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The relation among vague filters andResiduated Lattices

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Abstract:
The focus of this paper is to develop The relation between the vague filters and residuated lattices, and its essential properties are investigated. Characterizations of vague filters in residuated lattice are established. We discuss some properties of vague filters in terms of its level subsets. Also the notion of Extended pair of vague filter is introduced and characterize their properties.

1.Introduction:

The notion of residuated lattices is initiated in order to provide a reliable logical foundation for uncertain information processing theory and establish a logical system with truth value in a relatively general lattice. The concept of fuzzy set was introduced by Zadeh (1965) [19]. Since then this idea has been applied to other algebraic structures. Since the fuzzy set is single function, it cannot express the evidence of supporting and opposing. Hence the concept of vague set [6] is introduced in 1993 by W.L.Gau and Buehrer. D.J. In a vague set A, there are two membership functions: a truth membership functiont_A and a false membership function f_A, where t_A and $f_A$are lower bound of the grade of membership respectively andt_A(x) + f_A(x) ≤ 1. Thus the grade of membership in a vague set A is a subinterval [t_A(x), 1-f_A(x)] of [0, 1]. Vague set is an extension of fuzzy sets. The idea of vague sets is that the membership of every elements which can be divided into two aspects including supporting and opposing. With the development of vague set theory, some structure of algebras corresponding to vague set have been studied. R.Biswas [3] initiated the study of vague algebras by studying vague groups.T.Eswarlal [5] study the vague ideals and normal vague ideals in semirings. H.Hkam , etc[13] study the vague relations and its properties. Quotient algebras are basic tool for exploring the structures of algebras. There are close correlations among filters, congruences and quotient algebras.

2.Vague Filters on residuated Lattice

Definition 2.1:

A Vague set A of L is called a vague filter of L, if for any x, y ∈ L:

1. V_A(I) ≥V_A(x)
2. V_A(y) ≥ min(V_A(x → y), V_A(x))

Theorem 2.2:

Let A be a vague filter of L. Then, for any x, y ∈ L if x ≤ y, then V_A(x) ≤V_A(y) .

Proof:

Since x ≤ y, it follows that x → y = I. Since A is a vague filter of L, we have V_A(y) ≥ min(V_A(x → y), V_A(x)) and V_A(I) ≥V_A(x) for any x, y ∈ L.

Therefore V_A(y) ≥ min(V_A(x → y), V_A(x))
= min (V_A(I), V_A(x))≥ min (V_A(x), V_A(x)) = V_A(x).
Therefore V_A(x) ≤V_A(y).

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Theorem 2.3:

Let A be a vague set on L. Then A is vague filter of L, if and only if , for any x, y, z ∈ L V_A(I) ≥ V_A(x) and V_A(x → z) ≥ min(V_A(y → (x → z)) , V_A(y))

Proof:

Let A be vague filter of L, obviously V_A(I) ≥ V_A(x) and V_A(I) ≥ V_A(x) and V_A(x → z) ≥min(V_A(y → (x → z)) , V_A(y)) holds for any x, y, z ∈ L. Taking x = I in V_A(x → z) ≥min(V_A(y → (x → z)) , V_A(y)), we have V_A(z) = V_A(I → z) ≥ min(V_A(y → (I → z)), V_A(y)) = min(V_A(y → z),V_A(y)). Since V_A(I) ≥ V_A(x) holds, and so A is a vague filter of L.

Theorem 2.4:

Let A be a vague set on L. Then A is a vague filter of L, if and only if, for any x, y, z ∈ L, A satisfies if x ≤ y, then V_A(x) ≤ V_A(y)for any x, y ∈ L and V_A(x * y) ≥ min(V_A(x), V_A(y)) .

Proof:

Assume that A is a vague filter of L, obviously if x ≤ y, then V_A(x) ≤ V_A(y)holds for any x,y∈ L. Since x ≤ y → (x * y), we have V_A(y → (x * y)) ≥ V_A(x). By Definition 2.1 (2), it follows that V_A(x * y) ≥ min(V_A(y), V_A(y → (x * y))) ≥ min(V_A(y), V_A(x)).

Conversely, assume that if x ≤ y, then V_A(x) ≤ V_A(y)and V_A(x * y) ≥ min(V_A(x), V_A(y)) holds. Taking y = I, we get V_A(I) ≥ V_A(x) . As x * (x → y) ≤ y, thus V_A(y) ≥ V_A(x * (x → y)). Therefore V_A(y) ≥ min(V_A(x), V_A(x → y)). Hence A is a vague filter of L.

Remark 2.5:

A vague set on L is a vague filter of L, if and only if, for any x, y, z ∈ L :if x → (y → z) = I then V_A(z) ≥ min(V_A(x), V_A(y)).

Remark 2.6:

A vague set on L is a vague filter of L, if and only if, for any x, y, z ∈ L:

if a_n→ (a_{n-1}→ ….→ (a_1→ x) …..) = I, then V_A(x) ≥ min(V_A(a_n), …….,V_A(a_1))

Theorem 2.7:

A vague set on L is a vague filter of L, if and only if, for any x, y, z ∈ L, A satisfies Remark 2.5 and V_A((x → (y → z)) → z) ≥ min(V_A(x), V_A(y)).

Proof:

If A is a vague filter of L then Remark 2.5 holds. Since V_A((x → (y→ z)) → z) → z) ≥ min (V_A((x → (y → z)) → (y → z)), V_A(y)). As (x → (y → z)) → (y → z) = x ∨ (y → z) ≥ x , by Theorem 2.2 we have V_A((x → (y → z)) → (y → z)) ≥ V_A(x). Therefore, V_A((x → (y → z)) → z) ≥ min(V_A(x), V_A(y)).

Conversely, suppose V_A((x → (y → z)) → z) ≥ min(V_A(x), V_A(y)) is valid. Since V_A(y) = V_A(I → y) = V_A(((x → y) → (x → y)) → y) ≥ min(V_A(x → y), V_A(x)). Hence by Definition 2.1, A is s vague filter of L.

Theorem 2.8:

Let A be a vague set on L. Then A is a vague filter of L, for any x, y, z ∈ L, A satisfies Definition 2.1(1) and V_A(x → z) ≥ min(V_A(x → y), V_A(y → z)).

Proof:

Assume that A is vague filter of L. Since (x → y) ≤ (y → z) → (x → z), it follows from Theorem 2.2 thatV_A((y → z) → (x → z)) ≥ V_A(x → y). As A is a vague filter , so V_A(x → z)≥ min(V_A(y → z), V_A((y→ z)→(x → z))). We have V_A(x → z) ≥ min(V_A(y → z), V_A(x → z)).

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Conversely, if V_A(x → z) ≥ min(V_A(x → y), V_A(y → z)) for any x, y, z ∈ L, then V_A(I → z) ≥ min(V_A(I → y), V_A(y → z)) that is V_A(z) ≥ min(V_A(y), V_A(y → z)). Hence by definition 2.1 A is a vague filter of L.

Theorem 2.9:

Let A be a vague set on L. Then A is a vague filter of L, if and only if, for any α, β∈ [0, 1] and α + β≤ 1, the sets U(t_A, α) (≠φ) and L(1-f_A, β) (≠φ) are filters of L, where U(t_A, α) = {x ∈ L / t_A(x) ≥α}, L(1- f_A(x), β) = {x ∈ L/ 1- f_A(x) ≥β}.

Proof:

Assume A is a vague filter of L, then V_A(I) ≥V_A(x) . By the condition U(t_A, α) ≠φ, it follows that there exist a ∈ L such that t_A(a) ≥α , and so t_A(I) ≥α, hence I ∈ U(t_A, α) Let x, x → y ∈ U(t_A, α), then t_A(x) ≥α, t_A(x → y) ≥α. Since A is a filter of L, then t_A(y) ≥ min(t_A(x), t_A(x → y)) ≥ min (α, α) = α. Hence y ∈ U(t_A, α). Therefore U(t_A, α) is a filter of L. We will show that L(1- f_A(x), β) is a filter of L. Since A is a vague filter of L, then 1- f_A(I) ≥ 1- f_A(x). By the condition L(1- f_A(x), β) ≠φ, it follows that there exist a ∈ L such that 1- f_A(a) ≥β. Therefore we have 1- f_A(I) ≥ 1- f_A(a) ≥β. Hence I∈ L(1- f_A(x), β). Let x, x → y ∈ L(1- f_A(x), β), then 1- f_A(x) ≥β , 1- f_A(x → y) ≥β. Since A is a vague filter of L, then 1- f_A(y) ≥ min (1- f_A(x), 1- f_A(x → y)) ≥ min(β, β) = β. It follows that 1- f_A(y) ≥β, hence y ∈ L(1- f_A(x), β). Therefore L(1- f_A(x), β) is a filter of L. Conversely, suppose that U(t_A, α) ≠φ and L(1- f_A(x), β) ≠φ are filters of L, then, for any x ∈ L, x ∈ U(t_A, t_A(x)) and x ∈ L(1- f_A, 1- f_A(x)).

By U(t_A, t_A(x)) ≠φ and L(1- f_A, 1- f_A(x)) ≠φ are filters of L, it follows that I∈ U(t_A, t_A(x)) and I ∈ L(1- f_A, 1- f_A(x)), and so V_A(I) ≥V_A(x). For any x, y ∈ L, let α = min(t_A(x), t_A(x → y)) and β = min (1- f_A(x), 1- f_A(x → y)), then x, x → y ∈ U(t_A, α) and x, x → y ∈ L(1- f_A, β). And so y ∈ U(t_A, α) and y ∈ L(1- f_A(x), β).

Therefore t_A(y) ≥α = min(t_A(x), t_A(x → y)) and 1- f_A(y) ≥β = min(1- f_A(x), 1- f_A(x → y)). From theorem 3.2, we have A is a vague filter of L.

Theorem 2.10:

Let A, B be two vague filters of L, then A∩B is also a vague filter of L.

Proof:

Let x , y, z ∈ L such that z ≤ x → y, then z → (x → y) = I. Since A, B be two vague filters of L, we have V_A(y) ≥ min (V_A(z), V_A(x)) and V_B(y) ≥ min (V_B(z), V_B(x)). Since $V_{A\cap B}$(y) = min(V_A(y), V_B(y))≥ min(min(V_A(z), V_A(x)), min (V_B(z), V_B(x))) = min(min(V_A(z), V_B(z)), min(V_A(x), V_B(x)) = min($V_{A\cap B}$(z), $V_{A\cap B}$(x)). Since A, B be two vague filters of L, we have V_A(I) ≥V_A(x) and V_B(I) ≥V_B(x). Hence $V_{A\cap B}$(I) = min(V_A(I), V_B (I)) ≥ min(V_A(x), V_B(x)) = $V_{A\cap B}$(x) . Then A ∩ B is a vague filters of L.

Remark 2.11:
Let A_i be a family of vague sets on L, where i is an index set. Denoting C by the intersection of A_i, i.e. $\bigcap_{i\in I} A_i$, where V_C(x) = min(V_{A_1}(x), V_{A_2}(x), ……) for any x ∈ L.

Note 2.12:
Let $A_i$be a family of vague filters of L, where i ∈ I, I is an index set, then $\bigcap_{i\in I} A_i$ is also a vague filters of L.

Theorem 2.13:
Let A be a vague set on L. Then

a. For any α, β∈ [0, 1], if $A_{(\alpha,\beta)}$ is a filter of L. Then, for any x, y, z ∈ L,

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V_A(z) ≤ min(V_A(x → y), V_A(x)) imply V_A(z) ≤V_A(y).

b. If A satisfy Definition 2.1(1) and condition (a), then, for any α, β∈ [0, 1], $A_{(\alpha,\beta)}$ is a filter of L.

Proof:

a. Assume that $A_{(\alpha,\beta)}$ is a filter of L for any α, β∈ [0, 1].
Since V_A(z) ≤ min(V_A(x → y), V_A(x)), it follows that V_A(z) ≤V_A(x → y), V_A(z) ≤V_A(x).Therefore,x → y ∈$A_{(t_A(z),1-f_A(z))}$, x ∈$A_{(t_A(z),1-f_A(z))}$. As V_A(z) ∈ [0, 1], and $A_{(t_A(z),1-f_A(z))}$ is a filter of L, so y ∈$A_{(t_A(z),1-f_A(z))}$. Thus V_A(z) ≤V_A(y).

b. Assume A satisfy (a) and (b). For any x, y ∈ L, α, β∈ [0, 1], we havex → y ∈$A_{(\alpha,\beta)}$, x ∈$A_{(\alpha,\beta)}$,therefore t_A(x → y) ≥α, 1- f_A(x → y) ≥β and t_A(x) ≥α , 1- f_A(x) ≥β , and so min(t_A(x → y) , t_A(x)) ≥ min(α, α) = α. By (a), we have t_A(y) ≥α and 1- f_A(y) ≥β, that is, y ∈$A_{(\alpha,\beta)}$. Since V_A(I) ≥V_A(x) for any x ∈ L, it follows that t_A(I) ≥α and 1- f_A(I) ≥β, that is, I ∈$A_{(\alpha,\beta)}$. Then for any α, β∈ [0, 1], $A_{(\alpha,\beta)}$ is a filter of L.

Theorem 2.14:

Let A be a vague filter of L, then for any α, β∈ [0, 1], $A_{(\alpha,\beta)}$ (`≠φ) is a filter of L.

Proof:

Since $A_{(\alpha,\beta)}$≠φ, there exist α, β∈ [0, 1] such that t_A(x) ≥α , 1- f_A(x) ≥β . And A is a vague filter of L, we have t_A(I) ≥t_A(x) ≥α, 1- f_A(I) ≥ 1- f_A(x) ≥β, therefore I ∈$A_{(\alpha,\beta)}$. Let x, y ∈ L and x ∈$A_{(\alpha,\beta)}$, x → y ∈$A_{(\alpha,\beta)}$, therefore t_A(x) ≥α, 1- f_A(x) ≥β , t_A(x → y) ≥α, 1- f_A(x → y) ≥β. Since A is a vague filter of L, thus t_A(y) ≥ min(t_A(x → y), t_A(x)) ≥α and 1- f_A(y) ≥min(1- f_A(x → y), 1- f_A(x)) ≥β, it follows that y ∈$A_{(\alpha,\beta)}$. Therefore, $A_{(\alpha,\beta)}$ is a filter of L.

Remark 2.15:

From Theorem 2.14, the filter $A_{(\alpha,\beta)}$ is also called a vague – cut filter of L.

Theorem 2.16:

Any filter F of L is a vague –cut filter of some vague filter of L.

Proof:

Consider the vague set A of L:
A = {(x, t_A(x) / x ∈ L}, where If x ∈ F, V_A(x) = α. If x ∉ F, V_A(x) = 0. where α∈ [0, 1]. Since F is a filter of L, we have 1 ∈ F. Therefore V_A(I) =α≥V_A(x). For any x, y ∈L, if y ∈ F, then V_A(y) = α = min(α, α)≥ min (V_A(x → y), V_A(x)). If y ∉ F, then x ∉ F or x → y ∉ F. And so V_A(y) = 0 = min(0, 0) = min(V_A(x → y), V_A(x)). Therefore A is a vague filter of L.

Theorem 2.17:

Let A be a vague filter of L. Then F = {x ∈ L / t_A(x) = t_A(I), 1- f_A(x) = 1- f_A(I)} is a filter of L.

Proof:

Since F = {x ∈ L /t_A(x) = t_A(I), 1- f_A(x) = 1- f_A(I)}, obviously I ∈ F. Let x → y ∈ F, x ∈ F, so V_A(x → y) = V_A(x) = V_A(I). Therefore V_A(y) ≥ min(V_A(x → y), V_A(x)) = V_A(I) and V_A(I)≥V_A(y), then V_A(y) = V_A(I). Thus y ∈ F. It follows that F is a filter of L.

Case 1:(x = 1). We have $V_C \wedge V_{A\cup B}(1)$ = $V_C(1) \wedge {V_A}^{V_B} \,\tilde{\ast}\, {V_B}^{V_A}$ (1) = $V_C(1) \wedge (V_A(1) \vee V_B(1))$ = $(V_C(1) \wedge (V_A(1)) \vee (V_C(1) \wedge (V_B(1))$ = $(V_C \wedge V_A)^{V_C \wedge V_B} \,\tilde{\ast}\, ((V_C \wedge V_B)^{V_C \wedge V_A}(1)$.

Case 2:($x \neq 1$).

$\bigvee_{\substack{p*q \leq x \\ p \neq 1, q \neq 1}} \{V_C(x) \wedge V_A(p) \wedge V_C(x) \wedge V_B(q)\} \vee \{V_C(x) \wedge V_C(1) \wedge V_A{}^{V_B}(1) \wedge V_B(x)\} \vee \{V_C(x) \wedge V_C(1) \wedge V_B{}^{V_A}(1) \wedge V_A(x)\}\} = \bigvee_{\substack{p*q \leq x \\ p \neq 1, q \neq 1}} \{V_C(x) \wedge V_A(p) \wedge V_C(x) \wedge V_B(q)\} \vee \{V_C(1) \wedge (V_A(1) \vee V_B(1))\} \wedge [(V_C(x) \wedge V_B(x)) \vee (V_C(x) \wedge V_A(x))]\} = \bigvee_{\substack{p*q \leq x \\ p \neq 1, q \neq 1}} \{V_C(x) \wedge V_A(p) \wedge V_C(x)\} \wedge V_B(q)\} \vee \{[(V_C(1) \wedge (V_A(1)) \vee (V_C(1) \wedge V_B(1))] \wedge [(V_C(x) \wedge V_B(x)) \vee (V_C(x) \wedge V_A(x))]\} \leq \bigvee_{\substack{p*q \leq x \\ p \neq 1, q \neq 1}} \{(V_C \wedge V_A)^{V_C \wedge V_B}(p \vee x) \wedge (V_C \wedge V_B)^{V_C \wedge V_A}(q \vee x)\} \vee$ $[(V_C \wedge V_A)^{V_C \wedge V_B}(1)$ It holds that $\wedge$ $V_C(x) \wedge (V_C \wedge V_A)(p \vee x) = [(V_C \wedge V_B)^{V_C \wedge V_A}(1) \wedge (V_C \wedge V_B)(q \vee x)]$ $= \bigvee_{p*q \leq x} \{(V_C \wedge V_A)^{V_C \wedge V_B}(p \vee x) \wedge (V_C \wedge V_B)^{V_C \wedge V_A}(q \vee x)\}$.

CONCLUSION:

In this paper, we introduced the concept of vague filters and we discuss some properties of Vague filters in terms of its level subsets. Also by introducing the notion of extended vague filters, it is proved that the set of all vague filters forms a bounded distributive lattice.

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