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Modelling A Nonlinear Conical Tank System Using Supervised Learning Algorithms

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Abstract: Conical tank system (CTS) is a highly nonlinear process due to its varying cross-sectional area and it is a challenging task to design a controller for maintaining its level. Model based controllers are preferred to control such systems so that it will provide improved performance, greater robustness, better insight about system behaviour and cost efficiency. For designing model-based controller, the estimated model should be more accurate with less error. This paper proposes an efficient modelling of a CTS by comparing different supervised algorithms for System Identification (SI) such as Neural Network (NN) modelling, Auto Regressive Integrated Moving Average model (ARIMA) modelling and Long Short-Term Memory (LSTM) modelling. The models are developed using the experimental data obtained from the real time laboratory conical tank system and modelling through simulation using MATLAB and Google colab software tools. This study reveals that LSTM modelling gives the best performance wth less mean square error (MSE) when compared to other techniques and it is proposed to be used for designing the model based predictive controller to get better performance and robustness.

Keywords: Conical tank system, SI model, NN model, ARIMA model, LSTM model, MSE.

1. INTRODUCTION

Conical tanks are widely used in chemical and food processing industries because of their unique shape**.** The hole at the bottom of the tank and their smooth internal walls helps in better drainage of the contents. CTS is a highly nonlinear process and controlling the level of CTS is very difficult in industrial applications. Conventional PID controllers are not sufficient to control his process. In recent years model-based controllers are preferred to control such complicated systems to achieve better closed loop performance. Model-based controllers rely on perfect mathematical model of the system and hence an accurate model of CTS is essential in many industrial control applications.

The mathematical modelling of the conical tank can be obtained in many ways. The theoretical calculations can be made for simple system with less complications and it will not be an accurate one. Advanced algorithms using neural networks, machine learning and deep learning techniques are preferred for modelling nonlinear processes. In literature, many researchers proposed various intelligent algorithms for obtaining mathematical models. [1,2]

Fattah et al. (2018) discussed about the autoregressive integrated moving average models. The results obtained from the ARIMA model proved that the model developed was utilized to forecast the future demands in many industries. Gonzalez and W. Yu (2018) developed LSTM model that was used for modelling the nonlinear system and for the analysis of result. Greff et al. (2017) prposed different types of LSTM variants on three tasks such as speech recognition, handwriting recognition and polyphonic music modelling. Random search was used to optimise the hyper-parameters of LSTM. [3 -5]

Indhumathi et al. (2018) explained control of liquid level is difficult in a conical tank due to the variation in the area of cross section of the tank system with its change in shape. In this paper the model of the process is identified using black box modelling and approximated to be a first order plus dead time (FOPTD) model using linear regression . Marshiana, and Thirusakthimurugan (2018) proposed neural network based controller to control and train the non-linear data set of liquid level in order to optimize the network performance. Murugan and Ashok Kumar (2018) developed an iterative approach strives to adjust the number of hidden neurons of a NARXNN model. This approach systematically constructs various NARXNN models from simple architecture to complex architecture with different training functions and finds the optimum NARXNN model. [6,7]

Son et al. (2019) presented an evolving NN algorithm for modelling a nonlinear system and proved that the developed model had the ability to learn the dynamics of the system and to reduce the error to approximately zero. Yanwen et al. (2020) proposed the Long Short-Term Memory Network with the merge layer to predict the future states of the coupled Morris-Lecar (M-L) system with the chaotic itinerancy responses. In this proposal they used two LSTM models with single-branch and multi-branch. Boussaada et al. (2018) developed the solution predicts the direct solar radiation on a horizontal surface. The aim of this research work is to supply, with electricity, a race sailboat using exclusively various renewable sources. The result show good performance when NN trained network was used. [8,9]

This paper analyses various intelligent modelling techniques for conical tank process and proposes an efficient model that can be used for designing model-based control systems to improve the closed loop performance of CTS. The remaining sections of this paper is organised as follows: Section 2 gives the description about the CTS and the first principle model. Section 3 describes the empirical modelling techniques used in this study. Section 4 presents the simulation results with discussion followed by the conclusion in section 5.

2. System Description

2.1 CTS Specifications

Conical tank is a Single Input Single Output (SISO) system. The input variable is the inlet flow rate to the tank and output variable is the level of the tank. In conical tank, the term ‘conical’ describes the shape of the tank. This tank is nonlinear due to its varying cross section. The radius and the volume of the tank increases from the bottom to top of the tank. The specifications of the conical tank system are given in the table 1.

The experimental data set for the project is taken from the laboratory setup which is shown in Fig 1.

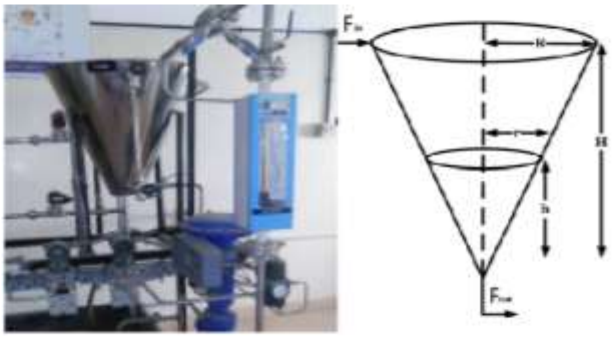

Figure 1. Laboratory setup for conical tank system

Table 1
Specifications of conical tank system

S.No.	Parameter	Value
1.	R	24 cm
2.	H	52 cm
3.	F_{in}(min)	600 LPH
4.	F_{in}(max)	950 LPH
5.	β	4.5946 cm^3/s

2.2 First Principle Model

According to the specifications given in the table 1.
The area of the conical tank is,

$$A = \pi r^2 \tag{1}$$

The radius (r) of the tank is a varying parameter. It is expressed as the ratio of the maximum radius (R) to the maximum height (H) of the Conical Tank,

$$\tan\theta = \frac{r}{h} = \frac{R}{H} \tag{2}$$

$$r = \frac{R}{H} h \tag{3}$$

According to Law of conservation of mass,
Inflow rate - Outflow rate = Accumulation

$$F_{in} - F_{out} = A\frac{dh}{dt} \tag{4}$$

$$F_{out} = C\sqrt{h} \tag{5}$$

$$\frac{dh}{dt} = Q_{in}\alpha h^{-2} - \beta h^{-3/2} \tag{6}$$

where,

$$\alpha = \frac{H^2}{\pi R^2} \tag{7}$$

$$\beta = C\alpha \tag{8}$$

At steady state,

$$\frac{dh_s}{dt} = \alpha Q_{ins} h^{-2} - \beta h_s^{-3/2} \tag{9}$$

$$0 = \alpha Q_{ins} h^{-2} - \beta h_s^{-3/2} \tag{10}$$

By using the Taylor series expansion,

$$Q(h, Q_{in}) = f(h_s, Q_{ins}) + \left(\frac{\partial f}{\partial h}\right)_{(h_s - Q_{ins})} (h - h_s) + \left(\frac{\partial f}{\partial Q_{ins}}\right)_{(h_s - Q_{ins})} Q(f_i - Q_{ins}) \tag{11}$$

$$Q(h, Q_{in}) = f(h_s, Q_{ins}) - 2Q_{in} h_s^{-3}(h - h_s) + h_s^{-2}(Q - Q_{ins}) \tag{12}$$

$$h^{-3/2} = h_s^{-3/2} - \left(\frac{3}{2}\right) h_s^{-5/2}(h - h_s) \tag{13}$$

The above nonlinear model is linearized to obtained as the first order transfer function which is given as follows,

$$\frac{2}{\beta} h_s^{5/2} \frac{dy}{dt} + y = \frac{2}{\beta} \alpha h_s^{1/2} U \tag{14}$$

Where,

$$\tau = \frac{2}{\beta} h_s^{5/2} \tag{15}$$

$$k = \frac{2}{\beta} \alpha h_s^{1/2} \tag{16}$$

$$\tau \frac{dy}{dt} + y = kU \tag{17}$$

$$G(s) = \frac{h(s)}{F_{in}(s)} \frac{Y(s)}{U(s)} = \frac{k}{(\tau s + 1)} \tag{18}$$

where,

K – Process gain
τ - dead time

By substituting the values in the above equation, the first principle modelling for the laboratory setup conical tank is,
For Zone 1,

$$\frac{h(s)}{F_{in}(s)} = \frac{0.9615}{250s+1} \tag{19}$$

(Range 4.5 to 34 cm)

For Zone 2,

$$\frac{h(s)}{F_{in}(s)} = \frac{1.5867}{602s+1} \qquad (20)$$

(Range 34.1 to 50 cm)

3. Empirical Modelling Techniques

System identification technique is used to generate the mathematical model of the process from the experimental data. The first principle approach is prone to errors because of modelling errors and model mismatch. In empirical modelling the system to be modelled is kept as a black box. Using real time experimental data of inputs and outputs from the process, the mathematical model is approximated through suitable algoritms.

3.1 Black Box Technique

In this work, two operating points (zone 1 and zone 2) are considered for the experimental conical tank system and the measured values of input flowrate and output level are obtained for those operating regions. The model of the conical tank system obtained using block box technique is given as follows,

For zone 1,

$$\frac{h(s)}{F_{in}(s)} = \frac{1}{270.49s+1} \qquad (21)$$

(Range 4.5 to 34 cm)

For zone 2,

$$\frac{h(s)}{F_{in}(s)} = \frac{1.495}{593.12s+1} \qquad (22)$$

(Range 34.1 to 50 cm)

3.2 ARIMA Model

The full form of ARIMA modelling is Auto Regressive Moving Average Model. The term 'Auto Regressive' in ARIMA model means it is a linear regression model that uses its own delay as predictors. This ARIMA model is also known as Box - Jenkins.

The general source formula for ARIMA model is:

$$\phi_p(B)(1-B)^d Y_t = \theta_q(B) \quad e_t \qquad (23)$$

where

Y_t - is the value of the time series data observed at the time t,

B - is the delay/ lag operator,

ϕ_p and θ_q - are the autoregressive and the moving average polynomials

e_t - is the difference between the observed value Y_t and the forecast $\widehat{Y_t}$ at the time t.

In ARIMA modelling for zone 1,p value is 0.0001590 and number of lags is 24. For zone 2, p value is 0.0002457 with 24 number of lags.

3.3 Neural Network Model

Artificial neural networks are forecasting algorithms that are based on simple mathematical models as shown in Fig 2. They allow complex nonlinear connections between the response variable and its predictors. A neural network is also a series of steps that aim to recognize fundamental connection in a set of data through a process that mimics the way the human brain operates. Neural networks can modify according to the altering input; so the network creates the best output without needing to redesign the output criteria.

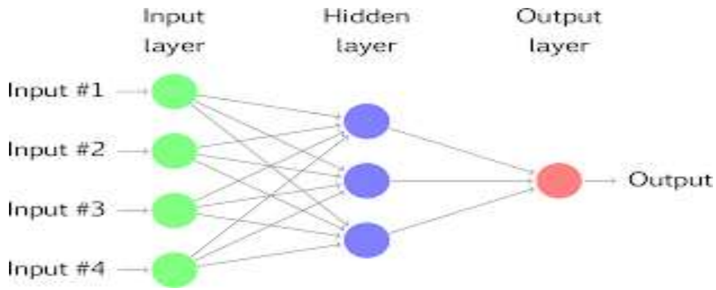

Figure 2. Simple neural network model

The design of neural network model consist of seven primary steps. They are

- Collect data.
- Create the network.
- Configure the network.
- Initialize the weights and biases.
- Train the network.
- Validate the network.
- Use the network.

3.4 LSTM Model

Long Short-Term Memory networks and usually called “LSTMs” which are a special kind of RNN as shown in Fig 3. It is explicitly designed in order to avoid the long-term dependency problem. Remembering information for long periods of time is practically their default behaviour of LSTM. LSTMs also have chain like structure, but the repeating block has a different structure [8].

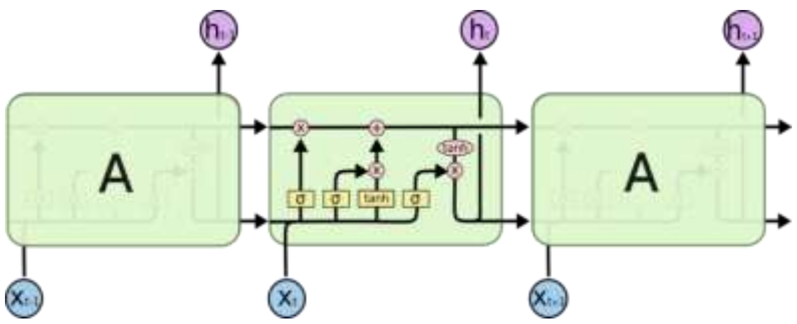

Figure 3. LSTM Network

The core idea behind LSTMs is the cell state, the horizontal line running at the top of the block. The cell state is kind of like a conveyor belt in motor. The cell state runs straight down the entire chain, with some slight linear interactions in the system. It’s very easy for information to just flow along the repeated structure without changing the information . The LSTM does have the capability to remove or add information to the cell state.

The three steps involved in LSTM

Step 1: The first step in LSTM to decide what information are going to discard from the cell state. This final opinion made by a sigmoid layer called the “forget gate layer”.

Step 2: The next step is to decide what kind of new information are going to store in the cell state. This step has two parts. First part, a sigmoid layer called the “input gate layer” decides which values will be updated. Next part, a “tanh” layer creates a vector of new candidate values, and update the state.

Step 3: The final step is to decide what will be the output. This output will be based on the cell state, but it will be a filtered version. First, a sigmoid layer will be run, which decides what parts of the cell state will be displayed as the output.

4. RESULTS AND DISCUSSION

4.1. Software Description

MATLAB and Google colab software are used for modelling the system using different techniques. MATLAB is a programming and numeric computing platform used by millions of engineers, students, scientists for analysing data, developing models,etc,. SI modelling and NN modelling are done in MATLAB.

Google Colab is an online platform which allow users to execute python code through browser especially for machine learning, deep learning and data analysis. ARIMA modelling and LSTM modelling are done in Google Colab.

The responses of model by various supervised learning algorithm and also the validation of the model based on the mean square error.

4.2 Response of First Principle Model for CTS

The response for first principle modelling of the conical tank system for zone 1 and zone 2 are shown in figures 4 and 5 respectively. From the responses, it is clear that the response settled at a particular set point without oscillations. For validation of model, mean square error is also calculated.

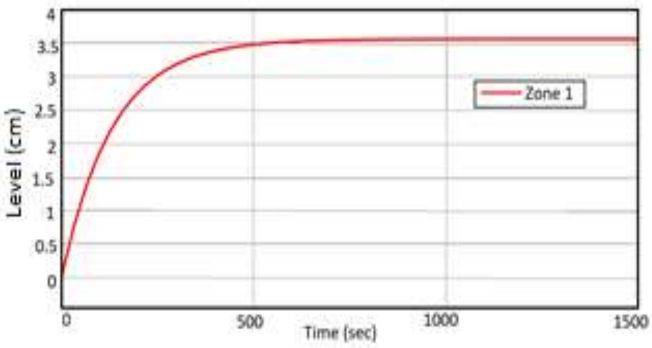

Figure 4. First Principle model response of the conical tank system for zone 1

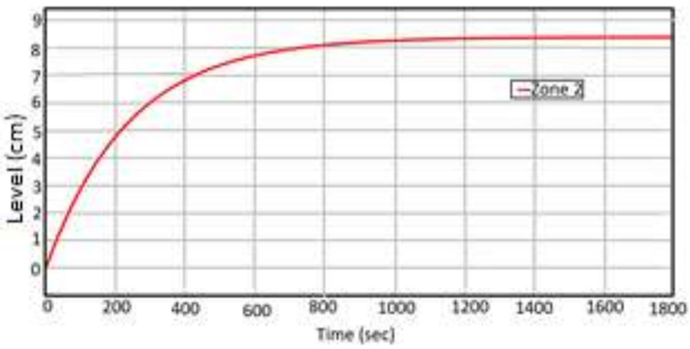

Figure 5. First Principle model response of the conical tank system for zone 2

4.3 Response of linear regression Model for CTS

Figure 6 shows response of the conical tank system for zone 1 and zone 2 using linear regression Model black box approach. From the response, it is clear that the actual response tracks black box response. For validation of model, mean square error is also calculated.

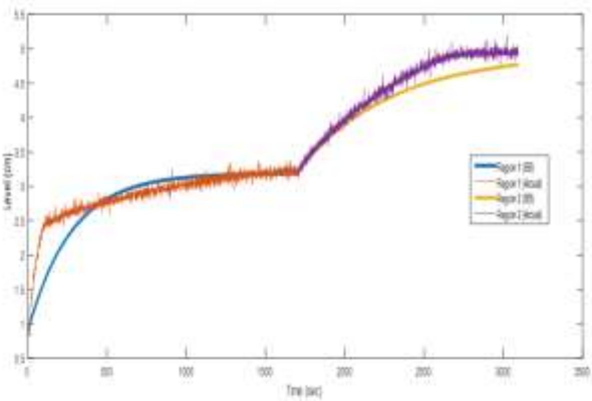

Figure 6. Actual response Vs black box response

4.4 Response of ARIMA model for CTS

Figures 7 and 8 show ARIMA model response of the conical tank system for zone 1 and zone 2. From the figures, it is shown that the response of ARIMA model does not have trend and seasonality in it. Also, the output response is not up to the desired level. For this modelling 1300 samples are used for training and 400 samples are used for testing of this algorithm.

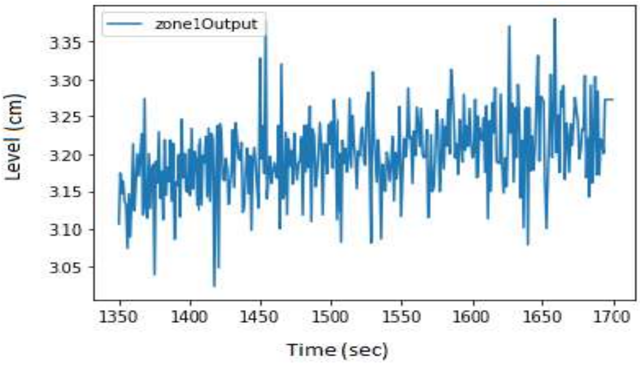

Figure 7. ARIMA model response of the conical tank system for zone 1

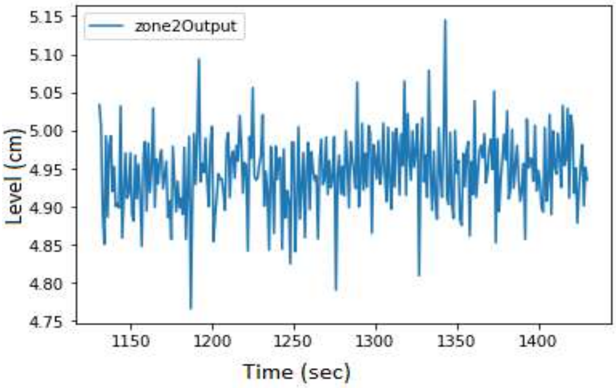

Figure 8. ARIMA model response of the conical tank system for zone 2

4.4 Response of Neural Network Model For CTS

Figures 9 and 10 show time response plots for the best neural network model (NARX) for zone 1 and zone 2. In this modelling 70% (1190 samples) for training, 15% (255 samples) for testing and 15% (255 samples) for validation of experimental data are used for designing model. And this model is analysed based on the mean square error.

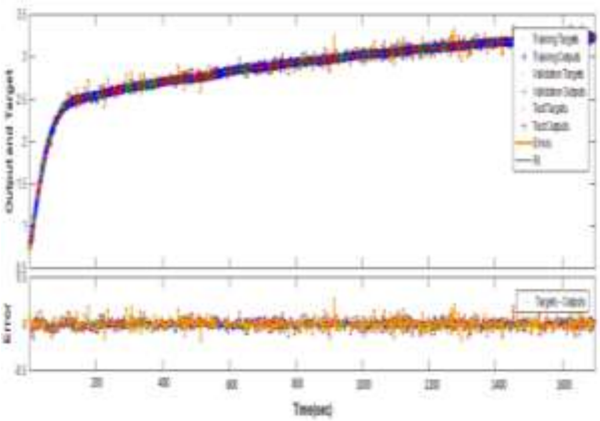

Figure 9. Neural network model response of the conical tank system for zone 1

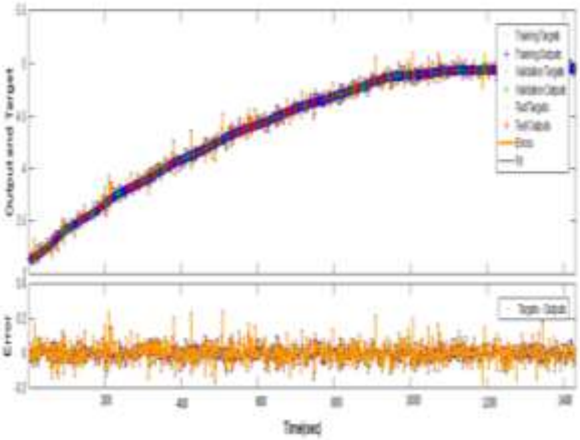

Figure 10. Neural network model response of the conical tank system for zone 2

4.5 RESPONSE OF LSTM MODEL FOR CTS

The response of LSTM model for zone 1 and zone 2 re shown I figures 11 and 12. In this modelling nearly 50% of data are used for training, remaining data are shuffled and used for testing. From this response it shows that the predicted value can able to track the actual value with little variations. This model is also analysed based on the mean square error.

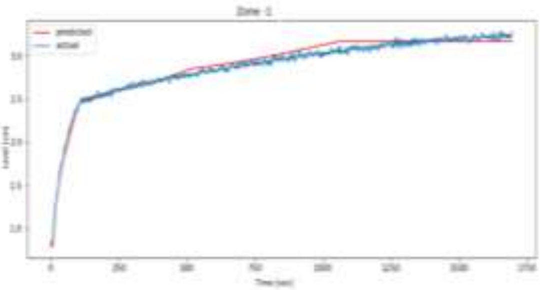

Figure 11. Response of LSTM model for the conical tank system for zone 1

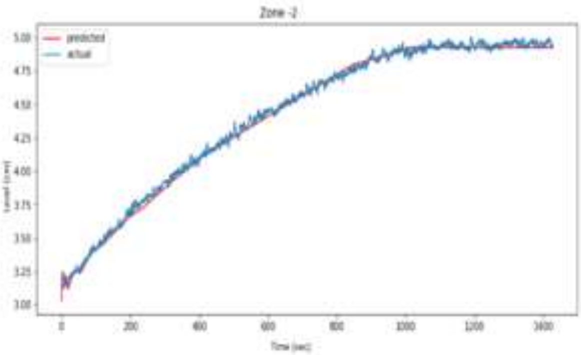

Figure 12. Response of LSTM model for the conical tank system for zone 2

4.5 Validation of Model

Mean square error is the square of difference between the actual value and the predicted value which is calculated for analyzing the best model. Table 2 shows the mean square error for all the models used in this study.

Table 2
Validation of model by mean square error

Different modelling	MSE in Zone 1	MSE in Zone 2
Linear Regression model	0.1301	0.1234
ARIMA model	0.0590	0.04934
Neural Network model	0.0207	0.0040
LSTM model	0.0029	0.0020

The best model is chosen based on which zone has the least value of MSE value which is nthi but LSTM model

5. CONCLUSION

In this paper four modelling approaches are tested by using different supervising algorithms for a real time conical tank. In black box modelling for conical tank system, it is observed that the mean square error is more when compare to other three methods. ARIMA modelling is a standard modelling in machine learning is implemented and observed that the response fails to follow the trend and seasonality, error is also larger. As these two methods of modelling does not yield good result, next level of advanced modelling in time series is neural network and deep learning. In neural network modelling the nonlinearity of the process is identified well and the error is minimum when compared to the previous models. In LSTM modelling the response and the mean square error value is the best when compared to other three methods. Hence LSTM can be used to develop the models for nonlinear systems and it can be used as a reference model in model-based controller design to predict the outputs accurately.

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