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On Fuzzy β^*-Continuity in Some Fuzzy Separation Axioms

R. Thangappan[1], K. Perarasan[2]

[1]Assistant Professor of Mathematics, Rajah Serfoji Government College (Autonomous), (Affiliated to Bharathidasan University, Trichirappalli-24) Thanjavur, Tamilnadu, India.
[2]Assistant Professor of Mathematics, Annai Vailankanni Arts and Science College, (Affiliated to Bharathidasan University, Trichirappalli-24) Thanjavur, Tamilnadu, India
Email: [1]thangappan.tnj@gmail.com, [2] rinomathz502@yahoo.com

***Abstract**: In this paper, we have freshly developed a concept of fuzzy β^* −Continuous function in fuzzy separation axioms, we has analysed some basic concepts of fuzzy topological space. Then we studied fuzzy β^* −open hence the author developed some theorems and properties using these open set. Further, we have introduced some theorems and for β^* − Continuity.*

***Keywords:** β −Continuity, β^* −open, fuzzy function.*
Mathematical subject Classification (2020): 22A10, 03E72.

1. INTRODUCTION:

The Method of fuzzy sets has imported Zedeh L.A [9]. Chang C.L [3] was popularized fuzzy topological space. Fuzzy continuous function and its properties has introduced by Guang-Quan, Z [4]. Mubarki, A [7] Introduced β^* − open set and β^* − Continuity in topological space. In all respect of this paper (X,τ), (Y,σ) (or simply X,Y) represent non-empty fuzzy topological spaces. Let μ be a fuzzy subset of a space X A function $f: X \to Y$ is called fuzzy δ −pre continuous [6] (resp. fuzzy δ − semi continuous [2] if $f^{-1}(\lambda)$ is fuzzy δ − pre open (resp. fuzzy δ − semi open) in X for every Fuzzy open set λ of Y. Thangappan R has introduced the concept of fuzzy β^* −open sets and fuzzy β^* −closed sets [8], further we study the relation between fuzzy β^* −open sets with various types of fuzzy open sets. We also import the new concepts of fuzzy β^* −continuous mappings and investigated their nature with separation axioms. Hence we explore some properties and theorems are matured.

2. PRELIMINARIES

Definition 2.01

A fuzzy set μ of (X,τ) is called as follows [1,4,5,6,9],

i. Fuzzy semi open if $\mu \le cl(int(\mu))$ [4].
ii. Fuzzy pre-open if $\mu \le Int(cl(\mu))(cl(int(\mu)) \le \mu)$ [5].
iii. Fuzzy α −open if $\mu \le int(cl(int(\mu))$ [6].
iv. Fuzzy β −open if $\mu \le cl(int(cl(\mu)))$ [1].

v. Fuzzy $\beta^* -$ Open set if $\mu \leq cl\left(int\left(cl(\mu)\right)\right) \vee int\left(cl_\delta(\mu)\right)$ ($\beta^* -$ Closed set if $\mu \geq int(cl(int(\mu))) \wedge cl(int_\delta(\mu))$[8].

Definition 2.02

A mapping $f:(X,\tau) \to (Y,\sigma)$ is said be

a. Fuzzy continuous [4] if $f^{-1}(\lambda)$ is fuzzy open in (X,τ) for each fuzzy open set λ in (Y,σ).
b. Fuzzy pre- continuous [2] if $f^{-1}(\lambda)$ is fuzzy open in (X,τ) for each fuzzy open set λ in (Y,σ).
c. Fuzzy $\alpha -$ continuous [5] if $f^{-1}(\lambda)$ is fuzzy open in (X,τ) for each fuzzy open set λ in (Y,σ).
d. Fuzzy $\beta -$ continuous [6] if $f^{-1}(\lambda)$ is fuzzy open in (X,τ) for each fuzzy open set λ in (Y,σ).

3. FUZZY β^* – CONTINUOUS FUNCTION IN SOME SEPARATION AXIOMS.

Definition 3.01

Let $f:(X,\varphi) \to (Y,\psi)$ be a continuous function and β^* be a non empty fuzzy open subset of (X,φ) then f is said to be Fuzzy $\beta^* -$ continuous if $f^{-1}(\lambda)$ is fuzzy $\beta^* -$ open in (X,φ) for each fuzzy open set λ in (Y,ψ).

Theorem 3.02

For mapping $f:(X,\varphi) \to (Y,\psi)$, the following statements are equivalent.

i. f Is fuzzy $\beta^* -$ continuous.
ii. For every fuzzy singleton $x_q \in X$ and every fuzzy open set v in Y such that $f(x_q) \leq v$, there exist fuzzy β^* –open set $u \leq X$ such that $x_q \leq u$ and $f(u) \leq v$.
iii. $f^{-1}(\lambda) = cl(int(clf^{-1}(\lambda))) \vee int(cl_\delta f^{-1}(\lambda))$ For each λ in Y.
iv. The inverse image of each fuzzy closed set in Y is $\beta^* -$ closed.
v. $f^{-1}cl(v) \geq int(cl(intf^{-1}(v))) \wedge cl(int_\delta f^{-1}(v))$ for each fuzzy set $v \leq Y$.
vi. $clf(\mu) \geq f(int\left(cl\left(int(\mu)\right)\right) \wedge cl(int_\delta(\mu)))$ for each fuzzy set $u \leq X$.

Proof:

$(i) \Rightarrow (ii)$: Let x_q be a singleton set in X and every fuzzy open set v in Y such that $f(x_q) \leq v$. since f is fuzzy $\beta^* -$ continuous. Then $x_q \in f^{-1}f(x_q) \leq f^{-1}(v)$. let $u = f^{-1}(v)$ is a fuzzy $\beta^* -$ open in X, so we have $x_q \leq u$. now $f(u) = f(f^{-1}(v)) \leq v$.

$(ii) \Rightarrow (iii)$: Let λ be any fuzzy open set in Y. Let x_q be any fuzzy point in X such that $f(x_q) \leq \lambda$. then $x_q \in f^{-1}(\lambda)$. since(ii), there exist a fuzzy $\beta^* -$ open $u \leq X$ such that $x_q \leq u$ and $f(u) \leq \lambda$. therefore, $x_q \in u \leq f^{-1}(\lambda) = cl\left(int\left(clf^{-1}(\lambda)\right)\right) \vee int\left(cl_\delta f^{-1}(\lambda)\right)$.

$(iii) \Rightarrow (iv)$: Let λ be any closed set in Y. Then $1-\lambda$ be any fuzzy open set in Y. from (iii),

$$f^{-1}(1-\lambda) \leq cl\left(int\left(clf^{-1}(1-\lambda)\right)\right) \vee int\left(cl_\delta f^{-1}(1-\lambda)\right)$$

$$\Rightarrow 1 - f^{-1}(\lambda) \leq cl\left(int\left(cl(1-f^{-1}(\lambda))\right)\right) \vee int\left(cl_\delta(1-f^{-1}(\lambda))\right)$$

$$\leq cl\left(int\left(1-clf^{-1}(\lambda)\right)\right) \vee int\left(1-cl_\delta f^{-1}(\lambda)\right)$$

$$= 1 - cl\left(int\left(clf^{-1}(\lambda)\right)\right) \vee 1 - int\left(cl_\delta f^{-1}(\lambda)\right)$$

And hence,

$$1 - f^{-1}(\lambda) \geq 1 - \left(int\left(cl\left(int f^{-1}(\lambda)\right)\right) \wedge cl\left(int_\delta f^{-1}(\lambda)\right)\right).$$

$$f^{-1}(\lambda) \geq int(cl(int\, f^{-1}(\lambda))) \wedge cl(int_\delta\, f^{-1}(\lambda))$$

$\Rightarrow f^{-1}(\lambda)$ Also fuzzy $\beta^* -$ closed set in X.

$(\boldsymbol{iv}) \Rightarrow (\boldsymbol{v})$: Let $v \leq Y$. Then $f^{-1}(cl(v))$ is fuzzy $\beta^* -$ closed set in X. $int(cl(int\, f^{-1}(v))) \wedge cl(int_\delta\, f^{-1}(v)) \leq int(cl(int\, f^{-1}(cl(v)))) \wedge cl\left(int_\delta\, f^{-1}(cl(v))\right) \leq f^{-1}(cl(v))$.

$(\boldsymbol{v}) \Rightarrow (\boldsymbol{iv})$: Let $u \leq X$. Put $v = f(u)$ in (v). Then $int(cl(int\, f^{-1}(f(u)))) \wedge cl(int_\delta\, f^{-1}(f(u))) \leq f^{-1}(clf(u))$, this implies $int(cl(int(\mu))) \wedge cl(int_\delta(\mu)) \leq f^{-1}(clf(u))$,

$f\left(int\left(cl\left(int(\mu)\right)\right) \wedge cl\left(int_\delta(\mu)\right)\right) \leq cl(f(u))$.

$(\boldsymbol{iv}) \Rightarrow (\boldsymbol{i})$: Let $v \leq Y$ be fuzzy open set. Put $u = I_Y - v$ and $u = f^{-1}(v)$ then $f(int(cl(int\, f^{-1}(f(v)))) \wedge cl\left(int_\delta\, f^{-1}(f(v))\right)) \leq cl(f(f^{-1}(v)) \leq cl(v) = v$. that is $f^{-1}(v)$ is fuzzy $\beta^* -$ closed in X, so f is fuzzy $\beta^* -$ continuous.

Definition 3.03

A mapping $f: (X, \varphi) \to (Y, \psi)$ is called fuzzy $\beta^* -$irresolute if $f^{-1}(\lambda)$ is fuzzy $\beta^* -$open in (X, φ) for every fuzzy $\beta^* -$open λ in (Y, ψ).

Theorem 3.04

Let (X, φ), (Y, ψ) and (Z, η) be a fuzzy topological space.

$(\boldsymbol{i})$. If $f: (X, \varphi) \to (Y, \psi)$ fuzzy $\beta^* -$ continuous and $g: (Y, \psi) \to (Z, \eta)$ fuzzy continuous, Then $g \circ f: (X, \varphi) \to (Z, \eta)$ is fuzzy $\beta^* -$ continuous.

$(\boldsymbol{i})$ If $f: (X, \varphi) \to (Y, \psi)$ fuzzy $\beta^* -$ irresolute and $g: (Y, \psi) \to (Z, \eta)$ is fuzzy $\beta^* -$ continuous, Then $g \circ f: (X, \varphi) \to (Z, \eta)$ is fuzzy $\beta^* -$ continuous.

Proof: Obvious.

Definition 3.05

A fuzzy topological space (X, τ) is called fuzzy $\beta^* - T_1$ if for each pair of distinct points p and q of X, there exists fuzzy $\beta^* -$open sets $\mathfrak{U}_1$ and $\mathfrak{U}_2$ such that $p \in \mathfrak{U}_1$ and $\in \mathfrak{U}_2$, where $q \notin \mathfrak{U}_1$ and $p \notin \mathfrak{U}_2$.

Theorem 3.06

If $f: (X, \varphi) \to (Y, \psi)$ is fuzzy $\beta^* -$ continuous injective function and Y is fuzzy T_1 Then X is fuzzy $\beta^* - T_1$.

Proof:

Assume, suppose that Y is fuzzy T_1. For any two distinct points p and q of X, there exists fuzzy open sets $\mathfrak{U}_1$ and $\mathfrak{U}_2$ in Y such that $f(p) \in \mathfrak{U}_1$, $f(q) \in \mathfrak{U}_2$, where $f(q) \notin \mathfrak{U}_1$, $f(p) \notin \mathfrak{U}_2$,since f is injective $\beta^* -$ continuous function. We have $f^{-1}(\mathfrak{U}_1)$ and $f^{-1}(\mathfrak{U}_2)$ are fuzzy $\beta^* -$open in X. So, from the definition X is fuzzy $\beta^* - T_1$.

Definition 3.07

A fuzzy topological space (X, τ) is called fuzzy $\beta^* -$Hausdorff (fuzzy $\beta^* - T_2$) if for each pair of distinct points p and q of X, there exists fuzzy $\beta^* -$open sets $\mathfrak{U}_1$ and $\mathfrak{U}_2$ such that $p \in \mathfrak{U}_1$ and $q \in \mathfrak{U}_2$.

Theorem 3.08

If $f: (X, \varphi) \to (Y, \psi)$ is fuzzy $\beta^* -$ continuous injective function and Y is fuzzy T_2

Then X is fuzzy $\beta^* - T_2$.
Proof: Obvious using definition 3.08.
Definition 3.09

A fuzzy topological space (X,τ) is said to be fuzzy β^* −normal space if every two disjoint fuzzy closed sets C and D of X. there exists disjoint fuzzy β^* −open sets $\mathfrak{U}_1$ and $\mathfrak{U}_2$ such that $C \leq \mathfrak{U}_1$ & $D \leq \mathfrak{U}_2$, $\mathfrak{U}_1 \wedge \mathfrak{U}_2 = 0$.
Theorem 3.10

If $f:(X,\varphi) \to (Y,\psi)$ is fuzzy β^* − continuous closed injective function and Y is fuzzy normal then X is fuzzy β^* −normal .
Proof:

Suppose that Y is fuzzy normal. Let C and D be closed fuzzy sets in X such that $C \wedge D = 0$. Since f is fuzzy closed injection $f(C)$ and $f(D)$ are fuzzy closed in Y and $f(C) \wedge f(D) = 0$. since Y is normal ,there exists fuzz y open sets $\mathfrak{U}_1$ and $\mathfrak{U}_2$ in Y such that $f(C) \leq \mathfrak{U}_1, f(D) \leq \mathfrak{U}_2$ and $\mathfrak{U}_1 \wedge \mathfrak{U}_2 = 0$. So we obtain, $\leq f^{-1}(\mathfrak{U}_1)$, $D \leq f^{-1}(\mathfrak{U}_2)$ and $f^{-1}(\mathfrak{U}_1 \wedge \mathfrak{U}_2) = 0$. Since f is fuzzy β^* − continuous, $f^{-1}(\mathfrak{U}_1)$ and $f^{-1}(\mathfrak{U}_2)$ are fuzzy β^* −open sets. Hence from the definition X is fuzzy β^* −normal.
Definition 3.11

A fuzzy topological space (X,τ) is said to be fuzzy β^* −regular space if for each closed set T of X and each $p \in X - T$, there exists disjoint fuzzy β^* −open sets $\mathfrak{U}_1$ and $\mathfrak{U}_2$ such that $p \in \mathfrak{U}_1$ and $Y \leq \mathfrak{U}_2$.
Theorem 3.12

If $f:(X,\varphi) \to (Y,\psi)$ is fuzzy β^* − continuous closed injective function and Y is fuzzy regular then X is fuzzy β^* −regular .
Proof:

Let T be a fuzzy closed set in Y with $q \notin T$. Take $q = f(p)$. Since Y is fuzzy regular, there exists disjoint fuzzy open sets $\mathfrak{U}_1$ and $\mathfrak{U}_2$ such that $p \in \mathfrak{U}_1$ and $q = f(p) \in f(\mathfrak{U}_1)$ and $T \leq f(\mathfrak{U}_2)$: $f(\mathfrak{U}_1)$ and $f(\mathfrak{U}_2)$ are disjoint fuzzy open sets . So we obtain $f^{-1}(T) \leq \mathfrak{U}_2$ since f is fuzzy β^* − continuous, $f^{-1}(T)$ is fuzzy β^* −closed set in X and $p \notin f^{-1}(T)$. Hence from the definition, X is fuzzy β^* − regular.

4. CONCLUSION

In this paper, we newly developed β^* −continuous function from the concepts of β^* −open and β^* −closed sets. Hence developed fuzzy β^* − space, fuzzy β^* − space, fuzzy β^* − normal and fuzzy β^* −regular space, further we given new definition and theorems.

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