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Further Results on Even Vertex Odd Mean Labeling of Graphs

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***Abstract:** A graph with p vertices and q edges is said to have an even vertex odd mean labeling if there exists an injective function $f: V(G) \to \{0, 2, 4, \dots, 2q-2, 2q\}$ such that the induced map $f^*: E(G) \to \{1, 3, 5, \dots, 2q-1\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex odd mean labeling is called an even vertex odd mean graph. Here we investigate the even vertex odd mean behaviour of some standard graphs.*

***Keywords.** even vertex odd mean labeling, even vertex odd mean graph.*

***2010 Mathematics Subject Classification Number.** 05C78.*

1. INTRODUCTION

Through out this paper, by a graph we mean a finite undirected simple graph. Let $G(V, E)$ be a graph with p vertices and q edges. For notation and terminology, we follow [4].
Let G be a graph and v be any vertex of G. If v' is a new duplicating vertex corresponding to the vertex v in G, then we may obtain a new graph H, whose vertex is $V(G) \cup \{v'\}$ and edge set is $E(G) \cup \{(u, v'): u \in N(v)\}$. $[P_m, C_n]$ is a graph obtained from a path P_m by attaching a cycle C_n at each vertex of P_m. (P_m, Q_3) is a graph obtained from a path P_m by attaching a cube Q_3 by an edge at each vertex of P_m. $T_p^{(n)}$ is a graph obtained from n copies of path on p vertices by joining a vertex in the i^{th} copy with the corresponding vertex in the $(i+1)^{th}$ copy, $1 \le i \le n-1$.
The graceful labeling of graphs was first introduced by Rosa in 1967 [1] and Gnanajothi introduced odd graceful graphs [3]. The concept of mean labeling was introduced and meanness of some standard graphs was studied by Somasundaram and Ponraj [6, 7, 9, 10]. Further, some more results on mean graphs are discussed in [8, 11, 12]. A graph G is said to be a mean graph if there exists an injective function $f: V(G) \to \{0,1,2, \dots, q\}$ such that the induced map f^* from $E(G)$ to $\{1,2,3, \dots, q\}$ defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection.
In [5], Manickam and Marudai introduced odd mean labeling of a graph. A graph G is said to be odd mean if there exists an injective function f from $V(G)$ to $\{0,1,2,3, \dots, 2q-1\}$ such that the induced map f^* from $E(G)$ to $\{1,3,5, \dots, 2q-1\}$ defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$

is a bijection. The concept of even mean labeling was introduced and studied by Gayathri and Gobi [2]. A function f is called an even mean labeling of a graph G with p vertices and q edges if f is an injection from the vertices of G to the set $\{2,4,6,\dots,2q\}$ such that when each edge uv is assigned the label $\frac{f(u)+f(v)}{2}$, then the resulting edge labels are distinct. A graph which admits an even mean labeling is said to be even mean graph.
In [13], Vasuki et al., introduced even vertex odd mean labeling of graphs. A graph G is said to have an even vertex odd mean labeling if there exists an injective function $f: V(G) \rightarrow \{0,2,4,\dots,2q-2,2q\}$ such that the induced map $f^*: E(G) \rightarrow \{1,3,5,\dots,2q-1\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex and odd mean labeling is called an even vertex odd mean graph. The graph shown in Figure 1, is an odd mean graph but not an even vertex odd mean graph. Every star graph is an even mean graph but $K_{1,n} (n \geq 3)$ is not an even vertex odd mean graph. These examples show that the notion of even vertex odd mean graph is independent of mean graph, odd mean graph and even mean graph.

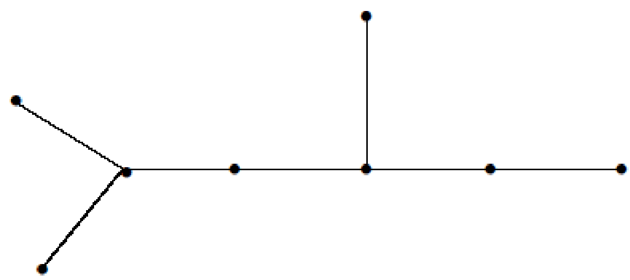

Figure 1. An odd mean graph but not an even vertex odd mean graph.

In this paper, we study the even vertex odd meanness of the graphs $C_p \circ K_1, [P_m: C_n], (P_m: Q_3), T_p^{(n)}, H_n \circ mK_1$, one step ladder and graphs obtained by duplicating a vertex of path, cycle and star.

Main Results

Proposition 2.1. *For each even integer* $p \geq 4, C_p \odot K_1$ *is an even vertex odd mean graph.*

Proof. In $C_p \odot K_1$, let $v_1, v_2, \dots, v_p$ be the vertices on the cycle and let u_i be the pendant vertex of v_i at each $i, 1 \leq i \leq p$.

Case (1). $p = 4m$, for $m \geq 1$.

Define $f: V(C_p \odot K_1) \to \{0,2,4,\dots,16m\}$ is defined as follows:

$$f(v_i) = \begin{cases} 4i-4, & 1 \leq i \leq 2m-1 \text{ and } i \text{ is odd} \\ 4i, & 2m+1 \leq i \leq 4m-1 \text{ and } i \text{ is odd} \\ 4i-2, & 2 \leq i \leq 4m \text{ and } i \text{ is even} \end{cases} \quad \text{and}$$

$$f(u_i) = \begin{cases} 4i-2, & 1 \leq i \leq 4m-1 \text{ and } i \text{ is odd} \\ 4i-4, & 2 \leq i \leq 2m \text{ and } i \text{ is even} \\ 4i, & 2m+2 \leq i \leq 4m \text{ and } i \text{ is even}. \end{cases}$$

Then the induced edge labeling is obtained as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} 4i-1, & 1 \leq i \leq 2m-1 \\ 4i+1, & 2m \leq i \leq 4m-1, \end{cases}$$

$f^*(v_{4m} v_1) = 8m-1$ and

$$f^*(u_i v_i) = \begin{cases} 4i-3, & 1 \leq i \leq 2m \\ 4i-1, & 2m+1 \leq i \leq 4m. \end{cases}$$

Thus f is an even vertex odd mean labeling of $C_p \odot K_1$.

Hence $C_p \odot K_1$ is an even vertex odd mean graph when $p = 4m$.

Case (2). $p = 4m+2$, for $m \geq 1$.

Define $f: V(C_p \odot K_1) \to \{0,1,2,\dots,16m+8\}$ as follows:

$$f(v_i) = \begin{cases} 4i-4, & 1 \leq i \leq 2m+1 \text{ and } i \text{ is odd} \\ 4i, & 2m+3 \leq i \leq 4m+1 \text{ and } i \text{ is odd} \\ 4i-2, & 2 \leq i \leq 2m \text{ and } i \text{ is even} \\ 4i+2, & i = 2m+2 \\ 4i-2, & 2m+4 \leq i \leq 4m+2 \text{ and } i \text{ is even} \end{cases} \quad \text{and}$$

$$f(u_i) = \begin{cases} 4i-2, & 1 \leq i \leq 2m+1 \text{ and } i \text{ is odd} \\ 4i-6, & i = 2m+3 \\ 4i-2, & 2m+5 \leq i \leq 4m+1 \text{ and } i \text{ is odd} \\ 4i-4, & 2 \leq i \leq 2m+2 \text{ and } i \text{ is even} \\ 4i, & 2m+4 \leq i \leq 4m+2 \text{ and } i \text{ is even}. \end{cases}$$

Then the induced edge labeling is obtained as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} 4i-1, & 1 \leq i \leq 2m \\ 4i+1, & i = 2m+1 \\ 4i+3, & i = 2m+2 \\ 4i+1, & 2m+3 \leq i \leq 4m+1, \end{cases}$$

$f^*(v_{4m+2} v_1) = 8m+3$ and

$$f^*(u_i v_i) = \begin{cases} 4i-3, & 1 \leq i \leq 2m+1 \\ 4i-1, & i = 2m+2 \\ 4i-3, & i = 2m+3 \\ 4i-1, & 2m+4 \leq i \leq 4m+2. \end{cases}$$

Thus f is an even vertex odd mean labeling of $C_p \odot K_1$. Hence $C_p \odot K_1$ is an even vertex odd mean graph.

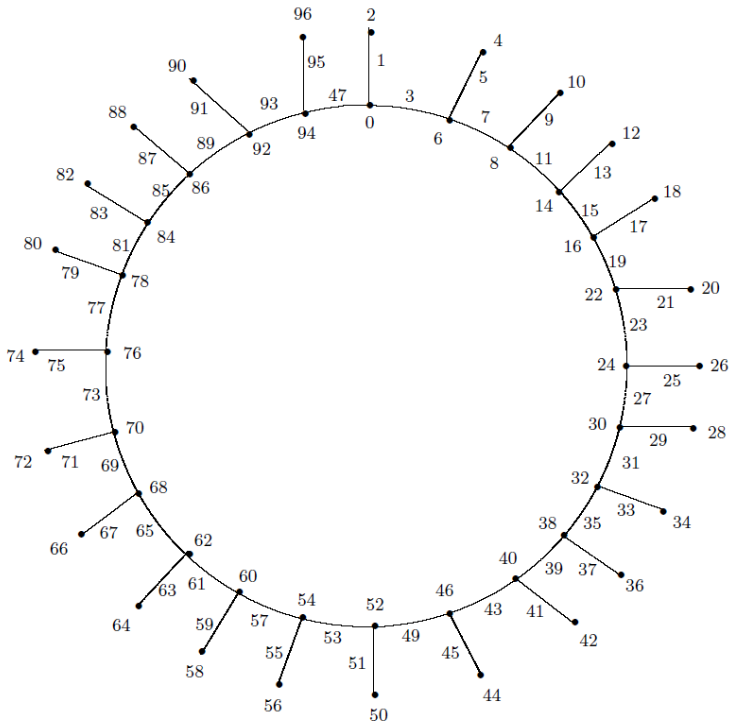

Figure 2. An even vertex odd mean labeling of $C_{24} \odot K_1$.

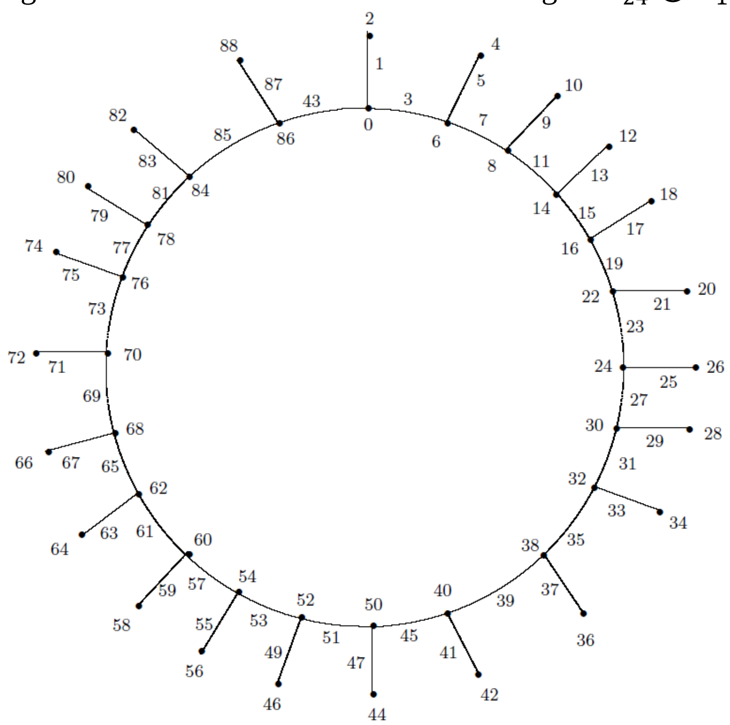

Figure 3. An even vertex odd mean labeling of $C_{22} \odot K_1$.

Proposition 2.2. $[P_m: C_n]$ *is an even vertex odd mean graph for* $n \equiv 0(mod\ \ 4)$ *and any* $m \geq 2$.

Proof. In $[P_m: C_n]$, let $v_1, v_2, \dots, v_m$ be the vertices on the path $P_m, v_{i,1}, v_{i,2}, \dots, v_{i,n}$ be the vertices of the i^{th} cycle C_n, for $1 \leq i \leq m$ and each vertex $v_{i,1}$ of the i^{th} cycle C_n is identified with the vertex v_i of the path $P_m, 1 \leq i \leq m$.
Suppose $n = 4t, t \geq 1$.
The labeling $f: V[P_m: C_n] \to \{0,2,4, \dots, 2m(n+2)-2\}$ is defined as follows:
For $1 \leq i \leq m$,

$$f(v_{i,j}) = \begin{cases} 2(n+1)(i-1)+2j-2, & 1 \leq\ j \leq\ 2t, i\ and\ j\ are\ odd \\ 2(n+1)(i-1)+2j+2, & 2t+1 \leq\ j \leq\ 4t, i\ and\ j\ are\ odd \\ 2(n+1)(i-1)+2j-2, & 1 \leq\ j \leq\ 4t, i\ is\ odd\ and\ j\ is\ even \\ 2(n+1)i-2j, & 1 \leq\ j \leq\ 2t, i\ is\ even\ and\ j\ is\ odd \\ 2(n+1)i-2j-4, & 2t+1 \leq\ j \leq\ 4t, i\ is\ even\ and\ j\ is\ odd \\ 2(n+1)i-2j, & 1 \leq\ j \leq\ 4t, i\ is\ even\ and\ j\ is\ even. \end{cases}$$

Then the induced edge labeling is obtained as follow:
For $1 \leq i \leq m$,

$$f^*(v_{i,j}v_{i,j+1}) = \begin{cases} 2(n+1)(i-1)+2j-1, & 1 \leq\ j \leq\ 2t-1\ and\ i\ is\ odd \\ 2(n+1)(i-1)+2j+1, & 2t \leq\ j \leq\ 4t-1\ and\ i\ is\ odd \\ 2(n+1)(i-1)+9, & j = 1\ and\ i\ is\ even \\ 2(n+1)(i-1)+2j-3, & 2 \leq\ j \leq\ 2t+1\ and\ i\ is\ even \\ 2(n+1)(i-1)+2j-1, & 2t+2 \leq\ j \leq\ 4t-1\ and\ i\ is\ even \end{cases} \quad \text{and}$$

$$f^*(v_{i,4t}v_{i,1}) = \begin{cases} 2(n+1)(i-1)+4t-1, & i\ is\ odd \\ 2(n+1)(i-1)+8t-1, & i\ is\ even. \end{cases}$$

Thus f is an even vertex odd mean labeling of $[P_m: C_n]$.
Hence $[P_m: C_n]$ is an even vertex odd mean labeling.

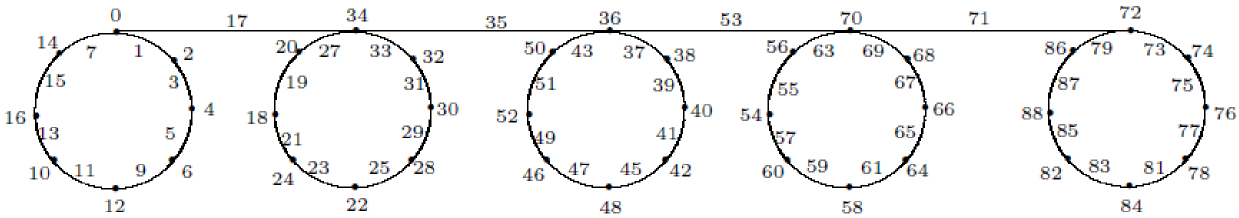

Figure 4. An even vertex odd mean labeling of $[P_5: C_8]$.

Proposition 2.3. $(P_m: Q_3)$ *is an even vertex odd mean graph for any positive integer* $m \geq 1$.

Proof. Let $v_{i,j}$, $1 \leq j \leq 8$ be the vertices in the i^{th} copy of Q_3, $1 \leq i \leq m$ and $u_1, u_2, \dots, u_m$ be the vertices of the path $P_m, \{u_i u_{i+1}: 1 \leq i \leq n-1\} \cup \{u_i v_{i,1}: 1 \leq i \leq m\} \cup \{v_{i,1}v_{i,2}, v_{i,1}v_{i,4}, v_{i,1}v_{i,6}, v_{i,2}v_{i,3}, v_{i,2}v_{i,7}, v_{i,3}v_{i,4}, v_{i,3}v_{i,8}, v_{i,4}v_{i,5}, v_{i,5}v_{i,6}, v_{i,5}v_{i,8}, v_{i,6}v_{i,7}, v_{i,7}v_{i,8}: 1 \leq i \leq m\}$ be the edge set of $(P_m: Q_3)$.
The labeling $f: V((P_m: Q_3)) \to \{0,2,4, \dots, 28m-2\}$ is defined as follows:

For $1 \leq i \leq m$, $f(u_i) = \begin{cases} 28(i-1), & i\ is\ odd \\ 28i-2, & i\ is\ even. \end{cases}$

For $1 \leq i \leq m$ and i is odd, $f(v_{i,j}) = \begin{cases} 28i - 26, & j = 1 \\ 28i - 24 + 2j, & 2 \leq j \leq 3 \\ 28i - 24, & j = 4 \\ 28i - 10, & j = 5 \\ 28i - 16 + 2j, & 6 \leq j \leq 7 \\ 28i - 8, & j = 8. \end{cases}$

For $1 \leq i \leq m$ and i is even, $f(v_{i,j}) = \begin{cases} 28i - 4, & j = 1 \\ 28i - 2j - 6, & 2 \leq j \leq 3 \\ 28i - 6, & j = 4 \\ 28i - 20, & j = 5 \\ 28i - 2j - 14, & 6 \leq j \leq 7 \\ 28i - 22, & j = 8. \end{cases}$

Then the induced edge label of $(P_m : Q_3)$ is obtained as follows:
For $1 \leq i \leq m-1$, $f^*(u_i u_{i+1}) = 28i - 1$.
For $1 \leq i \leq m$, $f^*(u_i v_{i,1}) = \begin{cases} 28i - 27, & i \text{ is odd} \\ 28i - 3, & i \text{ is even}. \end{cases}$

For $1 \leq i \leq m$ and i is odd	For $1 \leq i \leq m$ and i is even
$f^*(v_{i,1}v_{i,2}) = 28i - 23$	$f^*(v_{i,1}v_{i,2}) = 28i - 7$
$f^*(v_{i,1}v_{i,4}) = 28i - 25$	$f^*(v_{i,1}v_{i,4}) = 28i - 5$
$f^*(v_{i,1}v_{i,6}) = 28i - 15$	$f^*(v_{i,1}v_{i,6}) = 28i - 15$
$f^*(v_{i,2}v_{i,3}) = 28i - 19$	$f^*(v_{i,2}v_{i,3}) = 28i - 11$
$f^*(v_{i,2}v_{i,7}) = 28i - 11$	$f^*(v_{i,2}v_{i,7}) = 28i - 19$
$f^*(v_{i,3}v_{i,4}) = 28i - 21$	$f^*(v_{i,3}v_{i,4}) = 28i - 9$
$f^*(v_{i,3}v_{i,8}) = 28i - 13$	$f^*(v_{i,3}v_{i,8}) = 28i - 17$
$f^*(v_{i,4}v_{i,5}) = 28i - 17$	$f^*(v_{i,4}v_{i,5}) = 28i - 13$
$f^*(v_{i,5}v_{i,6}) = 28i - 7$	$f^*(v_{i,5}v_{i,6}) = 28i - 23$
$f^*(v_{i,5}v_{i,8}) = 28i - 9$	$f^*(v_{i,5}v_{i,8}) = 28i - 21$
$f^*(v_{i,6}v_{i,7}) = 28i - 3$	$f^*(v_{i,6}v_{i,7}) = 28i - 27$
$f^*(v_{i,7}v_{i,8}) = 28i - 5$	$f^*(v_{i,7}v_{i,8}) = 28i - 25.$

Thus f is an even vertex odd mean labeling of $(P_m : Q_3)$. Hence $(P_m : Q_3)$ is an even vertex odd mean graph.

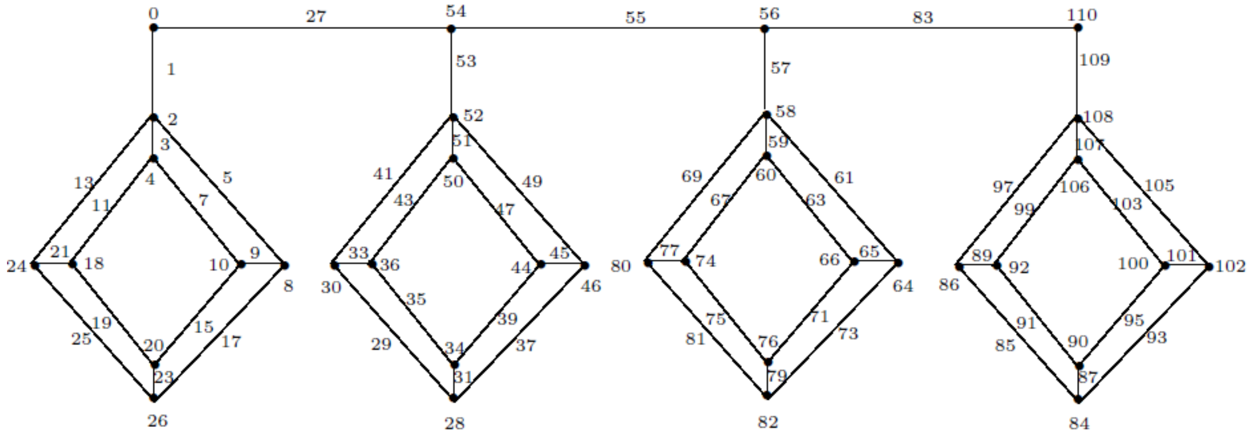

Figure 5. An even vertex odd mean labeling of $(P_4 : Q_3)$.

Proposition 2.4. *For all positive integers p and n, the graph $T_p^{(n)}$ is an even vertex odd mean graph.*

Proof. Let $v_i^{(j)}, 1 \le i \le p$ be the vertices of the j^{th} copy of the path on p vertices, $1 \le j \le n$. The graph $T_p^{(n)}$ is formed by adding an edge $v_i^{(j)} v_i^{(j+1)}$ between j^{th} and $(j+1)^{th}$ copy of the path at some $i, 1 \le i \le p$.

The labeling $f: V(T_p^{(n)}) \to \{0,2,4, \dots, 2np-2\}$ is defined as follows:

For $1 \le i \le p$ and $1 \le j \le n$ and $f(v_i^{(j)}) = \begin{cases} 2p(j-1) + 2i - 2, & j \text{ is odd} \\ 2pj - 2i, & j \text{ is even.} \end{cases}$

Then the induced edge labeling is obtained as follows:

For $1 \le j \le n$ and $1 \le i \le p-1$,

$f^*(v_i^{(j)} v_{i+1}^{(j)}) = \begin{cases} 2p(j-1) + 2i - 1, & j \text{ is odd} \\ 2pj - 2i - 1, & j \text{ is even} \end{cases}$ and

$f^*(v_i^{(j)} v_i^{(j+1)}) = 2pj - 1.$

Thus f is an even vertex odd mean labeling of the graph $T_p^{(n)}$. Hence $T_p^{(n)}$ is even vertex odd mean graph.

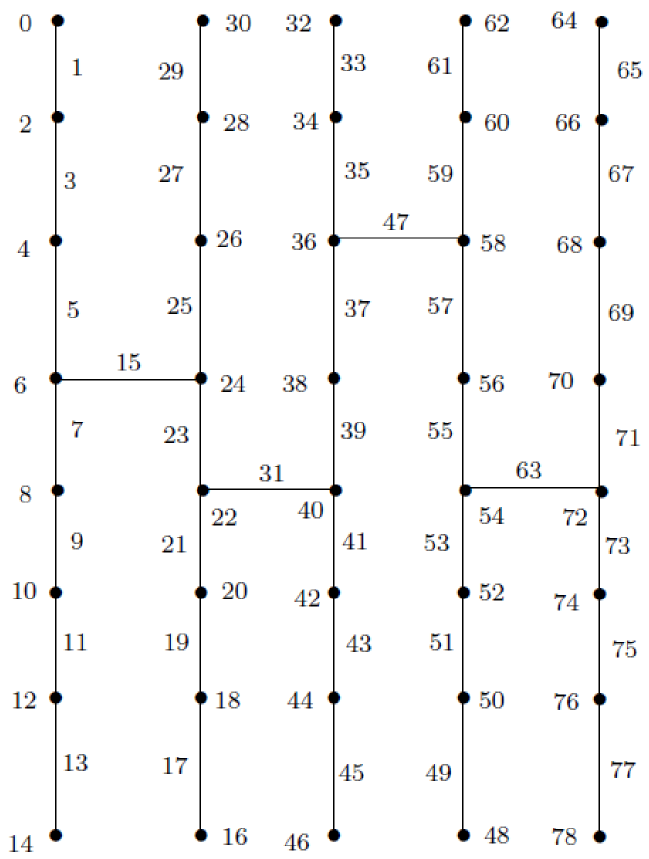

Figure 6. An even vertex odd mean labeling of $T_8^{(5)}$.

Proposition 2.5. *The graph $H_n \odot mK_1$ is an even vertex odd mean graph for all positive*

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integers m and n.

Proof. Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ be the vertices on the paths of length $n-1$. Let $x_{i,k}$ and $y_{i,k}$, $1 \le k \le m$, be the pendant vertices at u_i and v_i respectively for $1 \le i \le n$. Define $f: V(H_n \odot mK_1) \to \{0,2,4,\dots,4n(m+1)-2\}$ as follows:

For $1 \le i \le n$, $f(u_i) = \begin{cases} 2i(m+1)-2m, & i \text{ is odd} \\ 2i(m+1)-4, & i \text{ is even} \end{cases}$ and

$$f(v_i) = \begin{cases} 2f(u_i)+2n(m+1)+2m-4, & i \text{ is odd and } n \text{ is odd} \\ 2f(u_i)+2n(m+1)-2m+4, & i \text{ is even and } n \text{ is odd} \\ 2f(u_i)+2n(m+1), & n \text{ is even.} \end{cases}$$

For $1 \le i \le n$ and $1 \le k \le m$,

$$f(x_{i,k}) = \begin{cases} 2(m+1)(i-1)+4k-4, & i \text{ is odd} \\ 2(m+1)(i-2)+4k+2, & i \text{ is even} \end{cases} \text{ and}$$

$$f(y_{i,k}) = \begin{cases} 2f(x_{i,k})+2n(m+1)-2m+4, & i \text{ is odd and } n \text{ is odd} \\ 2f(x_{i,k})+2n(m+1)+2m-4, & i \text{ is even and } n \text{ is odd} \\ 2f(x_{i,k})+2n(m+1), & n \text{ is even.} \end{cases}$$

Then the induced edge labels are obtained as follows:

For $1 \le i \le n-1$,

$f^*(u_iu_{i+1}) = 2i(m+1)-1$ and

$f^*(v_iv_{i+1}) = f^*(u_iu_{i+1}) + 2n(m+1)$.

For $1 \le i \le n$ and $1 \le k \le m$,

$f^*(u_ix_{i,k}) = 2(m+1)(i-1)+2k-1$ and

$f^*(v_iy_{i,k}) = f^*(u_ix_{i,k}) + 2n(m+1)$.

When n is odd, $f^*\left(u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}\right) = 2n(m+1)-1$.

When n is even, $f^*\left(u_{\frac{n}{2}+1}v_{\frac{n}{2}}\right) = 2n(m+1)-1$.

Thus f is an even vertex odd mean labeling of $H_n \odot mK_1$. Hence $H_n \odot mK_1$ is an even vertex odd mean graph for all positive integers m and n.

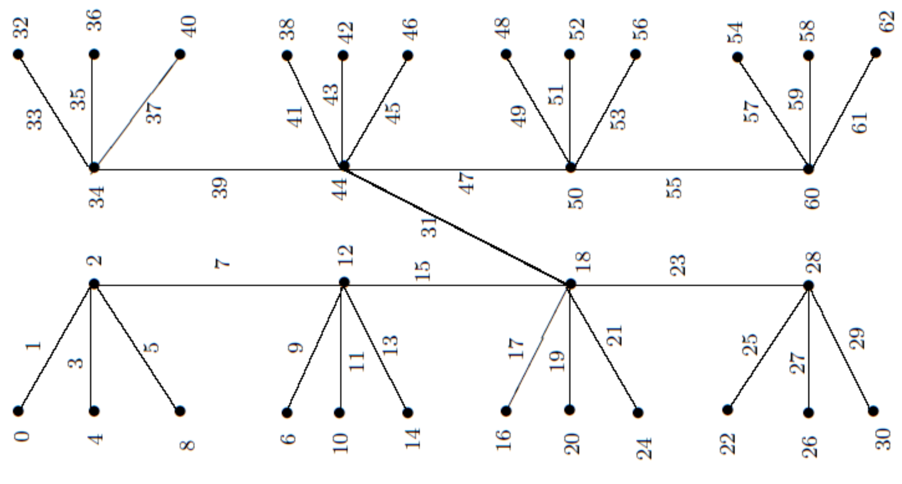

Figure 7. An even vertex odd mean labeling of $H_4 \odot 3K_1$.

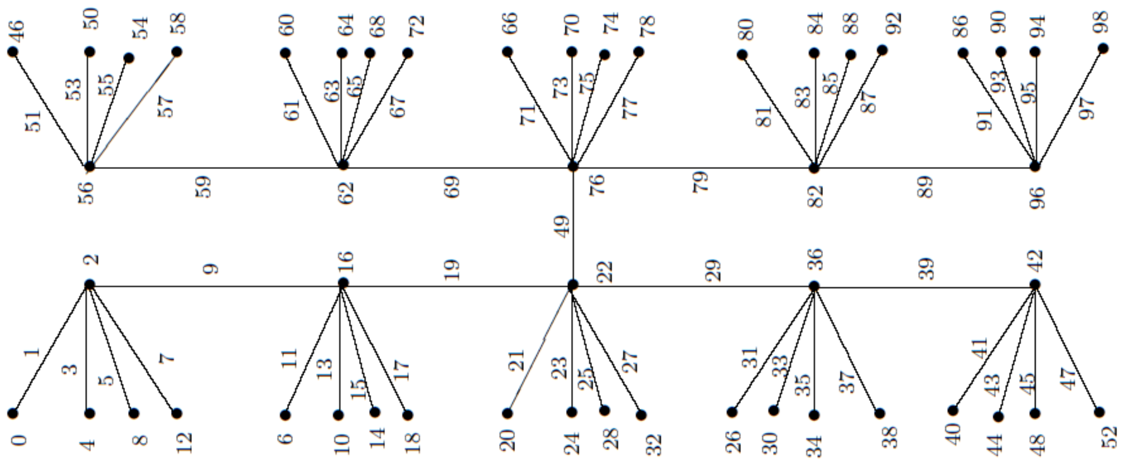

Figure 8. An even vertex odd mean labeling of $H_5 \odot 4K_1$.

Proposition 2.6. *Let G be the one sided step ladder. Then G is an even vertex odd mean graph.*

Proof. Let $v_{i,1}v_{i,2}, v_{i,3}, \dots, v_{i,(i+1)}$ be the vertices in the i^{th} row of the n step on sided step ladder $G, 1 \le i \le n$ and $v_{(n+1),1}, v_{(n+1),2}, \dots, v_{(n+1),(n+1)}$ be the vertices in the last row. The sum of vertices and edges in G are $\frac{n^2+5n+2}{2}$ and n^2+3n.
Define $f: V(G) \to \{0,2,4, \dots, 2(n^2+3n)\}$ as follows:

$$f(v_{i,j}) = \begin{cases} 2(i^2-1)+2(j-1) & 1 \le i \le n, \quad 1 \le j \le i+1 \\ 2(i^2-1)+2(j-1) & i = n+1, \quad 1 \le j \le n+1 \end{cases}$$

The induced edge labeling f^* is obtained as follows:
$f^*(v_{i,j}v_{i,(j+1)}) = 2(i^2-1)+2j-1, 1 \le i \le n, 1 \le j \le i$
$f^*(v_{(n+1),j}v_{(n+1),(j+1)}) = 2(n^2+2n)+2j-1, 1 \le j \le n$ and
$f^*(v_{i,j}v_{(i+1),j}) = 2i^2+2(i+j)-3, 1 \le i \le n, 1 \le j \le i+1.$
Thus f is an even vertex odd mean labeling of G.

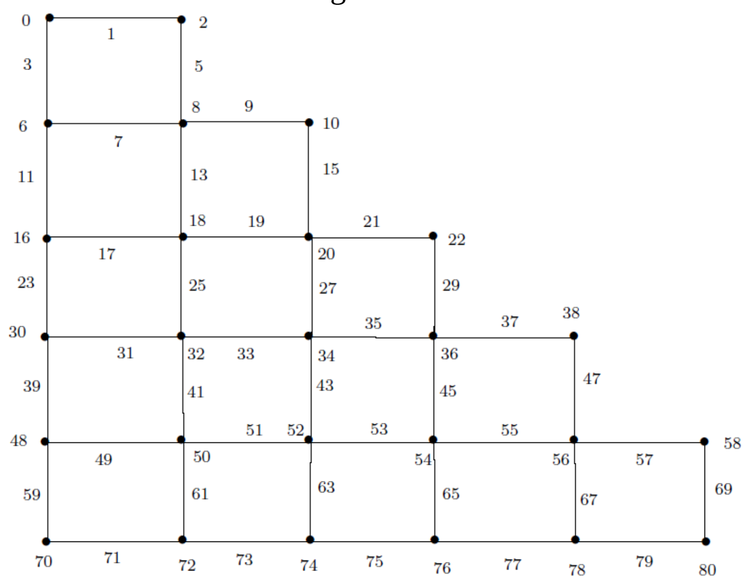

Figure 9. An even vertex odd mean labeling of one sided step ladder with 5 steps.

Proposition 2.7. *Let G be the graph obtained from a path P_n by duplicating a vertex v. Then G is an even vertex odd mean graph if and only if v is not a pendant vertex while n is odd.*

Proof. Let $v_1, v_2, v_3, \dots, v_n$ be the vertices of the path of length of length $n-1$. Let v_j' be the duplicating the vertex of v_j.

Case (i). $2 \leq j \leq n-1$.

Define $f: V(G) \to \{0,2,4,\dots,2n\}$ as follows:

$$f(v_i) = \begin{cases} 2i-2, & 1 \leq i \leq j-1 \\ 2i+2, & j \leq i \leq n \end{cases} \text{ and}$$

$f(v_j') = 2j-2$.

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} 2i-1, & 1 \leq i \leq j-2 \\ 2i+1, & i = j-1 \\ 2i+3, & j \leq i \leq n, \end{cases}$$

$f^*(v_{j-1} v_j') = 2j-3$ and

$f^*(v_j' v_{j+1}) = 2j+1$.

Thus f is an even vertex odd mean labeling of G.

Case (ii). $i = 1$ or $i = n$.

When n is odd, at least one edge having the labels of its vertices with same suit. Thus even vertex odd mean labeling does not exist while n is odd.

Let n be even.

Define $f: V(G) \to \{0,2,4,\dots,2n\}$ as follows:

$$f(v_i) = \begin{cases} 2i-2, & i = 1 \\ 2i+2, & 2 \leq i \leq n \text{ and } i \text{ is odd} \\ 2i-2, & 2 \leq i \leq n \text{ and } i \text{ is even} \end{cases} \text{ and}$$

$f(v_1') = 4$.

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} 1, & i = 1 \\ 2i+1, & 2 \leq i \leq n-1 \end{cases} \text{ and}$$

$f^*(v_1' v_2) = 3$.

Thus f is an even vertex and odd mean labeling of G.

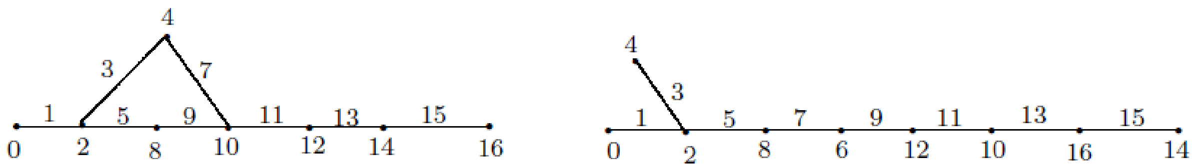

Figure 10. An even vertex odd mean labeling of duplicating of a vertex of a path.

Proposition 2.8. *Let G be the graph obtained from a cycle C_n by duplicating a vertex v. Then G is an even vertex odd mean graph if and only if n is even.*

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices on the cycle of length n in G. Let v_1' be the duplicating vertex of v_1 in G.

Case (i). $n \equiv 2 (mod\ 4)$.

Define $f: V(G) \to \{0,2,4,\dots,2n+4\}$ as follows: $f(v_i) =$

$$\begin{cases} 2(n+i-1), & 1 \le i \le 2 \\ 2i-6, & 3 \le i \le \frac{n}{2}+3 \\ 2i-2, & \frac{n}{2}+4 \le i \le n,\ i\ is\ odd\ and\ \frac{n}{2}+3 \equiv 0(mod\ 4) \\ 2i-6, & \frac{n}{2}+4 \le i \le n,\ i\ is\ even\ and\ \frac{n}{2}+3 \equiv 0(mod\ 4) \\ 2i-4, & \frac{n}{2}+4 \le i \le n,\ i\ is\ odd\ and\ \frac{n}{2}+3 \equiv 2(mod\ 4) \\ 2i, & \frac{n}{2}+4 \le i \le n,\ i\ is\ even\ and\ \frac{n}{2}+3 \equiv 2(mod\ 4) \end{cases}$$ and

$f(v_1') = 2n+4.$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} 2n+1, & i=1 \\ n+1, & i=2 \\ 2i-5, & 3 \le i \le \frac{n}{2}+2 \\ 2i-3, & \frac{n}{2}+3 \le i \le n-1, \end{cases}$$

$f^*(v_n v_1) = 2n-3,$

$f^*(v_1' v_2) = 2n+3$ and

$f^*(v_1' v_n) = 2n-1.$

Thus f is even vertex odd mean labeling of G.

Case (ii). $n \equiv 0(mod\ 4).$

Define $f: V(G) \to \{0,2,4,\dots,2n+4\}$ as follows:

$$f(v_i) = \begin{cases} 2n-4, & i=1 \\ 2n+2, & i=2 \\ 2i-6, & 3 \le i \le \frac{n}{2}+3 \\ 2i-2, & \frac{n}{2}+4 \le i \le n\ and\ i\ is\ even \\ 2i-6, & \frac{n}{2}+4 \le i \le n\ and\ i\ is\ odd \end{cases}$$ and

$f(v_1') = 2n+4.$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} 2n-1, & i=1 \\ n+1, & i=2 \\ 2i-5, & 3 \le i \le \frac{n}{2}+2 \\ 2i-3, & \frac{n}{2}+3 \le i \le n-1, \end{cases}$$

$f^*(v_n v_1) = 2n-3,$

$f^*(v_1' v_2) = 2n+3$ and

$f^*(v_1' v_n) = 2n+1.$

Thus f is an even vertex odd mean labeling of G.

Case (iii). n is odd.

For every vertex assignment, at least one edge is having the vertex label of same suit and hence the corresponding edge label is even. Hence no even vertex odd mean labeling exists when n is odd.

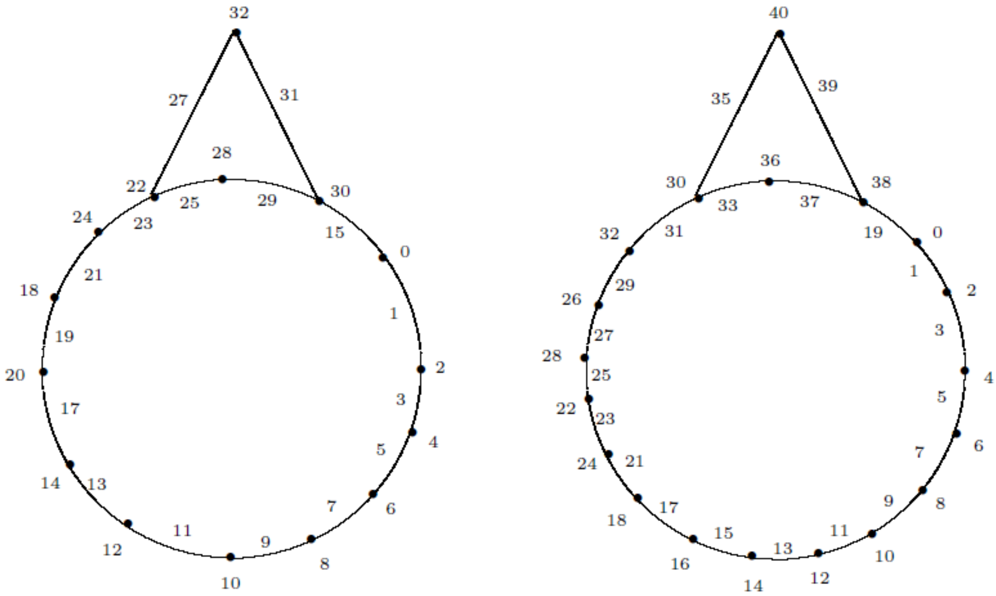

Figure 11. An even vertex odd mean labeling of duplicating a vertex of cycle.

Proposition 2.9. *Let G be a graph obtained from the star graph $K_{1,n}$ ($n \geq 2$) by duplicating a vertex. Then G is an even vertex odd mean graph of G if and only if central vertex of $K_{1,n}$ is duplicated.*

Proof. Let v_0 be the central vertex and $v_1, v_2, \ldots, v_n$ be the pendent vertices of $K_{1,n}$.
Case (i). v' is the duplicating vertex of v_j, for some j, $1 \leq j \leq n$.
In this case G is a $K_{1,n+1}$ graph. In order to get the edge label 1, the vertex label 0 and 2 are adjacent. So either 0 is the label of the central vertex and 2 is the label of the pendant vertex and vice versa. If 0 is the central vertex, then definitely 4 will be the label of a pendant vertex while $n \geq 2$ and 2 is obtained as an edge label. If 2 is the label of the central vertex, then 6 will be the label of the pendant vertex while $n \geq 2$ and 4 is obtained as an edge label. When $n = 1$, G is the path P_3, which admits an even vertex odd mean labeling.
Case (ii). v' is the duplicating vertex of the central vertex.
In this case, G is $K_{2,n}$ so that $\{v_0, v'\}$ is one of the partition.
Define $f: V(G) \rightarrow \{0,2,4,6, \ldots, 4n\}$ as follows:
$f(v_0) = 0, f(v_i) = 4i - 2, 1 \leq i \leq n$ and $f(v') = 4n$.
The induced edge labeling f^* is obtained as follows:
$f^*(v_0 v_i) = 2i - 1, 1 \leq i \leq n$ and
$f^*(v_i v') = 2n + 2i - 1, 1 \leq i \leq n$.
Thus f is an even vertex odd mean labeling of G.

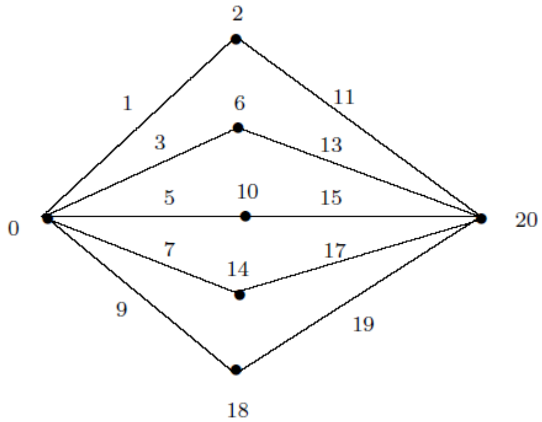

Figure 12. An even vertex odd mean labeling of duplicating the central vertex of star $K_{1,5}$.

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