

On Soft $g\#$ s-closed Mappings in Soft Topological Spaces

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Abstract : *The concept of soft $g\#$ s-closed mappings and soft $g\#$ s-open mappings have been introduced and some of its properties in soft topological spaces have been investigated here in this paper.*

Keywords: *Soft sets, Soft topology, soft-interior, soft-closure, soft-closed, soft generalized closed, soft $g\alpha$ -closed set, soft og -closed set, soft $g\#$ s-closed set, soft-sets mapping, soft closed mapping, soft $g\#$ s closed mapping and soft normal space.*

1. INTRODUCTION

In Mathematics, Soft set theory is a recently developing field. Molodstov(1) has introduced the concept of soft set theory and it deals the uncertainty in 1999. The theory of soft set has been initiated as a Mathematical Model to discuss the uncertainty.

The concept of soft set theory helped the researchers to develop a new concept Soft topological space. The concept soft topological space has been introduced in 2001 by Muhammed Shabir, Munazz Naz. These ideas of soft topological spaces induced the researchers to develop the concepts, soft generalized closed in soft topological spaces. K. Kannan introduced these concepts of soft generalized closed sets in 2012. The ideas of $g\#$ semi-closed sets in topological spaces have been introduced by Veerakumar, 2002. The soft $g\#$ semi-closed sets in soft topological space has been introduced by T. Rajendrakumar, V. Kaladevi, 2016.

In this article, the idea of soft $g\#$ s – closed mappings and soft $g\#$ s – open mappings have been introduced and some of its properties in soft topological spaces have been examined.

Basic Concepts : Throughout this present paper, (X, τ, E) denotes a soft topological space having no separation axioms. The notations $Cl(A, E)$, $Int(A, E)$ $sCl(A, E)$ and $\alpha Cl(A, E)$ respectively denotes the closure, interior, semi-closure and α closure for a soft set (A, E) .

Definition 2.1 [7] Let X denote the initial universe and E denotes the set of parameters and the power set of X is denoted by $P(X)$ and if A is the non-empty subset of E then the soft set over x is defined as a pair (F, E) , where F is defined as a function $F:A \rightarrow P(X)$.

Definition 2.2 [2] Let X denote the initial universe and E denotes the set of parameters. A soft set (F, E) over X is said to be **null soft set** denoted by \emptyset if for all $e \in E$, $F(e) = \phi$. A soft set (F, E) over X is said to be **absolute soft set** denoted by X if for all $e \in E$, $F(e) = X$.

Definition 2.3 [8] Let X be considered as an initial universe and the set of parameters defined on the initial universe X is denoted as E and let τ be a collection of soft sets over X , then τ is said to be a **soft topology** on X if

- i) \emptyset, X belongs to τ
- ii) The union of any number of soft open sets in τ belongs to τ
- iii) The finite intersection of soft open sets in τ belongs to τ

The triplet (X, τ, E) is said to be a **soft-topological space** over X .

The member of τ is said to be **soft-open set** in X . The soft-closed set is defined as the complement of a soft-open set over X .

Definition 2.4 [2] Let (X, τ, E) be considered as a soft topological space over X and the soft-set over X is considered as (F, E) . Then **soft interior of a soft-set (F, E) is defined as**

$Int(F, E) = \cup \{ (H, E) : (H, E) \text{ is soft open and } (H, E) \subseteq (F, E) \}$ **and a soft-closure of a soft-set (F, E) is defined as**

$Cl(F, E) = \cap \{ (H, E) : (H, E) \text{ is soft closed sets and } (H, E) \supseteq (F, E) \}$.

Definition 2.5 [3] Consider a soft-topological space (X, τ, E) . Let (A, E) be a subset of (X, τ, E) and (A, E) is said to be **soft semi-open set** if $(A, E) \subseteq Cl(Int(A, E))$ and a **soft semi-closed** if $Int(Cl(A, E)) \subseteq (A, E)$.

Definition 2.7 [4] A subset (A, E) of soft topological space (X, τ, E) is called **soft α -open** if $(A, E) \subseteq Int(Cl(Int(A, E)))$ and a **soft α -closed** if $Cl(Int(Cl(A, E))) \subseteq (A, E)$.

The **semi closure** [3] (respectively **pre closure**, **α -closure** and **β -closure**) of a soft set A of (X, τ, E) is the intersection of all soft semi -Closed (respectively soft pre-closed, soft α -closed and soft β -closed) sets that contain A and is denoted by $sCl(A)$ (respectively $pCl(A)$, $\alpha Cl(A)$ and $\beta Cl(A)$).

Definition 2.9 [2] A subset (A, E) of soft topological space (X, τ, E) is called **soft generalized closed** if $Cl(A, E) \subseteq (H, E)$, whenever $(A, E) \subseteq (H, E)$ and (H, E) is soft open set in X .

Definition 2.10 [1] A subset (A, E) of soft topological spaces (X, τ, E) is called **soft generalized semi closed**(soft gs - closed) if $sCl(A, E) \subseteq (H, E)$ whenever $(A, E) \subseteq (H, E)$ and (H, E) is soft open set in X .

Definition 2.11 [1] A subset (A, E) of soft topological spaces (X, τ, E) is called **soft α -generalized closed** (soft α g - closed) if $\alpha Cl(A, E) \subseteq (H, E)$ whenever $(A, E) \subseteq (H, E)$ and (H, E) is soft open set in X .

Definition 2.12 [1] A subset (A, E) of (X, τ, E) is called **soft generalized α closed** ($g\alpha$ -closed) if $\alpha Cl(A, E) \subseteq (H, E)$ whenever $(A, E) \subseteq (H, E)$ is soft α open set in X .

Definition 2.13 [10] Let (X, τ, E) be a soft topological space and (F, E) be a soft subset of X . The soft set (F, E) is said to be **soft g^*s -closed** if $scl(F, E) \subseteq (H, E)$ Whenever $(F, E) \subseteq (H, E)$ and (H, E) is soft gs -open.

Definition 2.14 [13] Let (X, τ, E) be a soft topological space. A soft set (A, E) of (X, τ, E) is called soft $g^{\#}s$ -closed if $sCl(A, E) \subseteq (H, E)$, whenever $(A, E) \subseteq (H, E)$ and (H, E) is soft α g -open in X .

Definition 2.15 [13] Let (A, E) be soft set in soft topological spaces. Then soft $g^{\#}s$ closure and soft $g^{\#}s$ interior of (A, E) are defined as follows:

- i) $g^{\#}s Cl(A, E) = \cap \{ (F, E) : (F, E) \text{ is soft } g^{\#}s \text{ closed set and } (A, E) \subseteq (F, E) \}$
- ii) $g^{\#}s Int(A, E) = \cup \{ (F, E) : (F, E) \text{ is soft } g^{\#}s \text{ open set and } (A, E) \supseteq (F, E) \}$

Definition 2.16 [14] Let (F, E) be a soft set over X . The soft set (F, E) is called a **soft point**, denoted by (x_e, E) , if for the element $e \in E$, $F(e) = \{x\}$ and $F(e^1) = \emptyset$ for all $e^1 \in E - \{e\}$.

Definition 2.17 [14] Let (X, τ, E) be a soft topological space over X and (G, E) be a soft set over X and $x \in X$. Then x is said to be a **soft interior point** of (G, E) , if there exists a soft open set (F, E) such that $x \in (F, E) \subseteq (G, E)$.

Definition 2.18 [14] Let (X, τ, E) be a soft topological space over X and (G, E) be a soft set over X and $x \in X$. Then (G, E) is said to be a **soft neighbourhood** of x , if there exists a soft open set (F, E) such that $x \in (F, E) \subseteq (G, E)$.

Definition 2.19 [12] Let (F, A) and (G, B) be two soft sets over U , then the Cartesian product of (F, A) and (G, B) is defined as, $(F, A) \times (G, B) = (H, A \times B)$, where $H: A \times B \rightarrow P(U \times U)$ and $H(a, b) = F(a) \times G(b)$, Where $(a, b) \in A \times B$.

i.e., $H(a, b) = \{ (h_i, h_j) : \text{where } h_i \in F(a) \text{ and } h_j \in G(b) \}$.

The cartesian product of three or more nonempty soft sets can be defined by generalizing the definition of the Cartesian product of two soft sets. The Cartesian product $(F_1, A) \times (F_2, A) \times \dots \times (F_n, A)$ of the nonempty soft sets $(F_1, A), (F_2, A), \dots, (F_n, A)$ is the soft set of all ordered n -tuple (h_1, h_2, \dots, h_n) where $h_i \in F_i(a)$.

Definition 2.20 [12] Let (F, A) and (G, B) be two soft sets over U , then a relation from (F, A) to (G, B) is a soft subset of $(F, A) \times (G, B)$.

In other words, a relation from (F, A) to (G, B) is of the form (H_1, S) where $S \subset A \times B$ and $H_1(a, b) = H(a, b)$, for all $(a, b) \in S$. Where $(H, A \times B) = (F, A) \times (G, B)$ as defined in the definition of Cartesian product. Any subset of $(F, A) \times (G, B)$ is called a relation on (F, A) .

Definition 2.21 [12] Let (F, A) and (G, B) be two non-empty soft sets. Then a soft set relation f from (F, A) to (G, B) is called a **soft set function** if every element in domain has a unique element in the range. If $F(a)$, & $G(b)$ then we write $f(F(a)) = G(b)$.

Definition 2.22 [12] A function f from (F, A) to (G, B) is called **injective(one- one)** if $F(a) \neq F(b) \Rightarrow f(F(a)) \neq f(F(b))$. i.e., f is called injective if each element of the range f appears exactly once in the function.

Definition 2.23 [12] A function f from (F, A) to (G, B) is called **surjective (onto)** if range of $f = (G, B)$.

Definition 2.24 [12] A function which is both injective and surjective is called a **bijjective function**.

Definition 2.25 [12] Let $f: (F, A) \rightarrow (G, B)$ and $g: (G, B) \rightarrow (H, C)$ be two soft set functions. Then $g \circ f: (F, A) \rightarrow (H, C)$ is also a soft set function defined by $(g \circ f)(F(a)) = g(f(F(a)))$. This is called **composition of two soft set functions**.

Definition 2.26 [12] Let f be an injective function from (F, A) to (G, B) . Then the inverse relation f^{-1} is called the **inverse function**.

Definition 2.27 [14] A soft continuous mapping at (x_e, E) between the two soft topological spaces (X, τ_1, E) and (Y, τ_2, E) is defined as $f: (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$, is such that for each soft neighbourhood (H, E) of $f((x_e, E))$ there exist a soft neighbourhood (F, E) of (x_e, E) Whenever $f((F, E)) \subset (H, E)$.

If f is soft continuous for all (x_e, E) , then f is called soft continuous mapping.

Definition 2.28 [14] A function $f: (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is said to be soft continuous if $f^{-1}((G, E))$ is soft open in (X, τ_1, E) for each soft open set (G, E) of (Y, τ_2, E) .

Definition 2.29 [14] Let (X, τ_1, E_1) and (Y, τ_2, E_2) be two soft topological spaces, and $f: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ be a mapping.

- a) If the image of $f((F, E_1))$ of each soft-open set (F, E_1) over (X, E_1) is a soft-open set in (Y, E_2) , then f is said to be a **soft-open mapping**.
- b) If the image of $f((H, E_1))$ of each soft closed set (H, E_1) over (X, E_1) is soft-closed set in (Y, E_2) , then f is said to be a **soft-closed mapping**.

Definition 2.30 [10] A mapping $f: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ is said to be **soft-g*s-closed mapping** if for each soft closed set (A, E_1) of (X, E_1) , $f((A, E_1))$ is soft -g*s-closed set in $(Y,$

E_2) and is called **soft-g*s-open mapping** if for each soft open set (B, E_1) of (X, E_1) , $f((B, E_1))$ is soft-g*s-open set in (Y, E_2) .

Definition 2.31 [10] Let (X, τ, E) be a soft-topological space, (F, E) and (G, E) be soft-closed sets in X such that $(F, E) \cap (G, E) = \phi$. If there exist soft-open sets (A, E) and (B, E) such that $(F, E) \subseteq (A, E)$, $(G, E) \subseteq (B, E)$ and $(A, E) \cap (B, E) = \phi$ then X is called a **soft-normal space**.

Definition 2.32 [10] A function $f: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ is said to be **soft-g-closed mapping**, if for every soft closed set (A, E_1) in (X, E_1) its image $f((A, E_1))$ is soft-g-closed in (Y, E_2) .

Definition 2.33 [10] A function $f: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ is said to be **soft-gs-closed mapping**, if for every soft closed set (A, E_1) in (X, E_1) its image $f((A, E_1))$ is soft-gs-closed in (Y, E_2) .

Definition 2.34 [10] A function $f: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ is said to be **soft-g α -closed mapping**, if for every soft-closed set (A, E_1) in (X, E_1) its image $f((A, E_1))$ is soft g α -closed in (Y, E_2) .

Soft g#s-closed mappings and soft g#s-open mappings

Definition 3.1 A mapping $f: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ is said to be **soft-g#s – closed mapping** if for each soft closed set (A, E_1) of x , $f((A, E_1))$ is soft-g#s – closed set in (Y, E_2) and is called **soft-g#s – open mapping** if for each soft open set (B, E_1) of (X, E_1) $f((B, E_1))$ is soft-g#s- open in (Y, E_2) .

Theorem 3.2 Every soft-closed mapping is soft-g#s - closed mapping.

Proof: Let $f: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ be a soft-closed mapping. Let (A, E_1) be a soft-closed set in (X, E_1) . Then $f((A, E_1))$ is soft-closed in (Y, E_2) . Since every soft-closed set is soft-g#s – closed, $f((A, E_1))$ is soft-g#s – closed. Hence f is soft-g#s – closed mapping.

Theorem 3.3 Every soft-g#s – closed mapping is soft- g-closed mapping.

Proof : Let $f: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ be a soft g#s- closed mapping and let (V, E_1) be a soft-closed set in (X, E_1) . Then $f((V, E_1))$ is soft-g#s– closed in (Y, E_2) . Since every soft g# s – closed set is soft-g closed set, $f((V, E_1))$ is soft-g-closed. Hence f is soft-g– closed mapping.

Theorem 3.4 Every soft g# s – closed mapping is soft gs closed mapping.

Proof : Let $f: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ be a soft-g#s- closed mapping and let (V, E_1) be a soft closed set in (X, E_1) . Then $f((V, E_1))$ is soft-g#s–closed in (Y, E_2) . Since every soft g# s – closed set is soft gs closed set, $f((V, E_1))$ is soft-gs-closed. Hence f is soft-gs–closed mapping.

Theorem 3.5 Every soft g# s – closed mapping is soft g α closed mapping.

Proof : Let $f: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ be a soft-g#s- closed mapping and let (V, E_1) be a soft closed set in (X, E_1) . Then $f((V, E_1))$ is soft-g#s–closed in (Y, E_2) . Since every soft-g#s– closed set is soft-g α -closed set, $f((V, E_1))$ is soft-g α -closed. Hence f is soft-g α – closed mapping.

Theorem 3.6 Every soft-g#-closed mapping is soft sg- closed mapping.

Proof : Let $f : (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ be a soft-g#s-closed mapping and let (V, E_1) be a soft closed set in X . Then $f((V, E_1))$ is soft-g#s-closed in (Y, E_2) . Since every soft-g#s-closed set is soft-sg-closed set, $f((V, E_1))$ is soft-sg-closed. Hence f is soft-sg-closed mapping.

The converse of the above theorems may not be closed

Example 3.7 Let $X = \{ x_1, x_2, x_3 \} = Y$ and $E_1 = \{ e_1, e_2, e_3 \} = E_2$. Then $\tau_1 = \{ \phi, X, (F, E_1) \}$ is a soft topology over (X, E_1) , $\tau_2 = \{ \phi, Y, (G, E_2) \}$ is a soft topology over (Y, E_2) . Now the soft sets over (X, E_1) and (Y, E_2) are defined as follows :

$$\begin{aligned} F_1(e_1) &= \{x_1\}; F_1(e_2) = \{x_1\} \\ F_2(e_1) &= \{x_2\}; F_2(e_2) = \{x_2\} \\ F_3(e_1) &= \{x_1, x_2\}; F_3(e_2) = \{x_1, x_2\} \end{aligned}$$

$$\begin{aligned} G_1(e_1) &= \{x_1\}; G_1(e_2) = \{x_1\} \\ G_2(e_1) &= \{x_1, x_2\}; G_2(e_2) = \{x_1, x_2\} \\ G_3(e_1) &= \{x_2, x_3\}; G_3(e_2) = \{x_2, x_3\}. \end{aligned}$$

If the mapping $f: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ is defined as $f(x_1) = x_1$; $f(x_2) = x_3$; $f(x_3) = x_2$. Considering the soft set $A_1(e_1) = \{x_3\}$; $A_1(e_2) = \{x_3\}$, it is clear that f is soft-g#s-closed mapping but not soft closed mapping.

Example 3.8 From the above example, considering the soft set $A_1(e_1)=\{x_1\}$; $A_1(e_2)=\{x_1\}$, it is clear that f is soft g-closed mapping but not soft-g#s- closed mapping.

Example 3.9 Let $X = \{ x_1, x_2, x_3 \} = Y$ and $E_1 = \{ e_1, e_2, e_3 \} = E_2$. Then $\tau_1 = \{ \phi, X, (F, E_1) \}$ is a soft topology over (X, E_1) , $\tau_2 = \{ \phi, Y, (G, E_2) \}$ is a soft topology over (Y, E_2) . Now the soft sets over (X, E_1) and (Y, E_2) are defined as follows :

$$\begin{aligned} F_1(e_1) &= \{x_2\}; F_1(e_2) = \{x_1\} \\ F_2(e_1) &= \{x_2\}; F_2(e_2) = \{x_2\} \\ F_3(e_1) &= \{x_2, x_3\}; F_3(e_2) = \{x_2, x_3\} \end{aligned}$$

$$\begin{aligned} G_1(e_1) &= \{x_1\}; G_1(e_2) = \{x_1\} \\ G_2(e_1) &= \{x_2, x_3\}; G_2(e_2) = \{x_2, x_3\} \\ G_3(e_1) &= \{x_3\}; G_3(e_2) = \{x_3\}. \end{aligned}$$

If the mapping $f: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ is defined as $f(x_1) = x_2$; $f(x_2) = x_3$; $f(x_3) = x_2$. Considering the soft set $H_1(e_1) = \{x_1\}$; $H_1(e_2) = \{x_1\}$ and the soft-set $B_1(e_1) = \{x_1, x_3\}$; $B_1(e_2) = \{x_1, x_3\}$ it is clear that f is soft $g\alpha$ closed, g_s closed and sg closed mapping but not soft-g#s-closed mapping.

Theorem 3.10 If $f : (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ be a soft closed mapping and $g : (Y, \tau_2, E_2) \rightarrow (Z, \tau_3, E_3)$ be a soft-g#s-closed mapping then $g \circ f : (X, \tau_1, E_1) \rightarrow (Z, \tau_3, E_3)$ is a soft-g#s-closed mapping.

Proof : Let (A, E_1) be a soft closed set in (X, E_1) . Then $f((A, E_1))$ is soft-closed in (Y, E_2) , since f is soft closed mapping. Now $g(f((A, E_1)))$ is a soft-g#s-closed set in (Z, E_3) , since g is soft-g#s-closed mapping. Hence $g \circ f : (X, \tau_1, E_1) \rightarrow (Z, \tau_3, E_3)$ is a soft-g#s-closed mapping.

Example 3.11 Let $X = Z = \{ x_1, x_2, x_3 \}$, $Y = \{ x_1, x_2, x_3, x_4 \}$ and $E_1 = \{ e_1, e_2 \} = E_2 = E_3$. Then $\tau_1 = \{ \phi, X, (F, E_1) \}$ is a soft topology over (X, E_1) and $\tau_2 = \{ \phi, Y, (G, E_2) \}$ is a soft topology over (Y, E_2) and $\tau_3 = \{ \phi, Z, (H_1, E_3), (H_2, E_3) \}$ be soft topology over (Z, E_3) . Here (F, E_1) is a soft set over (X, E_1) , (G, E_2) is a soft-set over (Y, E_2) . (H_1, E_3) and (H_2, E_3) are soft sets over (Z, E_3) defined as follows :

$$F(e_1) = \{ x_1 \}, F(e_2) = \{ x_1 \}, G(e_1) = \{ x_1, x_3 \}, G(e_2) = \{ x_1, x_3 \}, H_1(e_1) = \{ x_3 \}, H_1(e_2) = \{ x_3 \}, H_2(e_1) = \{ x_1, x_2 \} \text{ and } H_2(e_2) = \{ x_1, x_3 \}.$$

If the mapping $I: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ and $f: (Y, \tau_2, E_2) \rightarrow (Z, \tau_3, E_3)$ defined as $f(x_1) = x_1$, $f(x_2) = f(x_4) = x_2$ and $f(x_3) = x_3$. It is clear that each of I and f is of soft g#s closed mapping but $f \circ I$ is not soft g#s closed, since $(f \circ I)^{-1}(H_1, E_3) = \{ \{ x_3 \}, \{ x_3 \} \}$ is not a soft closed set over X .

Theorem 3.12 A mapping $f: (X, \tau_1, E_1) \rightarrow (Y, \tau_2, E_2)$ is soft g#s – closed mapping if and only if for each soft-subset (F, E_2) of (Y, E_2) and for each soft-open set (G, E_1) containing $f^{-1}((F, E_2))$ there is a soft-g#s-open set (H, E_2) of (Y, E_2) such that $(F, E_2) \subseteq (H, E_2)$ and $f^{-1}((H, E_2)) \subseteq (G, E_1)$.

Proof: Suppose f is a soft-g#s-closed mapping and let (F, E_2) be a soft subset of (Y, E_2) and (G, E_1) be soft-open set of (X, E_1) such that $f^{-1}(F, E_2) \subseteq (G, E_1)$.

Then $(H, E_2) = Y - f(X - (G, E_1))$ is a soft-g#s open set containing (F, E_2) such that $f^{-1}((H, E_2)) \subseteq (G, E_1)$. Conversely, suppose that (F, E_2) is soft-closed in X . Then $f^{-1}(Y - f((F, E_2))) \subseteq X - (F, E_2)$, $X - (F, E_2)$ is soft open in X . By hypothesis, there is a soft g#s open set (V, E_2) of Y such that $Y - f((F, E_2)) \subseteq (V, E_2)$ and $f^{-1}((V, E_2)) \subseteq X - (F, E_2) \Rightarrow (F, E_2) \subseteq X - f^{-1}((V, E_2)) \Rightarrow Y - (V, E_2) \subseteq f((F, E_2)) \subseteq f(X - f^{-1}((V, E_2))) \subseteq Y - (V, E_2) \Rightarrow f((F, E_2)) = Y - (V, E_2)$. Since $Y - (V, E_2)$ is a soft g#s closed set in Y , $f((F, E_2))$ is soft g#s closed set in Y .

Theorem 3.13 If a bijective mapping $f: (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is a soft g#s closed and $(H, E) = f^{-1}((K, E))$ for some soft closed set (K, E) of Y , then $f: (H, E) \rightarrow (Y, \tau_2, E)$ is soft g#s closed.

Proof: Let (F, E) be a soft closed set in (H, E) Then there exists a soft closed set (G, E) in X such that $(F, E) = (H, E) \cap (G, E) \Rightarrow f((F, E)) = f((H, E) \cap (G, E)) = f((G, E)) \cap (K, E)$.

Since f is soft g#s closed, $f((G, E))$ is soft g#s closed in Y . Then $f((G, E)) \cap (K, E)$ is soft g#s closed $\Rightarrow f((F, E))$ is soft g#s closed.

Theorem 3.14 If $f: (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is soft continuous, soft g#s closed map from a soft normal space X onto a soft space Y , then Y is soft normal.

Proof: Let (A, E) and (B, E) be two disjoint soft closed sets of Y . Then $f^{-1}((A, E))$, $f^{-1}((B, E))$ are disjoint soft closed sets of X . Since X is soft normal, there are disjoint soft open sets (U, E) and (V, E) of X such that $f^{-1}((A, E)) \subseteq (U, E)$ and $f^{-1}((B, E)) \subseteq (V, E)$.

By hypothesis, there exists soft g#s closed sets (C, E) and (D, E) in Y such that $(A, E) \subseteq (C, E)$ and $(B, E) \subseteq (D, E)$ and $f^{-1}((C, E)) \subseteq (U, E)$, $f^{-1}((D, E)) \subseteq (V, E)$. Since (U, E) and (V, E) are disjoint, $\text{int}((C, E))$ and $\text{int}((D, E))$ are also soft open sets. Since (C, E) is soft g#s open, (A, E) is soft closed, $(A, E) \subseteq (C, E) \subseteq \text{int}(C, E)$ also $(B, E) \subseteq \text{int}((D, E))$. Hence Y is soft normal.

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