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Variance Of Time To Recruitment In A Single Grade Marketing Organization With Non-Instantaneous Exits And Dependent Wastage Having Two Thresholds

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Abstract—Attrition occur whenever a marketing organization(manpower system) takes policy decisions such as revision of targets and pay structure. It is not wise to make recruitment frequently due to cost factor. Since decision and its consequences are not predictable, an appropriate policy for recruitment becomes necessary. In this paper, for a single grade manpower system with non-instantaneous exits due to policy decisions, variance of time to recruitment is obtained using a suitable policy for recruitment when (i) the system has an additional control limit for alertness and a breakdown threshold for cumulative wastage (ii) wastage due to exits form a sequence of exchangeable and constantly correlated exponential random variables or an order statistics is associated with the wastage process or wastage process is a geometric process (iii) the inter-policy decision times form a sequence of exchangeable and constantly correlated exponential random variables and (iv) the inter-exit times are independent and identically distributed random variables. The analytical results are numerically illustrated and relevant findings are presented.

Keywords— Single grade marketing organization, non-instantaneous exits, correlated wastage, order statistics, geometric process, policy for recruitment, control limit for alertness and breakdown threshold and variance of time to recruitment.

AMS Subject Classification: 60H30, 60K05, 90B70, 91D35

1. INTRODUCTION

Depletion of manpower due to policy decisions takes place in any manpower system. As recruitment involves cost frequent recruitment will not be a remedy for balancing the manpower loss. Hence the cumulative wastage is allowed till it reaches a level, called the breakdown (mandatory) threshold. As in the case of shock model approach for replacement of systems in reliability theory, recruitment is done in the system when the cumulative wastage exceeds this threshold. In [1, 2, 3] the authors have studied Markovian and renewal theoretic stochastic manpower models using suitable statistical techniques. In [4] the author has considered a single grade manpower system with non-instantaneous exits, only one control limit (mandatory threshold) for cumulative wastage and studied the problem of time to recruitment under different conditions on wastage, policy decisions and thresholds. In [5] the author has introduced an alertness level as an optional control limit (optional threshold) prior to the mandatory threshold and studied the problem of time to recruitment under different conditions on wastage, policy decisions and thresholds. Since exits are not always instantaneous, by considering separate epochs for policy decisions and exits in [7], the authors have determined the variance of time to recruitment from the Laplace transform of its density function when the inter-decision times and wastage process independently form sequences of independent exponential random variables using a recruitment policy involving two control limits. In [8] the authors have studied the work of [7] when the inter-decision times are correlated. In [9, 10] the authors have obtained variance of time to recruitment when the wastage process is correlated or order statistics is associated with wastage process and the inter-decision times form a geometric process and an order statistics, thereby extended the work in [5] for a manpower system with non-instantaneous exits. Recently, in [11] the authors have extended their work in [10] when wastage form a geometric process. The present paper extends the work of the authors in [8] when the wastage process is correlated or order statistics is associated with wastage process or form a geometric process.

2. MODEL DESCRIPTION

Assume that a single grade manpower system takes policy decisions at random epochs in $(0, \infty)$ and a random number of personnel quit this system leading to wastage in manpower. It is assumed that the exits are not instantaneous and wastage in manpower is linear and cumulative. Let X_i be the continuous random variable representing the wastage in manpower caused at the i^{th} exit point and S_k be the cumulative wastage in manpower in the first k exit points. It is assumed that $\left\{X_i\right\}_{i=1}^{\infty}$ form a sequence of exchangeable and constantly correlated exponential random variables or an order statistics where the sample of size r associated with this order statistics is selected from an exponential population with the common distribution is $M(t)=1-e^{-\alpha t}, \alpha>0$ or a geometric process. Let U_k be the continuous random variable representing the time between the (k-1)th and kth policy decisions. It is assumed that $\left\{U_k\right\}_{k=1}^{\infty}$ form a sequence of exchangeable and constantly correlated exponential random variables with mean $\frac{1}{\lambda},(\lambda>0)$ and correlation $R, -1 \le R < 1$. Let W_i be the continuous random variable representing the time between the (i-1)th and ith exits. Let $N_e(t)$ be the number of exit points lying in $(0, t]$. It is assumed that $N_e(t)$ is independent of X_i for each i and for all $t>0$. Let Y be the optional threshold level for the cumulative wastage in the system with probability density function $h_1(.)$, following an exponential distribution with mean $\frac{1}{\theta_1},(\theta_1>0)$. Let Z (Y < Z) be the mandatory threshold level for the cumulative wastage in the system with probability density function $h_2(.)$, following an exponential distribution with mean $\frac{1}{\theta_2}, (\theta_2 > 0)$. The organization may or may not go for recruitment when the cumulative wastage exceeds Y. It is assumed that the organization makes recruitment with probability 'p' when the cumulative wastage exceeds Y. It is also assumed that policy decisions produce exit with probability 'q' ($q \neq 0$). Let T be the random variable denoting the time to recruitment with probability distribution function L(.), density function $l(.)$, mean E(T) and variance V(T). Let $\bar{a}(.)$ be the Laplace transform of $a(.)$. Regarding the policy of recruitment, it is to be noted that when the cumulative wastage in manpower exceeds Y, the organization has the option to go or not to go for recruitment. However, recruitment is to be done when this cumulative wastage exceeds Z.

3. MAIN RESULT

From the recruitment policy, the tail distribution of time to recruitment is given by

$$P[T > t] = P\left(\widetilde{X_{N_e(t)}} \leq Y_1\right) + p\, P\left(Y_1 < \widetilde{X_{N_e(t)}} \leq Z_1\right). \quad (1)$$

Conditioning upon $N_e(t)$ from(1), we get

$$P[T > t] = \sum_{k=0}^{\infty} P[N_e(t) = k]\, P\left(\widetilde{X_k} \leq Y_1\right) + p \sum_{k=0}^{\infty} P[N_e(t) = k] P\left(\widetilde{X_k} > Y_1\right) P\left(\widetilde{X_k} \leq Z_1\right) \quad (2)$$

We now determine variance of time to recruitment for three different cases on wastage process.

Case (i): $\{X_i\}_{i=1}^{\infty}$ form a sequence of exchangeable and constantly correlated exponential random variable

with mean $\frac{1}{\alpha}, (\alpha > 0)$ and correlation $\rho,\ -1 \leq \rho < 1$.

Since Y_1 and $\widetilde{X_k}$ are independent, by conditioning upon Y_1 and using the result of [Gurland, (1995)], it can be shown that

$$P[T > t] = (1 - \rho) \sum_{k=0}^{\infty} a_k \left(G_{\widetilde{W_k}}(t) - G_{\widetilde{W_{k+1}}}(t)\right). \quad (3)$$

where

$$a_k = A_k + pB_k\left(1 - (1-\rho)A_k\right).$$

Here
$$A_k = \frac{(u\theta_1 + 1)}{(u\theta_1 + 1)^k [(1 - \rho + k\rho)(u\theta_1 + 1) - k\rho]}$$
and
$$B_k = \frac{(u\gamma_1 + 1)}{(u\gamma_1 + 1)^k [(1 - \rho + k\rho)(u\gamma_1 + 1) - k\rho]}.$$

Since $l_T(t) = -\frac{d}{dt} P[T > t]$ and using convolution theorem for Laplace transform we get

$$\overline{l_T}(s) = (\rho - 1)\left(1 - \overline{g_w}(s)\right) \sum_{k=0}^{\infty} a_k \left(\overline{g_w}(s)\right)^k. \quad (4)$$

An explicit result connecting the distributions of inter-exit times and inter-policy decision times is given by

$G_w(x) = q\sum_{n=1}^{\infty}(1-q)^{n-1} F_{\widetilde{U_n}}(x).$
(5)
From (5) we get
$\overline{g_W}'(0) = -\frac{v}{(1-R)q}$, where $\overline{f_{\widetilde{U_n}}}'(0) = -\frac{n}{\lambda}$ and $v = (1-R)/\lambda$.
(6)
and

$\overline{g_W}''(0) = q\sum_{n=1}^{\infty}(1-q)^{n-1}\overline{f_{\widetilde{U_n}}}''(0).$
(7)
Again using the result of [Gurland, (1995)], it can be shown that
$\overline{g_W}''(0) = \frac{2v^2}{(1-R)^2 q^2}\left[1+(1-q)R^2\right]$.
(8)
It is known that $E(T^r) = (-1)^r\left[\frac{d^r}{ds^r}\overline{l_T}(s)\right]_{s=0}$, $r = 1, 2, 3, \ldots$
(9)
From (4), (6) and (9) we get
$E(T) = K_1 \frac{(1-\rho)v}{(1-R)q}$.
(10)
From (4), (6), (8) and (9) we get
$E(T^2) = 2(1-\rho)\left(\frac{v}{(1-R)q}\right)^2\left[K_1\left\{1+(1-q)R^2\right\}+K_2\right].$
(11)
where $K_1 = \sum_{k=0}^{\infty} a_k$ and $K_2 = \sum_{k=0}^{\infty} k a_k$.
Equation (11) together with (10) give variance of time to recruitment for Case(i).

Case(ii): $\left\{X_i\right\}_{i=1}^{\infty}$ form an order statistics with $M_{X_{(j)}}(.)$ and $m_{X_{(j)}}(.)$ are the distribution and the probability density function of the j^{th} order statistics selected from the sample of size r from an exponential population. From the theory of order statistics [10], it is known that
$m_{X_{(j)}}(t) = j\binom{r}{j}[M_X(t)]^{j-1} m_X(t)[1-M_X(t)]^{r-j}, \ j = 1, 2, .., r.$
(12)
Suppose $m_X(t) = m_{X_{(1)}}(t).$
By proceeding as in Case(i) we get

$$P[T > t] = 1 + (K_5 - 1)\sum_{k=1}^{\infty} G_{\widetilde{W}_k}(t)K_5^{k-1} + p\left\{1 + (K_6 - 1)\sum_{k=1}^{\infty}[G_{\widetilde{W}_k}(t)K_6^{k-1} - [1 + (K_5K_6 - 1)]\sum_{k=1}^{\infty}[G_{\widetilde{W}_k}(t)]\,(K_5K_6)^{k-1}\right\}.$$

(13)

$$\overline{l_T}(s) = (1-K_5)\overline{g_W}(s)\sum_{k=1}^{\infty}(K_5\ \overline{g_W}(s))^{k-1} + p\ \left\{(1-K_6)\overline{g_W}(s)\sum_{k=1}^{\infty}(K_6\ \overline{g_W}(s))^{k-1} - (1-K_5K_6)\overline{g_W}(s)\sum_{k=1}^{\infty}(K_5K_6\ \overline{g_W}(s))^{k-1}\right\}.$$

(14)

$$E(T) = K_7\,\frac{v}{(1-R)q}\,.$$

(15)

and

$$E(T^2) = \frac{2v^2}{(1-R)^2q^2}\left[K_7\left\{1+(1-q)R^2\right\}+K_8\right].$$

(16)

Here the constants in (15) and (16) are given below:

$$K_5 = \frac{\alpha\, r}{\alpha\, r+\theta_1} \text{ and } K_6 = \frac{\alpha\, r}{\alpha\, r+\gamma_1}.$$

$$K_7 = \left[\frac{1}{(1-K_5)} + p\left\{\frac{1}{(1-K_6)} - \frac{1}{(1-K_5K_6)}\right\}\right].$$

(17)

i.e $K_7 = \dfrac{(\alpha\, r+\theta_1)}{\theta_1} + \dfrac{p(\alpha\, r+\gamma_1)\alpha\, r\theta_1}{\gamma_1\left[\alpha\, r(\theta_1+\gamma_1)+\theta_1\gamma_1\right]}.$

$$K_8 = \frac{K_5}{(1-K_5)^2} + p\left\{\frac{K_6}{(1-K_6)^2} - \frac{K_5K_6}{(1-K_5K_6)^2}\right\}.$$

(18)

i.e $K_8 = \alpha\, r\left[\dfrac{(\alpha\, r+\theta_1)}{\theta_1^{\,2}} + p\,(\alpha\, r+\gamma_1)\left(\dfrac{1}{\gamma_1^{\,2}} - \dfrac{\alpha\, r(\alpha\, r+\theta_1)}{(\alpha\, r(\theta_1+\gamma_1)+\theta_1\gamma_1)^2}\right)\right].$

Equation (16) together with (15) give variance of time to recruitment when $m_X(t) = m_{X_{(1)}}(t)$.

Suppose $m_X(t) = m_{X_{(r)}}(t)$.

By proceeding as in Case(i) we get

$$E(T) = K_7 \frac{v}{(1-R)q} .$$

(19)

and

$$E(T^2) = \frac{2v^2}{(1-R)^2 q^2}\left[K_7\left\{1+(1-q)R^2\right\}+K_8\right] .$$

(20)

$$K_9 = \frac{(r)!\ \alpha^r}{(\theta_1+\alpha)(\theta_1+2\alpha)\ldots(\theta_1+r\alpha)} .$$

(21)

$$K_{10} = \frac{(r)!\ \alpha^r}{(\gamma_1+\alpha)(\gamma_1+2\alpha)\ldots(\gamma_1+r\alpha)} .$$

(22)

Here (21) and (22) give the values of K_5 and K_6 in K_7 and K_8 .

Equation (20) together with (19) give variance of time to recruitment when $m_X(t) = m_{X_{(r)}}(t)$.

Case(iii): $\{X_i\}_{i=1}^{\infty}$ form a geometric process with rate d and we assume X_1 to follow an exponential distribution

with mean $\frac{1}{\alpha}, \alpha > 0$.

By proceeding as in Case(i) we get

$$P[T > t] = \sum_{k=0}^{\infty}\left[G_{\widetilde{W_k}}(t) - G_{\widetilde{W_{k+1}}}(t)\right]\overline{m_{\widetilde{X_k}}}(\theta_1)$$
$$+ p\sum_{k=0}^{\infty}[G_{\widetilde{W_k}}(t) - G_{\widetilde{W_{k+1}}}(t)][\overline{m_{\widetilde{X_k}}}(\gamma_1) - \overline{m_{\widetilde{X_k}}}(\theta_1)\overline{m_{\widetilde{X_k}}}(\gamma_1)] .$$

(23)

$$\overline{l_T}(s) = [\overline{g_W}(s) - 1]\sum_{k=0}^{\infty}\left(\overline{g_W}(s)\right)^k b_k.$$

(24)

where

$$b_k = \left[\overline{m_{\widetilde{X_k}}}(\theta_1) + p\,\overline{m_{\widetilde{X_k}}}(\gamma_1)\left(1 - \overline{m_{\widetilde{X_k}}}(\theta_1)\right)\right], k = 0,1,2 \ldots$$

and

$$\overline{m_{\widetilde{X_k}}}(\theta_1) = \alpha\prod_{i=1}^{k}\left(\frac{d^{i-1}}{\theta_1 + \alpha\ d^{i-1}}\right),\ \overline{m_{\widetilde{X_k}}}(\gamma_1) = \alpha\ \prod_{i=1}^{k}\left(\frac{d^{i-1}}{\gamma_1+\alpha\ d^{i-1}}\right).$$

$$E(T) = K_{11}\left(\frac{v}{(1-R)q}\right).$$
(25)
and
$$E(T^2) = \frac{2v^2}{(1-R)^2 q^2}\left[\left\{1+(1-q)R^2\right\}K_{11} + K_{12}\right].$$
(26)

where $K_{11} = \sum_{k=1}^{\infty} b_k$ and $K_{12} = \sum_{k=1}^{\infty} k b_k$ and (24) gives the values of $b_k{}^{'}s$ in K_{11} and K_{12}.
Equation (26) together with (25) give variance of time to recruitment for Case(iii).

4. NUMERICAL ILLUSTRATION

The influence of nodal parameters on the performance measures namely mean and the variance of time to recruitment are studied numerically for all the three Cases.
Case(i):
Effect of ρ and R on the performance measures by fixing all the other parameters $\alpha = 0.5$, $\lambda = 0.3$, $\theta_1 = 0.6$, $\gamma_1 = 0.4$, p=0.5 and q=0.5 are given below.

Table-1

ρ	R	E(T)	V(T)
- 0.3	0.6	16.358	889.574
- 0.4	0.6	15.667	722.275
- 0.5	0.6	15.479	679.853
0.5	- 0.4	16.386	844.042
0.6	- 0.4	17.037	945.744
0.7	- 0.4	18.035	1119.929
- 0.3	0.7	16.358	917.927
- 0.3	0.8	16.358	950.642
- 0.3	0.9	16.358	987.719
0.5	- 0.6	16.386	887.739
0.5	- 0.7	16.386	916.141
0.5	- 0.8	16.386	948.914

Case(ii):
Effect of R and r on the performance measures by fixing all the other parameters $\alpha = 0.5$, $\lambda = 0.3$, $\theta_1 = 0.6$, $\gamma_1 = 0.4$, p=0.5 and q=0.5 are given below.

Table-2

R	r	$m_X(t) = m_{X_{(1)}}(t)$		$m_X(t) = m_{X_{(r)}}(t)$	
		E(T)	V(T)	E(T)	V(T)
0.5	4	39.603	1547.682	10.514	135.229

	5	47.674	2252.140	9.664	113.322
	6	55.741	3089.163	9.132	100.993
0.6	4	39.603	1576.724	10.514	142.940
	5	47.674	2287.101	9.664	120.409
	6	55.741	3130.040	9.132	107.690
- 0.7	4	39.603	1611.047	10.514	152.051
	5	47.674	2328.419	9.664	128.785
	6	55.741	3178.348	9.132	115.604
- 0.8	4	39.603	1650.650	10.514	162.565
	5	47.674	2376.093	9.664	138.449
	6	55.741	3234.089	9.132	124.736

Case(iii):

Effect d and R on the performance measures by fixing all the other parameters $\alpha = 0.5$, $\lambda = 0.3$, $\theta_1 = 0.6$, $\gamma_1 = 0.4$, p=0.5 and q=0.5 are given below.

Table-3

d	$R = -0.4$		$R = 0.5$		$R = 0.6$	
	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
0.80	15.190	561.497	15.190	570.611	15.190	581.750
0.85	18.037	723.256	18.037	734.078	18.037	747.305
0.90	22.637	1003.317	22.637	1016.899	22.637	1033.500
1	42.804	2081.829	42.804	2113.219	42.804	2056.146
1.02	48.540	2268.481	48.540	2304.077	48.540	2239.357
1.04	53.766	2374.228	53.766	2413.656	53.766	2341.968

5. FINDINGS

From the above tables, the following observations are presented which agree with reality. From the Table-1, we find that

1. When ρ increases and keeping all the other parameters fixed, the mean and variance of time to recruitment increase.
2. When R increases and keeping all the other parameters fixed, the mean of time to recruitment remains unchanged and variance of time to recruitment increases.

From the Table-2, we find that

1. When r increases and keeping all the other parameters fixed, the mean and variance of time to recruitment increase for the case $m_X(t) = m_{X_{(1)}}(t)$. But, the mean and variance of

time to recruitment decrease for the case $m_X(t) = m_{X_{(r)}}(t)$.

2. When R increases and keeping all the other parameters fixed, the mean of time to recruitment remains unchanged and variance of time to recruitment increases for both the cases $m_X(t) = m_{X_{(1)}}(t)$ and $m_X(t) = m_{X_{(r)}}(t)$.

From the Table-3, we find that

1. When d (either $d < 1 or\ d \leq 1$) increases and keeping all the other parameters fixed, the mean and variance of time to recruitment increase.
2. When R increases, either $d < 1$ or $d \geq 1$ and keeping all the other parameters fixed, the mean of time to recruitment remains unchanged and variance of time to recruitment increases.

6. CONCLUSION

The present work contributes to the existing literature in the sense that the models discussed in this paper are new in the context of considering (i) Non –instantaneous exits (ii) a chance factor for any decision to have exit points and (iii) provision of an alertness level as an additional control limit. The assumption on the non –instantaneous exits gives a provision for elongating the time to recruitment to a desirable limit.

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