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A Study On Hiv Infection Under Intuitionistic Fuzzy Environment

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Abstract : The Mathematical Modelling Of Disease Control And Analysis Of The Diseases Behaviour Of Hiv Dynamics Model Is Discussed . Homotopy Perturbation Method (Hpm) Is Applied For An Analytical Approximation Of Hiv Infection. Semi Analytical Representation For Uninfected Cells, Infected Cells And Free Virus Particles Are Derived Using Homotopy Perturbation Method. The Hpm Yields A Rapidly Convergent, Easily Computable And Readily Verifiable Sequence Of Analytic Approximations That Are Convenient For Parametric Simulations.

Keywords: Hiv Dynamic Model, Intuitionistic Fuzzy Differential Equations, Triangular Intuitionistic Fuzzy Number, Homotopy Perturbation Method

1. INTRODUCTION

Over The Past Few Years, Human Immune Deficiency Virus (Hiv) Continues A Significant Menace To Human Lives. Human Hiv Infection Causes Acquired Immune Deficiency Virus (Aids), A Disease That Has Devastated People Around The World. There Has Been Tremendous Research Work On How To Eradicate The Disease. Mathematical Modelling Of Viral Infections Has Resulted To A Better Knowledge Of The Structure Of Viruses And Has Assisted Predict And Control The Spread Of Bacterial Illnesses Such As Hiv, Hepatitis B Virus (Hbv), Hepatitis C Virus (Hcv), And Dengue Fever. Hiv Infection Analysis Involves Three Main Parameters, Namely Number Of Uninfected Cells, Infected Cells And Free Virus Cells.

Researchers Have Suggested Many Hiv-Dynamic Models (Nowak& May Model 1991, Perelson *Et Al.*1996; Wu *Et Al.*1998; Perelson & Nelson 1999; Ngarakana Gwasira *Et Al.* 2011; Huang *Et Al.* 2003). In Nonlinear Hiv Model Analysis, Investigating Viral Fitness And Assessing The Parameters Of Viral Fitness Are Studied By (Miao *Et Al.* 2009).

In (Najariyan *Et Al.* 2011) Optimal Control Of Hiv Infection Through Fuzzy Dynamical Systems Is Studied By Fuzzifying The Parameters Age, Gender, Feed. Depicting The Amount Of Immune Neurons And Viral Load In People With Hiv Disease Is Represented As A Fuzzy Mathematical Model In Zarei *Et Al.* (2012). In Addition, The Authors Analyzed The Immune Cell Level's Dynamic Behaviour And Viral Load Within The Group Of Patients With Weak, Moderate, And Strong Immune Systems.There Are Enormous Applications Of Ifs (Atanassov's 1983) In Various Scientific Areas Such As Medical Diagnosis (De *Et Al.* 2001), Pattern Recognition (Li &Chang 2002) Microelectronic Fault Analysis (Shu *Et Al.* 2006), Decision-Making Issues (Ye 2009), Drug Selection (Kharal 2009), And So On. (Melliani And Chadli 2000, 2001) Addressed Linear And Partial Differential Equations In An Intuitionistic Fuzzy Environment. Numerical Solution With Intuitionistic Treatment Using

The Runge-Kutta Technique Was Developed By (Abbasbandy And Allahviranloo 2002). (Snehlata And Amit Kumar 2012) Presented A Time-Dependent Intuitionistic Linear Differential Equation And Suggested A Technique For Solving It. (Sankar Prasad Mondal And Tapan Kumar Roy 2014) Addressed The Strong And Weak Solution Of Intuitionistic Fuzzy Differential Equations. In Addition, They Discussed The System Of Differential Equations With Initial Value As Triangular Intuitionistic Fuzzy Number. Second Order Linear Differential Equations With Boundary Conditions As Generalized Trapezoidal Intuitionistic Fuzzy Number Was Discussed By (Mondal And Roy 2015). There Are Few Numerical Methodology Implementations Such As Intuitionistic Fuzzy Euler And Taylor Methods, Runge-Kutta Of Order Four, Adams Bashforth, Adams-Moulton And Predictor-Corrector Methods In Intuitionistic Fuzzy Differential Equations (Nirmala *Et Al.* 2015, Ben Amma *Et Al.*2016, Parimala *Et Al.* 2017 Ben Amma *Et Al.* 2018)

In This Paper, The Three Main Parameters Of Hiv Dynamical System Namely Uninfected Cells, Infected Cells And Free Virus Particles Are Represented In Intuitionistic Fuzzy Environment. Using (I)-Gh Differentiable And (Ii) Gh Differentiable, The Intuitionistic Fuzzy Hiv Model Is Converted As A Set Of Ordinary Nonlinear Differential Equations The Main Purpose Of This Paper Is To Derive Approximate Semi Analytical Expressions For For Uninfected Cells, Infected Cells, And Free Virus Particles Are Derived And To Discuss The Influence Of Empirical Parameters On Concentrations

2. BASIC DEFINITIONS

Intuitionistic Fuzzy Set (Ifs) (Atanassov 1983)

Intuitionistic Fuzzy Set Is A Generalization Of Fuzzy Set Proposed By Atanassov (1983) To Represent Impression In Better Way. A Fuzzy Set Is Characterized By The Degree Of Belongingness. The Intuitionistic Fuzzy Set Is Characterized By Both The Degree Of Belongingness And The Non-Belongingness.

Let A Set X Be Fixed. Ifs $\tilde{A}$ In X Denoted As

$$\tilde{A} = \{< x, \mu_{\overline{A}}(x), \gamma_{\overline{A}}(x) >: x \epsilon X\},$$

Where The Degree Of Membership Is $\mu_{\overline{A}}(x): X \to [0,1]$ And Degree Of Non-Membership Is $\gamma_{\overline{A}}(x): X \to [0,1]$. For Every Element Of $x \in X$, $0 \le \mu_{\overline{A}}(x) + \gamma_{\overline{A}}(x) \le 1$ And The Degree Of Indeterminacy Of $x\ to\ \tilde{A}$ Is $\pi_{\overline{A}}(x) = 1 - \mu_{\overline{A}}(x) - \gamma_{\overline{A}}(x)$.

$(\boldsymbol{\alpha}, \boldsymbol{\beta}) - \boldsymbol{Cut}$ (Atanassov 1983)

A Set Of $(\alpha, \beta) - cut$,Generated By An Ifs $\tilde{A}$ Where α, β Are Fixed Numbers And Lies Between 0 And 1, With $\alpha + \beta \le 1$ Is Defined As

$$[\tilde{A}]_{\alpha,\beta} = \left\{\left(x, \mu_{\overline{A}}(x), \gamma_{\overline{A}}(x)\right): x \in X, \mu_{\overline{A}}(x) \ge \alpha, \gamma_{\overline{A}}(x) \le \beta, \alpha, \beta \in [0,1]\right\}$$

$[\tilde{A}]_{\alpha,\beta} = \{\overline{A}_\alpha$, Where $\overline{A}_\alpha$ Is The Crisp Set Of Elements $x \in atleast\ to\ the\ degree\ \alpha$ And $\overline{A}_\beta$ Is The Crisp Set Of Elements $x \notin to\ atmost\ to\ the\ degree\ \beta$.

Intuitionistic Fuzzy Number (Ifn) (Grzegrorzewski 2003)

An Ifn $\tilde{A}$ Is An Intuitionistic Fuzzy Subset Of The Real Line R Defined As Follows

(i) There Is Any $x_0 \in R\ such\ that\ \mu_{\overline{A}}(x_0) = 1\ and$
$\gamma_{\overline{A}}(x_0) = 0. (Here\ x_0\ is\ the\ mean\ value\ of\ \tilde{A})$

(ii) A Convex Set For The Membership Function $\mu_{\overline{A}}(x)$, I.E.,

$$\mu_{\overline{A}}(\lambda x_1 + (1-\lambda)x_2) \ \ge Min\ \left(\mu_{\overline{A}}(x_1), \mu_{\overline{A}}(x_2)\right) \forall x_{1,} x_2 \in R, \lambda \in [0,1]$$

(iii) A Concave Set For The Non-Membership Function $\gamma_{\overline{A}}(x)$, I.E.,

$$\gamma_{\overline{A}}(\lambda x_1 + (1-\lambda)x_2)) \geq Max\left(\gamma_{\overline{A}}(x_1), \gamma_{\overline{A}}(x_2)\right) \forall x_{1,} x_2 \in R, \lambda \in [0,1].$$

The $(\boldsymbol{\alpha}, \boldsymbol{\beta}) - \boldsymbol{Cut}$ Representation Of Ifn $\widetilde{A}$(Li 2007)

The $(\alpha, \beta) - cut$ Representation Of Ifn $\tilde{A}$ Is

$$[\tilde{A}]_{\alpha,\beta} = \{\overline{A}_\alpha, \overline{A}_\beta \; such\; that \alpha + \beta \leq 1\} = \left\{\left[\underline{A}_1^\alpha, \overline{A}_1^\alpha\right];\ \left[\underline{A}_1^\beta, \overline{A}_1^\beta\right] such\; that\; \alpha + \beta \leq 1\right\},$$

$Where\ \underline{A}_1^\alpha\ is\ \ \alpha - cut\ \ of\ \ \mu_{\overline{A}}(x);\ \underline{A}_1^\beta\ is\ \ \beta - cut\ of\ \gamma_{\overline{A}}(x).$

Traingular Intutitonistic Fuzzy Number (Tifn)(Burillo *Et Al.*1994)

A Tifn $\tilde{A}_{TIFN} = (a_1, a_2, a_3; a_1', a_{2,} a_3')$ Is A Subset Of Ifn In Real Numbers Has Membership Function And Non-Membership Function As Defined Follows:

$$\mu_{\overline{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}\ for\ a_1 \leq x \leq a_{2,} \\ \dfrac{a_3 - x}{a_3 - a_2}\ for a_2 \leq x \leq a_{3,} \\ 0 \qquad other\ wise \end{cases} \gamma_{\overline{A}}(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1{}'}\ for\ a_1{}' \leq x \leq a_{2,} \\ \dfrac{x - a_2}{a_3{}' - a_2}\ for a_2 \leq x \leq a_3{}' \\ 1 \qquad other\ wise \end{cases}$$

The $(\alpha, \beta) - cut$ For Tifn Is

$$[\tilde{A}]_{\alpha,\beta} = \{[a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]; [a_2 - \beta(a_2 - a_1{}'), a_3 - \beta(a_3{}' - a_2)]\}, \alpha, \beta \in [0,1] \text{Where } \alpha + \beta \leq 1.$$

Addition, Subtraction And Scalar Multiplication Of Two Intuitionistic Fuzzy Number Based On $(\boldsymbol{\alpha}, \boldsymbol{\beta})$- Cuts Methods: (Li 2007)

For Arbitrary Tifns $\tilde{u} = (a_1, a_2, a_3; a_1', a_2, a_3')$,
$\tilde{v} = (b_1, b_2, b_3; b_1', b_2, b_3')$ And For Any Scalar k

(a) Addition

$$\tilde{u} \oplus \tilde{v} = (a_1 + b_1, a_2 + b_2, a_3 + b_3; a_1' + b_1', a_2 + b_2, a_3' + b_3')$$

(b) Subtraction

$$\tilde{u} \ominus \tilde{v} = (a_1 - b_3, a_2 - b_2, a_3 - b_1; a_1' - b_3', a_2 + b_2, a_3' - b_1')$$

(C)Scalar Multiplication

$$k \odot \tilde{u} = \begin{cases} (ka_1, ka_2, ka_3; ka_1', ka_2, ka_3'), \mathrm{K} \geq 0 \\ (ka_3, ka_2, ka_1; ka_3', ka_2, ka_1'), \mathrm{K} < 0 \end{cases}$$

Intuitionistic Fuzzy Number Valued Function (Lei Wang & Sicong Guo 2016)

If $P = [a, b]$ Be A Closed Interval In The Real Line R Then
$\tilde{g} : [a, b] \to IFN(\tilde{A})$ Is Called An Intuitionstic Fuzzy Number Valued Function On P, Here $\tilde{g}$ Is Called An n Dimensional Vector Of Intuitionistic Fuzzy Number – Valued Function On P If $\tilde{f} : [a, b] \to \left(IFN(\tilde{A})\right)^n$.

Distance Between Intuitionistic Fuzzy Numbers (Lei Wang & Sicong Guo 2016)

Let $\tilde{A} = \{x, \mu_{\overline{A}}(x), \gamma_{\overline{A}}(x)\}$, $\tilde{B} = \{x, \mu_{\overline{B}}(x), \gamma_{\overline{B}}(x)\}$ Be Two Tifn, And $[\tilde{A}]_{\alpha,\beta} = \left[\underline{A}_1^\alpha, \overline{A}_1^\alpha\right];\ \left[\underline{A}_2^\beta, \overline{A}_2^\beta\right]$ And $[\tilde{B}]_{\alpha,\beta} = \left[\underline{B}_1^\alpha, \overline{B}_1^\alpha\right]; \left[\underline{B}_2^\alpha, \overline{B}_2^\alpha\right]$ Such That $0 \leq \alpha + \beta \leq 1$, Then The Distance Between Intuitionstic Fuzzy Numbers $\tilde{A}$ And $\tilde{B}$ Is Defined As

$$d(\tilde{A},\tilde{B}) = \frac{1}{4}\begin{bmatrix} \int_0^1 |\underline{A}_1^\alpha - \underline{B}_1^\alpha| d\alpha + \int_0^1 |\overline{A}_1^\alpha - \overline{B}_1^\alpha| d\alpha \\ + \int_0^1 |\underline{A}_2^\beta - \underline{B}_2^\beta| d\beta + \int_0^1 |\overline{A}_2^\beta - \overline{B}_2^\beta| d\beta \end{bmatrix}.$$

Clearly $M = (IFN(\tilde{A}), d)$ Is Metric.

Intuitionistic Hukuhara-Difference Of Intuitionistic Fuzzy Numbers (Lei Wang & Sicong Guo 2016)

Let $\tilde{x},\tilde{y} \epsilon IFN(\tilde{A})$. If There Exists $\tilde{z} \in IFN(\tilde{A})$Such As $\tilde{x} = \tilde{y} \oplus \tilde{z}$, Then $\tilde{z}$ Is Called As Intuitionstic Hukuhara-Difference Of Intuitionstic Fuzzy Numbers $\tilde{x}$ And $\tilde{y}$. It Is Denoted By $\tilde{x}-^{ih}\tilde{y}$.

Intuitionistic Strongly Generalized Differentiability (Lei Wang & Sicong Guo 2016)

Let $f:(a,b) \to IFN(\tilde{A})$ And $x_0 \in [\mathrm{A},\mathrm{B}]$. It Can Be Said That $f$is Intuitionistic Differential At x_0, If There Exists An Element $f'(x_0) \in IFN(\tilde{A})$, Such As

(i) For All $h > 0$ Is Sufficiently Small,

$\exists f(x_0+h)-^{ih}f(x_0), \exists f(x_0)-^{ih}f(x_0-h)$ And The Limits Exists In The Metric M

$$\lim_{h\searrow 0}\frac{f(x_0+h)-^{ih}f(x_0)}{h} = \lim_{h\searrow 0}\frac{f(x_0)-^{ih}f(x_0-h)}{h} = f'(\mathrm{X}_0)$$

Or

(Ii) For All $h > 0$is Sufficiently Small,

$\exists f(x_0)-^{ih}f(x_0+h), \exists f(x_0-h)-^{ih}f(x_0)$ And The Limits Exist In The Metric M

$$\lim_{h\searrow 0}\frac{f(x_0)-^{h}f(x_0+h)}{-h} = \lim_{h\searrow 0}\frac{f(x_0-h)-^{h}f(x_0)}{-h} = f'(x_0)$$

Or

(Iii) For All $h > 0$ Is Sufficiently Small,

$\exists f(x_0+h)-^{ih}f(x_0), \exists f(x_0- h)-^{ih}f(x_0)$ And The Limits Exist In The Metric M

$$\lim_{h\searrow 0}\frac{f(x_0+h)-^{ih}f(x_0)}{h} = \lim_{h\searrow 0}\frac{f(x_0-h)-^{ih}f(x_0)}{-h} = f'(x_0)$$

Or

(iv) For All $h > 0$ Is Sufficiently Small,

$\exists f(x_0)-^{ih}f(x_0+h)$, $\exists f(x_0)-^{ih}f(x_0-h)$ And The Limits Exist In The Metric M

$$\lim_{h\searrow 0}\frac{f(x_0)-^{ih}f(x_0+h)}{-h} = \lim_{h\searrow 0}\frac{f(x_0)-^{ih}f(x_0-h)}{h} = f'(x_0)$$

Also, It Can Be Said That $f\ (x)$ Is (I) –Gh Differentiable At x_0 If

$$[f'(x_0)]_{\alpha,\beta} = \{[f_{\mathrm{L}}'(x_0,\alpha), f_{\mathrm{R}}'(x_0,\alpha)]; [f_{\mathrm{L}}'(x_0,\beta), f_{\mathrm{R}}'(x_0,\beta)]$$

And, F(X) Is (Ii) –Gh Differentiable At x_0 If

$$[f'(x_0)]_{\alpha,\beta} = \{[f_{\mathrm{R}}'(x_0,\alpha), f_{\mathrm{L}}'(x_0,\alpha)]; [f_{\mathrm{R}}'(x_0,\beta), f_{\mathrm{L}}'(x_0,\beta)]$$

Intuitionistic Fuzzy Differential Equations With Intuitionistic Fuzzy Derivative (Lei Wang & Sicong Guo 2016)

Consider The Intuitionistic Fuzzy Initial Values Problem As

$$\begin{cases} X'(t) = G(t, X(t)), t \in [t_{0,}T] \\ X(0) = X_0 \end{cases}$$

Where $G: I \times S \ \to S$ Is A Continuous Intuitionistic Fuzzy Mapping And $X_0 \in S$. The Interval I May Be Like $[0,T]$ For Some $T > 0\ or\ I = [0,\infty)$ And S Be Set Of Intuitionistic Fuzzy Numbers.

3. HIV MODEL FORMULATION

The Nowak & May Model (1991) Is A Standard Model For Hiv Infection. According To This Model, The Mathematical Equations Are Represented As Follows:

$$\frac{dn}{dt} = r - an - \beta nv \tag{1}$$

$$\frac{di}{dt} = \beta nv - bi \tag{2}$$

$$\frac{dv}{dt} = ki - sv \tag{3}$$

Where The Parameters In (1-3) Are Considered As:
n - Number Of Uninfected Cells, i- Number Of Infectedcells, v - Number Of Free Virusparticle,r - Constant Rate Of Uninfected Cells Produced, an – Death Rate Of Uninfected Cells,βnv – Rate Of Production Of Infected Cells By Uninfected Cells With Free Virus,bi - Death Rate Of Infected Cells,ki- Rate Of Produced Of Infected Cells By Free Cells,sv - Rate Of Free Virus Is Die From Infected Cells.

3.1 Hiv Model Formulation Using Intuitionistic Fuzzy Differential Equations

In Practice, It Is Difficult To Examine The Count Of Infected Cells, Uninfected Cells And Free Virus Particles Exactly Due To Impression In Measurement. Najariyan *Et Al.* (2011) Considered This Vagueness And Modelled Hiv Infection In Fuzzy Environment Using Discretization Method. Since There Is A Possibility Of Change In Cells Count. This Fuzzy Discussion Also Having Lack To Produce The Results In Optimal Control Of Hiv Infection. As The Uninfected Cells/ Free Virus Particles May Be Attacked By Infected Cells, The Counts Of These Cells Keep Varying. Due To This Change, The Count Of Cells Need Not Be Precisely Defined. This Impression In Change In Count Of Infected Cells, Uninfected Cells And Free Virus Particles Can Be Best Represented Using Ifs. Hence, For This Hiv Model, The Cell Counts Are Considered In Intuitionistic Fuzzy Environment.
As A Result, The Equations (1) - (3) Representing Hiv Dynamic Nowak & May Model Is Rewritten As,

$$\frac{d\bar{n}}{dt} = r - a\bar{n} - \beta\bar{n}\bar{v} \tag{4}$$

$$\frac{d\bar{\imath}}{dt} = \beta\bar{n}\bar{v} - b\bar{\imath} \tag{5}$$

$$\frac{d\bar{v}}{dt} = k\bar{\imath} - s\bar{v} \tag{6}$$

Here, The Count Of Uninfected Cells, Infected Cells And Free Virus Particles Are Considered As Intuitionistic Fuzzy Variables Namely $\bar{n}$, $\bar{\imath}$, $\bar{v}$ Respectively. Using, (I)-Gh Differentiable And (Ii)-Gh Differentiable To Equation (4) - (6). The Following System Of Ordinary Differential Equations Are Obtained.
For Degree Of Belongingness

$$\frac{d\overline{n}^{\alpha}}{dt} = r - a\underline{n}^{\alpha} - \beta\underline{n}^{\alpha}\underline{v}^{\alpha} \tag{7}$$

$$\frac{d\overline{i}^{\alpha}}{dt} = \beta\,\overline{n}^{\alpha}\overline{v}^{\alpha} - b\underline{i}^{\alpha} \tag{8}$$

$$\frac{d\overline{v}^{\alpha}}{dt} = k\overline{i}^{\alpha} - s\underline{v}^{\alpha} \tag{9}$$

And

$$\frac{d\underline{n}_{\alpha}}{dt} = r - a\overline{n}^{\alpha} - \beta\,\overline{n}^{\alpha}\overline{v}^{\alpha} \tag{10}$$

$$\frac{d\underline{i}^{\alpha}}{dt} = \beta\,\overline{n}^{\alpha}\overline{v}^{\alpha} - b\overline{i}^{\alpha} \tag{11}$$

$$\frac{d\underline{v}_{\alpha}}{dt} = k\underline{i}^{\alpha} - s\overline{v}^{\alpha} \tag{12}$$

(Or)

$$\frac{d\overline{n}^{\alpha}}{dt} = r - a\overline{n}^{\alpha} - \beta\,\overline{n}^{\alpha}\overline{v}^{\alpha} \tag{13}$$

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$\frac{d\bar{i}^{\alpha}}{dt} = \beta\ \underline{n}_{\alpha}\underline{v}_{\alpha} - b\bar{i}_{\alpha}$ (14)

$\frac{d\bar{v}^{\alpha}}{dt} = k\underline{i}^{\alpha} - s\bar{v}^{\alpha}$ (15)

And

$\frac{d\underline{n}_{\alpha}}{dt} = r - a\underline{n}^{\alpha} - \beta\underline{n}^{\alpha}\underline{v}^{\alpha}$ (16)

$\frac{d\underline{i}^{\alpha}}{dt} = \beta\ \bar{n}^{\alpha}\bar{v}^{\alpha} - b\bar{i}_{\alpha}\beta\ \bar{n}^{\alpha}\bar{v}^{\alpha} - b\bar{i}^{\alpha}$ (17)

$\frac{d\underline{v}_{\alpha}}{dt} = k\bar{i}^{\alpha} - s\underline{v}^{\alpha}$ (18)

Similarly, For Degree Of Non-Belongingness The Differential Equations Are Given By

$\frac{d\bar{n}^{\beta}}{dt} = r - a\underline{n}^{\beta} - \beta\underline{n}^{\beta}\underline{v}^{\beta}$ (19)

$\frac{d\bar{i}^{\beta}}{dt} = \beta\ \bar{n}^{\beta}\bar{v}^{\beta} - b\underline{i}^{\beta}$ (20)

$\frac{d\bar{v}^{\beta}}{dt} = k\bar{i}^{\beta} - s\underline{v}^{\beta}$ (21)

And

$\frac{d\underline{n}_{\beta}}{dt} = r - a\bar{n}^{\beta} - \beta\ \bar{n}^{\beta}\bar{v}^{\beta}$ (22)

$\frac{d\underline{i}^{\beta}}{dt} = \beta\ \bar{n}^{\beta}\bar{v}^{\beta} - b\bar{i}^{\beta}$ (23)

$\frac{d\underline{v}_{\beta}}{dt} = k\underline{i}^{\beta} - s\bar{v}^{\beta}$ (24)

(Or)

$\frac{d\bar{n}^{\beta}}{dt} = r - a\bar{n}^{\beta} - \beta\ \bar{n}^{\beta}\bar{v}^{\beta}$ (25)

$\frac{d\bar{i}^{\beta}}{dt} = \beta\ \underline{n}_{\beta}\underline{v}_{\beta} - b\bar{i}_{\beta}$ (26)

$\frac{d\bar{v}^{\beta}}{dt} = k\underline{i}^{\beta} - s\bar{v}^{\beta}$ (27)

And

$\frac{d\underline{n}_{\beta}}{dt} = r - a\underline{n}^{\beta} - \beta\underline{n}^{\beta}\underline{v}^{\beta}$(28)

$\frac{d\underline{i}^{\beta}}{dt} = \beta\ \bar{n}^{\beta}\bar{v}^{\beta} - b\bar{i}^{\beta}$ (29)

$\frac{d\underline{v}_{\beta}}{dt} = k\bar{i}^{\beta} - s\underline{v}^{\beta}$ (30)

For Analysis, The Following Initial Conditions Are Considered

$\bar{n}(0) = (850 + 150\alpha, 1150 - 150\alpha, 750 + 250\beta, 1050 + 50\beta)$ (31)

$\bar{i}(0) = (3 + 2\alpha, 7 + 2\alpha, 2.5 + 2.5\beta, 6.5 + 1.5\beta)$ (32)

$\bar{v}(0) = (6750 + 250\ \alpha, 7250 + 250\alpha; 6650 + 350\beta, 7150 + 150\beta)$(33)

The Parameters And Initial Values (Najariyan *Et Al.* 2011) Are Shown Respectively In Table 1 And 2.

Table1 Parameters Of The Microscopic Hiv Model

$r = 7$	$a = 0.007$	$\beta = 42163 \times 10^{-11}$
$b = 0.0999$	$s = 0.2$	$k = 90.67$

Table 2 Initial Conditions Of The Crisp Model

$n(0)$	1000
$i(0)$	5

$v(0)$	7000
$t\ initial$	0 Time Units
$t\ final$	500 Time Units

*3.2**analytical Expression Of The Cells Using Homotopy Perturbation Method (Hpm)***

The Hiv Model Equations (7) To (30) Are Based On Non-Steady State System Of Equations Containing Non-Linear Terms. Hpm Method (He 1999) Is Used To Solve The System Of Equations.

The Solution Of Hiv Dynamic Model Equations (7) – (9) For Uninfected Cells, Infected Cells And Free Virus Particles Using (I)-Gh Differentiable Are Given Below (31)- (33).

$$\overline{n}^{\alpha}(t) = (1150-150\alpha)e^{-at} + \left[\frac{-r}{a} - \frac{\beta}{s}(1150-150\alpha)(7250+\ 250\alpha)\right]e^{-at} + \frac{r}{a} + \frac{\beta}{s}(1150-150)(7250+250\alpha)e^{-(s+a)t} \quad (31)$$

$$\overline{i}^{\alpha}(t) = (3+2\alpha)e^{-bt} + \left[-\frac{\beta}{b}(1150-150\alpha)(7250+250\alpha)\right]e^{-bt} + \frac{\beta}{b}(1150-150\alpha)(7250+250\alpha)e^{-(s+a)t} \quad (32)$$

$$\overline{v}^{\alpha}(t) = (7250+250\alpha)e^{-st} + \frac{k}{s-b}(7+2\alpha)e^{-bt} - \frac{k}{s-b}(7+2\alpha)e^{-st} \quad (33)$$

Solution Of Equations (10)-(12) (Degree Of Belongingness) For (Ii)-Gh Differentiable Using Hpm

Continuing So With (Ii)-Gh Differentiable In Equations (10) - (12) Is, The Final Solution Of Hiv Dynamic Model For Uninfected Cells, Infected Cells And Free Virus Particles Are Given In Equations (34) – (36).

$$\underline{n}_{\alpha}(t) = (1150-150\alpha)e^{-at} + \left[\frac{-r}{a} - \frac{\beta}{s}(1150-150\alpha)(7250-250\alpha)\right]e^{-at} + \frac{r}{a} + \frac{\beta}{s}(1150-150)(7250+250\alpha)e^{-(s+a)t} \quad (34)$$

$$\underline{i^{\alpha}}(t) = (7+2\alpha)e^{-bt} + [-\beta(1150-150\alpha)(7250+250\alpha)]e^{-bt} + \alpha(1150-150\alpha)(7250+250\alpha)e^{-(s+a)t} \quad (35)$$

$$\underline{v}_{\alpha}(t) = (7250+250\alpha)e^{-st} + \frac{k}{s-b}(7+2\alpha)e^{-bt} - \frac{k}{s-b}(7+2\alpha)e^{-st} \quad (36)$$

Solution Of Equations (19) - (21) (Degree Of Non-Belongingness) For (I)- Gh Differentiable Using

$$\overline{n}^{\beta} = (750+250\beta)e^{-at} + \left[-\frac{r}{a} - \frac{\beta(750+250\beta)(6650+350\beta)}{s}\right]e^{-at} + \frac{r}{a} + \frac{\beta(750+250\beta)e^{-at}(6650+350\beta)e^{-st}}{s} \quad (37)$$

$$\overline{i}^{\beta} = (2.5+2.5\beta)e^{-bt} + \left[\frac{\beta(1050+50\beta)(7150+150\beta)}{a+s-b}\right]e^{-bt} + \frac{r}{a} + \frac{\beta(1050+50\beta)e^{-at}(7150+150\beta)e^{-st}}{b-a-s} \quad (38)$$

$$\overline{v}^{\beta} = (6650+350\beta)e^{-st} + \left[\frac{k(6.5+1.5\beta)}{b-s}\right]e^{-st} + \frac{k(6.5+1.5\beta)e^{-bt}}{s-b} \quad (39)$$

Solution Of Equations (22) - (24) (Degree Of Non-Belongingness) For (Ii)- Gh Differentiable Using Hpm

$$\underline{n}^{\beta} = (1050+50\beta)e^{-at} + \left[-\frac{r}{a} - \frac{\beta(1050+50\beta)(7150+150\beta)}{s}\right]e^{-at} + \frac{r}{a} + \frac{\beta(1050+50\beta)e^{-at}(7150+150\beta)e^{-st}}{s} \quad (40)$$

$$\underline{i}^{\beta} = (6.5+1.5\beta)e^{-bt} + \left[\frac{\beta(750+250\beta)(6650+350\beta)}{a+s-b}\right]e^{-bt} + \frac{r}{a} + \frac{\beta(750+250\beta)e^{-at}(6650+350\beta)e^{-st}}{b-a-s} \quad (41)$$

$$\underline{v}^{\beta} = (7150+150\beta)e^{-st} + \left[\frac{k(2.5+2.5\beta)}{b-s}\right]e^{-st} + \frac{k(2.5+2.5\beta)e^{-bt}}{s-b} \quad (42)$$

4. DISCUSSION

The Present Study Has Applied Intuitionistic Fuzzy Environment To A Hiv Dynamic Nowak & May Model (1991). The Three Main Parameters, Uninfected Cells, Infected Cells And Free Virus Particles Are Considered As Ifs. Using (I)-Gh Differentiable And (Ii)-Gh Differentiable The Intuitionistic Fuzzy Hiv Model Is Converted As A Set Of Ordinary Nonlinear Differential Equations. In Addition, The Semi Analytical Expressions For Uninfected Cells, Infected Cells And Free Virus Particles Are Derived By Homotopy Perturbation Method.

Figures (1)-(6) Represents Plot Of The Solution For Degree Of Belongingness And Degree Of Non-Belongingness Using (I)- Gh Differentiable Of Uninfected Cells $\overline{n}^{\alpha}(t), \underline{n}_{\alpha}(t)$, Infected Cells $\overline{i}^{\alpha}(t), \underline{i}_{\alpha}(t)$ And Free Virus Particles $\overline{v}^{\alpha}(t), \underline{v}_{\alpha}(t)$ For Possible Values Of Parameters As r, a, b, k, sas Given In Table (1), (2) For Values Of $\alpha = 0,0.5,1$and Fixed Value Of Some Experimental Parameter.

Figures (7)-(12) Represent Plot Of The Solution For Degree Of Belongingness And Degree Of Non-Belongingness Using (Ii)-Gh Differentiable Of Uninfected Cells $\overline{n}^{\beta}(t), \underline{n}_{\beta}(t)$, Infected Cells $\overline{i}^{\beta}(t), \underline{i}_{\beta}(t)$ And Free Virus Particles $\overline{v}^{\beta}(t), \underline{v}_{\beta}(t)$ For Possible Values Of Parameters As r, a, b, k, s As Given In Table (1), (2) For Values Of $\beta =$ 0,0.5,1and Fixed Value Of Some Experimental Parameter

From These Figures, It Is Observed That, As α Value Decreases The Count Of Uninfected Cells Also Decreases For (I)-Gh Differentiable Where As It Is Notes That The Alpha Cut Decreases With Decrease In Uninfected Cells, Infected Cells And Number Free Virus Particles. It Is Similar Way To Discuss For β.

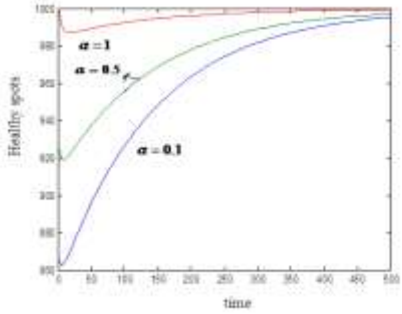

Figure 1 Plot Of Uninfected Cells nversus Time Using Equation (31) For Possible Values Of Parameters r, a, b, k, s. The Uninfected Cells Are Computed For Values Of $\alpha = 0,0.5,1$ And Fixed Value Of Some Experimental Parameter.

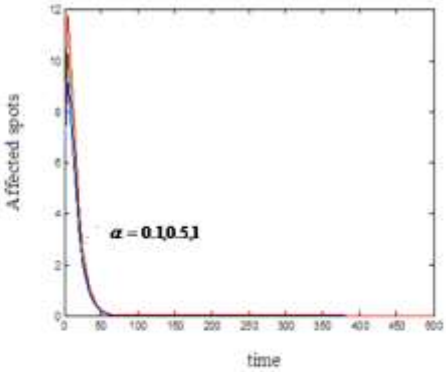

Figure 2 Plot Of Infected Cells i Versus Time Using Equation (32) For Possible Values Of Parameters r, a, b, k, s. The Infected Cells Are Computed For Values Of $\alpha = 0,0.5,1$ And Fixed Value Of Some Experimental Parameter.

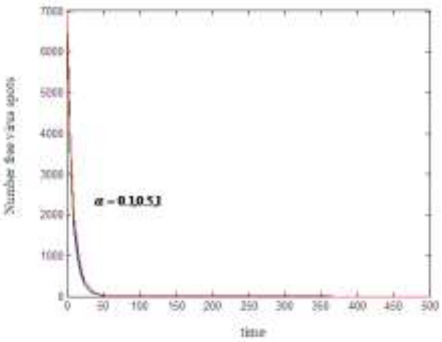

Figure 3 Plot Of Free Virus Particles v Versus Time Using Equation (33) For Possible Values Of Parameters r, a, b, k, s. The Free Virus Particles Are Computed For Values Of $\alpha =$ 0,0.5,1and Fixed Value Of Some Experimental Parameter.

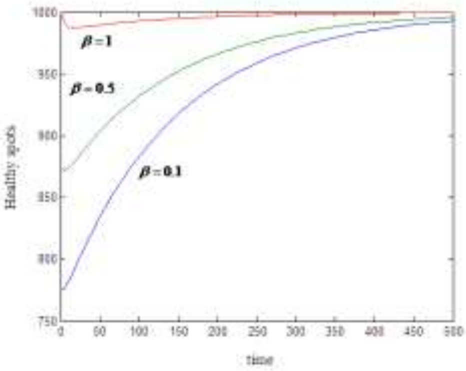

Figure 4 Plot Of Uninfected Cells n Versus Time Using Equation (37) For Possible Values Of Parameters r, a, b, k, s. The Uninfected Cells Are Computed For Values Of $\beta = 0{,}0.5{,}1$ And Fixed Value Of Some Experimental Parameter.

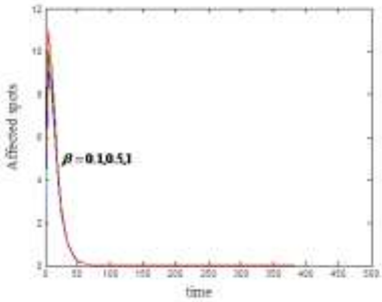

Figure 0 Plot Of Affected Cells i Versus Time Using Equation (38) For Possible Values Of Parameters r, a, b, k, s. The Affected Cells Are Computed For Values Of $\beta = 0{,}0.5{,}1$ And Fixed Value Of Some Experimental Parameter.

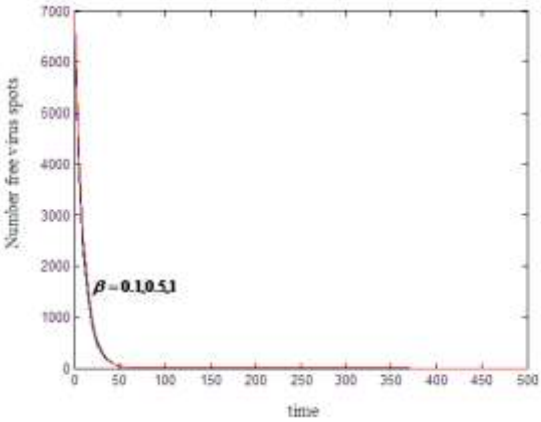

Figure 5 Plot Of Free Virus Particles v Versus Time Using Equation (39) For Possible Values Of Parameters r, a, b, k, s. The Free Virus Particles Are Computed For Values Of $\beta = 0{,}0.5{,}1$and Fixed Value Of Some Experimental Parameter.

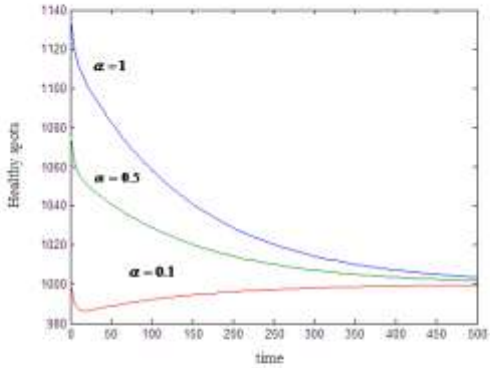

Figure 6 Plot Of Uninfected Cells n Versus Time Using Equation (34) For Possible Values Of Parameters r, a, b, k, s. The Uninfected Cells Are Computed For Values Of $\alpha = 0{,}0.5{,}1$ And Fixed Value Of Some Experimental Parameter.

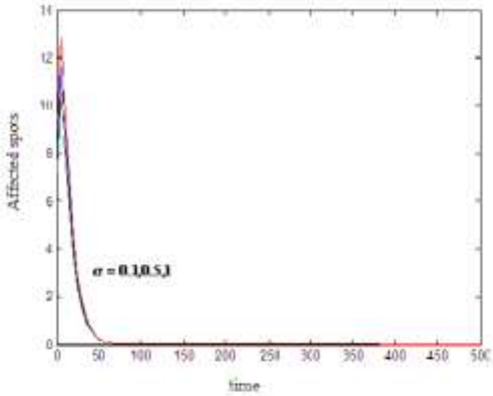

Figure 7 Plot Of Infected Cells i Versus Time Using Equation (35) For Possible Values Of Parameters r, a, b, k, s. The Infected Cells Are Computed For Values Of $\alpha = 0{,}0.5{,}1$ And Fixed Value Of Some Experimental Parameter.

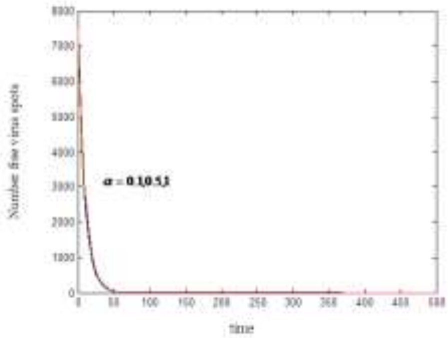

Figure 8 Plot Of Free Virus Particles v Versus Time Using Equation (36) For Possible Values Of Parameters r, a, b, k, s. The Free Virus Particles Are Computed For Values Of $\alpha = 0{,}0.5{,}1$ And Fixed Value Of Some Experimental Parameter.

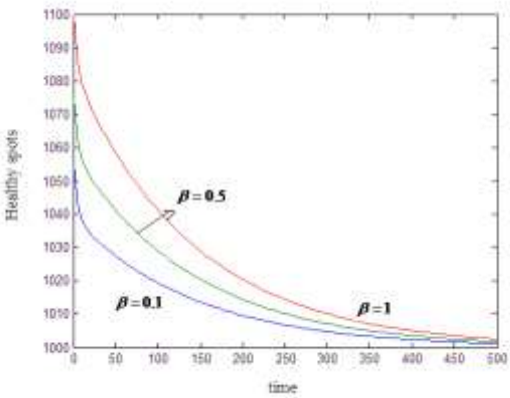

Figure 9 Plot Of Uninfected Cells n Versus Time Using Equation (40) For Possible Values Of Parameters r, a, b, k, s. The Uninfected Cells Are Computed For Values Of $\beta = 0{,}0.5{,}1$ And Fixed Value Of Some Experimental Parameter.

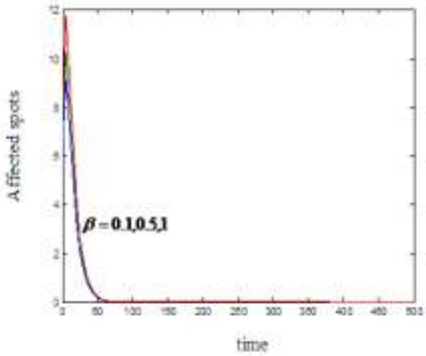

Figure 10 Plot Of Infected Cells i Versus Time Using Equation (41) For Possible Values Of Parameters r, a, b, k, s. The Infected Cells Are Computed For Values Of $\beta = 0{,}0.5{,}1$and Fixed Value Of Some Experimental Parameter.

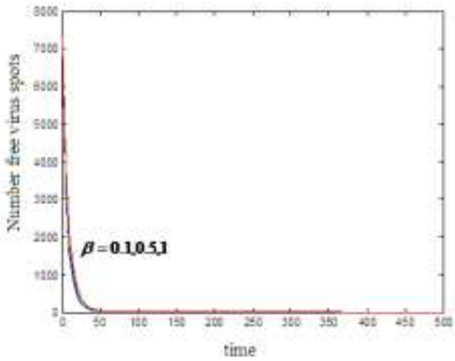

Figure 11 Plot Of Free Virus Particles v Versus Time Using Equation (42) For Possible Values Of Parameters r, a, b, k, s. The Free Virus Particles Are Computed For Values Of $\beta = 0{,}0.5{,}1$and Fixed Value Of Some Experimental Parameter

5. CONCLUSION

In This Paper, Optimal Control Of Hiv Infection Is Discussed In Intuitionistic Fuzzy Environment. The Semi Analytical Expression For Uninfected Cells, Infected Cells, And Free Virus Particles Are Derived By Using Homotopy Perturbation Method. This Technique

Provides Approximate And Simple Solutions That Are Quick, Easy To Compute, And Efficiently Correct. These Estimated Findings Are Used To Plot Of The Uninfected Cells, Infected Cells And Free Virus Particles For Different Values Of $\alpha - cut$, $\beta - cut$ Are Presented.

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