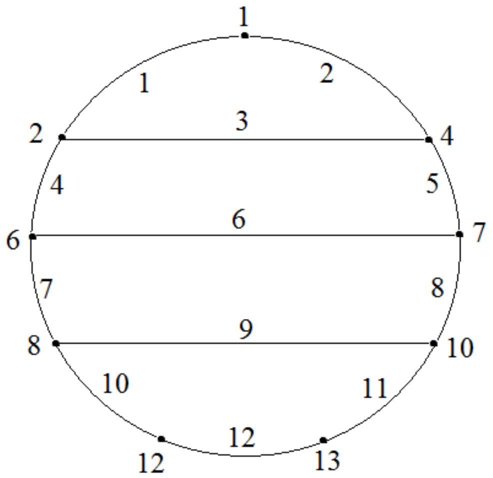

Figure 1.14: *F*-centroidal mean labelling of PC_9

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