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A sα- CLOSED SETS IN TOPOLOGICAL SPACES

E.Balaji., K. Senthamiselvi, R.ManoRanjini, Kavitha R & M.Nagarajan

Nehru Institute of Technology

nitbalaji@nehrucolleges.com

Abstract:

In this paper, we introduce sα–closed sets in topological spaces. Properties of these sets are investigated and we introduce six new topological spaces namely, (i,j)-$T^{\sim}$, (i,j)-$T^{\sim s}$,(i,j)-$^{\sim}T$, (i,j)-$^{\sim s}T$, (i,j)-T^{α} , (i,j)-$T^{\alpha\sim}$ spaces as applications. Further, we introduce and study (i,j)-sα–continuous and (i,j)-sα-irresolute maps.

Key Words: (i,j)-sα-closed sets, (i,j)-$T^{\sim}$, (i,j)-$T^{\sim s}$, (i,j)-$^{\sim}T$, (i,j)-$^{\sim s}T$, (i,j)-T^{α} , (i,j)-$T^{\alpha\sim}$ spaces, (i,j)-sα –continuous and (i,j)-sα-irresolute maps.

Introduction:

A triple (X ,τ_1, τ_2) where X is a nonempty set and τ_1 and τ_2 are topologies on X is called a topological space and Kelly initiated the study of such spaces. Levine introduced and studied semi-open sets and generalized closed sets in 1963 and 1970 respectively. S.P. Arya and T. Nourdefined generalized semi-closed sets (briefly gs-closed sets) in 1990 for obtaining some characterizations of s-normal spaces. Njåstadand Abd El-Monsef et. al introduced α-sets (called as α-closed sets) and semi-preopen sets respectively. Semi-preopen sets are also known as β-sets. Maki et.al. introduced generalized α-closed sets (briefly gα-closed sets)and α-generalized closed sets (briefly αg-closed sets)in 1993 and 1994 respectively.

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2. PREREQUISITES

Throughout this paper (X,τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) represent non-empty b topological spaces on which no separation axioms are assumed unless otherwise mentioned. If A is a subset of X with topology τ then cl(A), int(A) and C(A) denote the closure of A, the interior of A and the complement of A in X respectively. We recall the following definitions, which will be used often throughout this paper.

DEFINITION 2.1:

A subset A of a space (X, τ) is called

(1) a preopensetif $A \subseteq int(cl(A))$ and a preclosed set if $cl(int(A)) \subseteq A$.

(2) a semi-open set if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.

(3) an α-open setif $A\subseteq int(cl(int(A)))$ and a α-closed set if $cl(int(cl(A)))\subseteq A$.

(4) a semi-preopen set (=β-open) if $A \subseteq cl(int(cl(A)))$ and a semi-preclosed set (=β-closed) if $int(cl(int(A))) \subseteq A$.

The semi-closure (resp. α-closure) of a subset A of (X, τ) is denoted by scl(A) (resp. αcl(A) and spcl(A))and is the intersection of all semi-closed (resp. α-closed and semi-preclosed) sets containing **A.**

DEFINITION 2.2 :

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A subset A of a space (X, τ) is called

(1) a generalized closed (briefly g-closed) set[2][10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(2) a generalized semi-closed (briefly gs-closed) set[3][3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(3) a generalized semi-preclosed (briefly gsp-closed) set[12][9] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(4) an α-generalized closed (briefly αg-closed) set[8][12] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(5) a generalized α-closed (briefly gα-closed) set[7][13] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).

DEFINITION 2.3: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

(1) ***semi-continuous***[1][**11**] if $f^{-1}(V)$ is semi-open in (X, τ) for every open set V of (Y, σ).

(2) ***pre-continuous***[11][**14**] if $f^{-1}(V)$ is pre-closed in (X, τ) for every closed set V of (Y, σ).

(3) ***α-continuous***[12][**15**] if $f^{-1}(V)$ is α-closed in (X, τ) for every closed set V of (Y, σ).

(4) ***β-continuous***[5][**1**] if $f^{-1}(V)$ is semi-preopen in (X, τ) for every open set V of (Y, σ).

(5) ***g-continuous***[13][**4**] if $f^{-1}(V)$ is g-closed in (X, τ) for every closed set V of (Y, σ).

DEFINITION 2.4:

A topological space (X, τ) is said to be

1.a $T_{1/2}$ spaceif every g-closed set in it is closed.

2.a T_b space if every gs-closed set in it is closed.

3.an ${}_{\alpha}T_b$ space if every αg-closed set in it is closed.

DEFINITION 2.5: **A** subset A of a topological space (X, τ_1, τ_2) is called:

1.(i,j)-g–closed if τ_j–$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i

.2(i,j)-g*–closed if τ_j–$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in τ_i

3.(i,j)-rg–closed if τ_j–$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in τ_i

4. (i,j)-gpr–closed if τ_j–$pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in τ_i

The family of all (i,j)-g–closed sets (resp. (i,j)-g*–closed, (i,j)-rg–closed, (i,j)-gpr–closed) subsets of a topological space (X, τ_1, τ_2) is denoted by D(i, j) (resp. D*(i, j), $D_r(i, j)$, $\xi(i, j)$).

DEFINITION 2.6:

A subset A of a topological space (X, τ_1, τ_2) is called:

1.(i,j)-$T_{1/2}$ spaceif every (i,j)-g-closed sets is τ_j- closed.

2. (i,j)-T_b space if every (i,j)-gs-closed set is τ_j - closed.

3.(i,j)-${}_{\alpha}T_b$ space if every (i,j)-αg-closed set is τ_j - closed**.**

DEFINITION 2.7:

A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

(1) τ_j- *semi-continuous*[1][**11**] if $f^{-1}(V)$ is semi-open in (X, τ_1, τ_2) for every open set V of (Y, σ_1, σ_2).

(2) τ_j- *α-continuous*[12][**15**] if $f^{-1}(V)$ is α-closed in (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2).

(3) τ_j- σ_k – continuous if $f^{-1}(V) \in \tau_j$, for every $V \in \sigma_k$.

(4)(i,j)-*gs-continuous*[14][**7**] if $f^{-1}(V)$ is gs-closed in $(X, \tau_1, \tau_2,)$ for every closed set V of (Y, σ_1, σ_2).

(5) (i,j)-*gsp-continuous*[14][**7**] if $f^{-1}(V)$ is gsp-closed in $(X, \tau_1, \tau_2,)$ for every closed set V of (Y, σ_1, σ_2).

3. sα –closed sets in topological spaces

In this section we introduce the concept of sα–closed sets in topological spaces and discuss the related properties**.**

Definition 3.1: A Subset A of a space (X,τ_i,τ_j) is called a (i,j)-sα-closed set if τ_j-scl(A)$\subseteq$U whenever A$\subseteq$U and U is α-open in τ_i

Remark 3.2: By setting $\tau_i = \tau_j$ in Definition 3.1, a (i,j)-sα-closed set is a sα-closed set.

Theorem 3.3:

1. If A is τ_j – closed subset of (X,τ_i,τ_j) then A is (i,j)-sα-closed.
2. If A is τ_j –semi closed subset of (X,τ_i,τ_j) then A is (i,j)-sα-closed.
3. If A is τ_j –α closed subset of (X,τ_i,τ_j) then A is (i,j)-sα-closed.
4. Every (i,j)-gα-closed set is (i,j)-sα-closed.
5. Every (i,j)-w-closed set is (i,j)-sα-closed.

Proof: Straight forward. Converse of the above need not be true as in the following examples. **Example 3.4:** Let X={a,b,c} , τ_1 = {ϕ,X,{a,b}}, τ_2 = {ϕ,X,{a}} then {b} is (1,2)-sα –closed but not τ_2 –closed.

Example 3.5: Let X={a,b,c} , τ_1 = {ϕ,X,{a,b}}, τ_2 = {ϕ,X,{a}} then {a,c} is (1,2)-sα – closed but not τ_2 –semi closed **.Example 3.6:** Let X={a,b,c} , τ_1 = {ϕ,X,{a},{b,c}}, τ_2 = {ϕ,X,{a},{a,c}} then{a,b} is (1,2)-sα -closed but not τ_2 -α -closed

Example 3.7: Let X={a,b,c} , τ_1 = {ϕ,X,{a}}, τ_2 = {ϕ,X,{a},{b}, {a,b}} then{a} is (1,2)-sα -closed but not (1,2) -gα -closed .

Example 3.8: Let X={a,b,c} , τ_1 = {ϕ,X,{a,b}}, τ_2 = {ϕ,X,{a}} then{b} is (1,2)-sα -closed but not (1,2) -w-closed.

Thus the class of (i,j)-sα -closed sets properly contains the classes of τ_j –closed sets, τ_j -α-closed sets,τ_j –semi-closed sets, (i,j)-gα-closed sets, (i,j)-w-closed sets.

(6) ***gs-continuous***[14][**7**] if $f^{-1}(V)$ is gs-closed in (X, τ) for every closed set V of (Y, σ).
(7) ***αg-continuous***[2][**10**] if $f^{-1}(V)$ is αg-closed in (X, τ) for every closed set V of (Y, σ).
(8) ***gα-continuous***[7][**13**] if $f^{-1}(V)$ is gα-closed in (X, τ) for every closed set V of (Y, σ).
(9) ***gsp-continuous***[16][**9**] if $f^{-1}(V)$ is gsp-closed in (X, τ) for every closed set V of (Y, σ).
(10) ***αg-irresolute***[10][**6**] if $f^{-1}(V)$ is αg-closed in (X, τ) for every αg-closed set V of (Y, σ).
(11) ***pre-semi-open***[15][**5**] if f(U) is semi-open in (Y, σ) for every semi-open set U in (X, τ).

Theorem 3.9: In a topological space (X,τ_i,τ_j), every (i,j)-sα-closed set is :
1.(i,j)-gs-closed and
2.(i,j)-gsp-closed.
Proof: follows from the definitions.
The following examples show that the reverse implications of above proposition are not true.

Example 3.10: Let X={a,b,c}, τ_1 = {ϕ,X,{a},{a,c}}, τ_2 ={ϕ,X,{a},{b,c}} then {b} is (1,2)-gs-closed but not (1,2)-sα -closed**.**

Example 3.11: Let X={a,b,c}, τ_1 ={ϕ, X, {a}}, τ_2 ={ϕ, X,{a},{b,c}} then {b} is (1,2)-gsp-closed but not (1,2) -sα -closed.

So the class of (i,j)- sα-closed sets is properly contained in the classes of (i,j)-gs-closed and (i,j)-gsp-closed sets .

The following examples shows that (i,j)-sα-closedness is independent from (i,j)- αg-closedness, (i,j)-rg-closedness, (i,j)-gp-closedness , (i,j)-gpr-closedness**.**

Example 3.12: Let X={a,b,c}, τ_1 ={ϕ,X,{a}}, τ_2 ={ϕ,X,{a},{b},{a,b}} then the set {a,b} is (1,2)- αg-closed set, (1,2)-rg-closed set, (1,2)-gp-closed set, (1,2)-gpr-closed set but not (1,2)-sα -closed.

Proposition 3.13: If A is (i,j)-sα-closed set such that $A \subseteq B \subseteq \tau_j$-Scl(A) then B is also (i,j)-sα-closed.
Proof: Follows

Proposition 3.14: If A is (i,j)-sα-closed then τ_j-Scl(A) – A contains no non-empty τ_i-α-closed set.
Proof: Let A be an (i,j)-sα-closed set and F be a non-empty τ_i -α-closed subset such that $F \subseteq \tau_j$-Scl(A) – A = τ_j-Scl(A)$\cap A^c$ ∴ $F \subseteq \tau_j$-Scl(A) and $F \subseteq A^c$Since F^c is τ_i -α-open and A is (i,j)-sα -closed we have, τ_j-Scl(A) ⊆ F^c i.e F ⊆ $(\tau_j\text{-Scl}(A))^c$Hence $F \subseteq \tau_j$-Scl(A) ∩ $(\tau_j\text{-Scl}(A))^c$ = ϕ
∴ τ_j-Scl(A) – A contains no non-empty τ_i -α-closed set
Corollary 3.15: If A is (i,j)-sα-closed set in (X, τ_i, τ_j), then A is τ_j –semi-closed iff τ_j-Scl(A) – A is τ_i -α-closed.
Proof:
<u>**Necessity:**</u> If A is τ_j-semi-closed then τ_j-Scl(A)=A i.e τ_j-Scl(A) – A = ϕ and hence τ_j-Scl(A) – A is τ_i -α-closed. [by prop.3.14]
<u>**Sufficiency:**</u> If τ_j-Scl(A)–A is τ_i -α-closed then by proposition 3.14 we have,τ_j-Scl(A) – A = ϕ [since A is (i,j)-sα -closed] ∴ τ_j-Scl(A) = A. Hence A is τ_j – semi-closed.
Proposition 3.16: For each element x of (X, τ_i, τ_j), {x} is τ_i -α-closed (or) $\{x\}^c$ is (i,j)-sα -closed.
Proof: If {x} is not τ_i -α-closed then the only τ_i -α-open set containing X-{x} is XThus X-{x} is (i,j)-sα -closed. i.e $\{x\}^c$ is (i,j)-sα-closed. Hence Proved.
Proposition 3.17: If A is an τ_i -α-open and (i,j)-sα-closed set of (X, τ_i , τ_j) then A is τ_j – semi-closed.Proof: Let A be τ_i -α-open and (i,j)-sα-closed. Since A is (i,j)-sα-closed, we have τ_j –scl(A)⊆ U whenever A⊆U and U is τ_i -α-open ⇒τ_j –scl(A) = A ⇒A is τ_j –semi-closed.
Remark 3.18: An (i,j)-sα-closed set need not be (j,i)-sα-closed.Proof: Consider the Example Let X={a,b,c} , τ_1 = {ϕ,X,{c},{a,b}}, τ_2 = {ϕ,X,{a}} then {a,c} is (1,2)-sα –closed but not (2,1)-sα –closed

<u>4. Applications of (i,j)-sα-closed Set</u>

In this chapter we introduce six new spaces namely (i,j)-$T^{\sim}$space, (i,j)-$T^{\sim s}$ space,(i,j)-$^{\sim}T$ space, (i,j)-$^{\sim s}T$ space, (i,j)-T^{α} space, (i,j)-$T^{\alpha\sim}$ space.
We now introduce a new space (i,j)-$T^{\sim}$space.
Definition 4.1: A space (X, τ_i , τ_j) is called an (i,j)-$T^{\sim}$space if every (i,j)-sα-closed set is τ_j – closed.
Proposition 4.2: Every (i,j)-T_b space is an (i,j)-$T^{\sim}$ space but not conversely.
Proof: follows
The converse of above proposition need not be true which is shown by the following example.
Example 4.3: Consider the example X={a,b,c},τ_1={ϕ,X,{a}},τ_2 ={ϕ,X,{a},{b,c}} then (X,τ_1 ,τ_2) is (1,2)-$T^{\sim}$ space but not (1,2)-T_b –space.

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Characterization of (i,j)-$T^{\sim}$space

Theorem 4.4: If (X, τ_i , τ_j) is an (i,j)-$T^{\sim}$space, then every singleton of X is either τ_i -α-closed or τ_j -open

Proof: Let $x \in X$ and suppose that {x} is not τ_i -α-closed. Then X-{x} is (i,j)-sα-closed set since X is the only τ_i -α-open set containing X-{x}.So X-{x} is τ_j-closed.(i.e) {x} is τ_j -open

Remark 4.5: (X, τ_1) space is not generally $T^{\sim}$space even if (X,τ_1 ,τ_2 is (1,2)-$T^{\sim}$space shown in the following example.

Example 4.6: Consider the example X={a,b,c},τ_1={ϕ,X,{a}},τ_2 ={ϕ,X,{a},{b,c}} then (X,τ_1 ,τ_2) is (1,2)-$T^{\sim}$ space but (X,τ_1) is not $T^{\sim}$–space.

We now introduce a new space (i,j)-$T^{\sim s}$

Definition 4.7: A space (X, τ_i , τ_j) is called (i,j)-$T^{\sim s}$ space if every (i,j)-sα-closed set is τ_j – semi closed.

Proposition 4.8: Every (i,j)-T_b space is an (i,j)-$T^{\sim s}$ space but not conversely.

Proof: follows.

Example 4.9: Let X={a,b,c}, τ_1 ={ϕ,X,{a}}, τ_2 ={ϕ,X,{a},{b},{a,b}} then (X, τ_1, τ_2) is (1,2)-$T^{\sim s}$ space but not (1,2)-T_b space**.**

Proposition 4.10: Every (i,j)-$T_{1/2}$ space is an (i,j)-$T^{\sim s}$ space but not conversely.

Proof: follows.

Example 4.11: Let X={a,b,c}, τ_1 ={ϕ,X,{a}}, τ_2 ={ϕ,X,{a},{b,c}} then (X, τ_1, τ_2) is (1,2)-$T^{\sim s}$ space but not (1,2)-$T_{1/2}$ space**.**

Proposition 4.12: Every (i,j)-$T^{\sim}$ space is (i,j)-$T^{\sim s}$ space but not conversely.

Proof: Follows

Example 4.13: Let X={a,b,c}, τ_1 ={ϕ,X,{a}}, τ_2 ={ϕ,X,{a},{b}, {a,b}}. Then (X, τ_1 , τ_2) is (1,2)-$T^{\sim s}$ space but not (1,2)-$T^{\sim}$ space

Characterization of (i,j)-$T^{\sim s}$ space

Theorem 4.14: For a space(X,τ_i ,τ_j) the following are equivalent.

1. (X, τ_i , τ_j) is a (i,j)-$T^{\sim s}$ space
2. Every singleton of X is either τ_i -α-closed or τ_j –semi open.

Proof **: To Prove (1)⇒(2)** Let $x \in X$ and suppose that {x} is not τ_i -α-closed. Then X-{x} is (i,j)-sα-closed set since X is the only τ_i -α-open set containing X-{x}. Therefore X-{x} is τ_j – semi-closed.(i.e) {x} is τ_j - semi-open

To Prove (2)⇒(1) Let A be a (i,j)-sα-closed set of (X, τ_i , τ_j).Clearly $A \subseteq \tau_j$ – scl(A)**.**Let $x \in X$. by (2) {x} is either τ_i -α-closed or τ_j - semi-open

Case (i) Suppose {x} is τ_i -α-closed. If $x \notin A$, then τ_j – scl (A)-A contains the τ_i -α-closed set {x} and A is (i,j)-sα-closed set. Hence we arrive at a contradiction. Thus $x \in A$.

Case (ii) Suppose that {x} is τ_j - semi-open. Since $x \in \tau_j$ – scl(A), then $\{x\} \cap A \neq \phi$. So $x \in A$. Thus in any case $x \in A$. So τ_j – scl(A)$\subseteq$A ∴ A=τ_j – scl(A) (or) equivalently A is τ_j - semi-closed. Thus (X, τ_i , τ_j) is an (i,j)- $T^{\sim s}$ space.

Definition 4.15: A space (X, τ_i , τ_j) is called strongly pairwise $T^{\sim s}$ space if it is both (1,2)-$T^{\sim s}$ and (2,1)- $T^{\sim s}$

Proposition 4.16: If(X, τ_1 , τ_2) is strongly pairwise T_b space then it is strongly pairwise $T^{\sim s}$ space but not conversely.

Proof: follows

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Example 4.17: Let X={a,b,c}, τ_1 ={ϕ,X,{a},{a,b}}, τ_2 ={ϕ,X,{a}} then (X, τ_1, τ_2) is strongly pairwise $T^{\sim s}$ space but not strongly pairwise T_b space**.**

We introduce another new space (i,j)-˜T space

Definition 4.18: A space (X, τ_i , τ_j) is called (i,j)-˜T space if every (i,j)-sα-closed set is τ_j –α-closed

Proposition 4.19: Every (i,j)-T_b space is (i,j)-˜T space but not conversely.

Proof: Let (X,τ_i ,τ_j) be a (i,j)-T_b space and A be a (i,j)-sα-closed set of (X,τ_i ,τ_j). Since(X, τ_i , τ_j) is a (i,j)-T_b space, A is τ_j –closed. Since every τ_j –closed set is τ_j –α-closed set. Implies A is τ_j –α-closed∴ (X, τ_i , τ_j) is a (i,j)-˜T space.

Example 4.20: Let X={a,b,c}, τ_1 ={ϕ,X,{a},{b},{a,b}}, τ_2 ={ϕ,X} then (X, τ_1 , τ_2) is (1,2)-˜T space but not (1,2)-T_b space.

Proposition 4.21: Every (i,j)-˜T space is (i,j)-$T^{\sim s}$ space but not conversely.

Proof: follows

Example 4.22: Let X={a,b,c}, τ_1 ={ϕ,X,{a}}, τ_2 ={ϕ,X,{a},{b},{a,b}} then (X, τ_1 , τ_2) is (1,2)-$T^{\sim s}$ space but not (1,2)-˜Tspace.

Proposition 4.23: Every (i,j)-T˜space is (i,j)-˜T space but not conversely.

Proof: follows.

Example 4.24: Let X={a,b,c}, τ_1 ={ϕ,X,{a},{b},{a,b}}, τ_2 ={ϕ,X} then (X, τ_1 , τ_2) is (1,2)-˜T space but not (1,2)-T˜space.

Theorem 4.25: If (X, τ_i , τ_j) is a (i,j)-˜Tspace, then every singleton of X is either τ_i –α-closed or τ_j –α-open.Proof:Suppose that (X, τ_i , τ_j) is a (i,j)-˜Tspace. Suppose that {x} is not τ_i –α-closed for some x∈X. Then X-{x} is not τ_i –α-open.Then X is the only τ_i –α-open set containing X-{x.

So X-{x} is a (i,j)-sα-closed. Since (X, τ_i , τ_j) is a (i,j)-˜Tspace, X-{x} is τ_j –α-closed or equivalently {x} is τ_j –α-open.

We now introduce a new space (i,j)-$^{\sim s}$T space

Definition 4.26: A space (X,τ_i ,τ_j) is called a (i,j)-$^{\sim s}$T space if every (i,j)-gs-closed set is (i,j)-sα-closed.

Proposition 4.27: Every (i,j)-$T_{1/2}$ space is a (i,j)-$^{\sim s}$Tspace but not conversely.Proof: Let (X, τ_i , τ_j) be a (i,j)-$T_{1/2}$ space .Let A be a (i,j)-gs-closed set. Since (X, τ_i , τ_j) is (i,j)-$T_{1/2}$ space, A is τ_j –semi-closed. Therefore A is (i,j)-sα-closed. Hence (X, τ_i , τ_j) is a (i,j)-$^{\sim s}$T space. Hence proved.

Example 4.28: Let X={a,b,c}, τ_1 ={ϕ,X,{a,c},{c}}, τ_2 ={ϕ,X,{a}} then (X,τ_1 ,τ_2) is (1,2)-$^{\sim s}$Tspace but not (1,2)-$T_{1/2}$ space**.**

Proposition 4.29: Every (i,j)-T_b space is (i,j)-$^{\sim s}$T space but not conversely.

Proof: follows

Example 4.30: Let X={a,b,c}, τ_1 ={ϕ,X,{c},{a,b}}, τ_2 ={ϕ,X,{a}} then (X, τ_1 , τ_2) is (1,2)-$^{\sim s}$T space but not (1,2)-T_b space.

Theorem 4.31: A space (X, τ_i, τ_j) is a (i,j)-$T_{1/2}$ –space if and only if (X, τ_i, τ_j) is (i,j)-$^{\sim s}$T and (i,j)- $T^{\sim s}$ space .Proof: follows**.**

Theorem 4.32: A space (X, τ_i, τ_j) is a (i,j)-T_b –space if and only if (X, τ_i, τ_j) is (i,j)-$^{\sim s}T$ and (i,j)- $T^{\sim}$ space .Proof: follows**.**

We now introduce a new space (i,j)-T^{α} space

Definition 4.33: A space (X, τ_i , τ_j) is called (i,j)-T^{α} space if every (i,j)-sα-closed set is (i,j)-gα-closed.

Proposition 4.34: Every (i,j)- $T^{\sim}$ space is (i,j)- T^{α} space but not conversely.

Proof: follows

Example 4.35: Let X={a,b,c}, τ_1 ={ϕ,X,{a}}, τ_2 ={ϕ,X,{c},{a,b}} then (X, τ_1 , τ_2) is (1,2)- T^{α} space but not (1,2)- $T^{\sim}$ space.

Proposition 4.36: Every (i,j)-$^{\sim}T$ space is (i,j)- T^{α} space but not conversely.

Proof: follows

Example 4.37: Let X={a,b,c}, τ_1 ={ϕ,X,{a,b}}, τ_2 ={ϕ,X,{c},{a,b}} then (X, τ_1 , τ_2) is (1,2)- T^{α} space but not (1,2)-$^{\sim}T$ space.

We now introduce a new space (i,j)-$T^{\alpha\sim}$space

Definition 4.38: A space (X, τ_i , τ_j) is called (i,j)-$T^{\alpha\sim}$ space if every (i,j)-sα-closed set is (i,j)-w-closed.

Proposition 4.39: Every (i,j)-T_b space is (i,j)- $T^{\alpha\sim}$space but not conversely.

Proof: follows

Example 4.40: Let X={a,b,c}, τ_1 ={ϕ,X,{a,b}}, τ_2 ={ϕ,X,{a},{b,c}} then (X, τ_1 , τ_2) is (1,2)- $T^{\alpha\sim}$ space but not (1,2)- T_b space.

Proposition 4.41: Every (i,j)-$T^{\alpha\sim}$ space is (i,j)- T^{α} space but not conversely.

Proof: follows

5. sα–continuous maps in topological spaces.

We introduce the following definition.

Definition 5.1 : A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called (i ,j)- sα –continuous if $f^{-1}(V)$ is (i ,j)-sα– closed set of (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2).

Proposition 5.2: If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is τ_j -σ_k –continuous then it is (i ,j)- sα –continuous but not conversely.

Proof : follows from the definitions.

Example 5.3: Let X = {a,b,c}, τ_1 = {ϕ, X,{a,b}}, τ_2 ={ϕ, X, {a}} and Y = {p, q}, σ_1 = {ϕ, Y, {p}}, σ_2 = {ϕ, Y, {q}}. Define a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by f(a) = q, f(b) = f(c) = p. then f is (1 ,2)- sα –continuous but not τ_1 -σ_2 –continuous.

Proposition 5.4: If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (i ,j)- sα –continuous, then it is

(i ,j)- gs –continuous and (i ,j)- gsp –continous but not conversely.

Proof : follows from the definitions.

The converses are not true which is shown by the following examples.

Example 5.5: Let X = {a,b,c}, τ_1 = {ϕ, X,{a}}, τ_2 ={ϕ, X, {a},{b,c}} and Y = {p, q},

σ_1 = {ϕ, Y, {p}}, σ_2 = {ϕ, Y, {q}}. Define a map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) by

f(a) = f(c) =q, f(b) = p. then f is (1 ,2)- gs –continuous but not (1 ,2)- sα –continuous.

Example 5.6: Let X = {a,b,c}, τ_1 = {ϕ, X,{a}}, τ_2 ={ϕ, X, {a},{b,c}} and Y = {p, q},

σ_1 = {ϕ, Y, {p}}, σ_2 = {ϕ, Y, {q}}. Define a map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) by

f(a) = f(c) = q, f(b) = p. then f is (1 ,2)- gsp –continuous but not (1 ,2)- sα –continuous.

Remark 5.7: (i ,j)- g–continuous and (i ,j)-sα-continuous are independent which are shown by the following example. Let X = {a,b,c}, τ_1 = {ϕ, X,{a}}, τ_2 ={ϕ, X, {a},{b,c}} and Y = {p, q},

σ_1 = {ϕ, Y, {p}}, σ_2 = {ϕ, Y, {q}}. Define a map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) by

f(a) = f(c) = q, f(b) = p. then f is (1 ,2)- g –continuous but not (1 ,2)- sα –continuous.

Conclusions:

In this paper we introduced the concepts of sα-closed sets, $T^{\tilde{}}$space, $T^{\sim s}$ space, $^{\sim}T$ space, $^{\sim s}T$ space, T^{α} space, $T^{\alpha\sim}$space, sα-continuous and sα -irresolute maps for topological spaces and investigate some of their properties.

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