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Solving Fuzzy Linear Programming Problem With A Fuzzy Polynomial Barrier Method

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Abstract: In this paper, we emphasis a fuzzy polynomial barrier function for solving a fuzzy linear programming problem and develop an algorithm which gives a better rate of convergence to the fuzzy optimal solution to the problem. Some numerical example is involve in display the efficiency of the new algorithmic procedure.

Keywords: Fuzzy polynomial barrier method, fuzzy linear programming problem, triangular fuzzy number,
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1. INTRODUCTION

The fuzzy set theory has been useful to control theory and management sciences and various field. A decision -making problems are often uncertain (or) vague in some ways to apply real-world. In 1965, Zadeh fuzzy set theory was presented. Tanaka and Asai [16] is proposed by the concept of fuzzy linear programming problem at the general level.Dubois. D and Prade.H [2] presented the notion of fuzzy numbers. The function principle was presented by Hsieh and Chen [7] to treat fuzzy arithmetical operations of fuzzy numbers. We describe fuzzy basic solution (FBFS) for the FLP problems which recognized by[9]and[10], Mahdavi-Amiri and Nasseri for the FVLP problem and also describe the duality of the fuzzy linear programming problems. Defuzzification is the process of transforming fuzzy values to crisp values. The Fuzzy polynomial barrier method is an alternative class of algorithms for fuzzy constrained optimization. The Fuzzy polynomial barrier method is another method to solve the fuzzy linear programming problem. The Fuzzy polynomial barrier method is a procedure for approximating of the fuzzy constrained problems by addition of the fuzzy objective function a term that proposes a high cost for violation of the fuzzy constraints. The polynomial barrier function method was introduced by Frisch[5] and then developed by Fiacco[3] and Mc cormick[4]. Wright[17] and Parwardi et al. [14] give high cost to infeasible points in their works. A continuous fuzzy polynomial barrier function whose point value increases to infinity as the point approaches the boundary of the feasible region of the fuzzy optimization problem. Related with the fuzzy polynomial barrier method is a positive increasing parameter σ or γ that determines the level to which the fuzzy unconstrained problem methods the original fuzzy constrained problems.

2. PRELIMINARIES

2.1 Fuzzy Sets

A fuzzy set z is defined by $\tilde{\mathbf{z}}=\{(x,\mu_z(x)): x\in z, \mu_z(x)\in[0,1]\}$. A pair $(x,\mu_z(x))$, the first element x belongs to the classical set A and the second element $\mu_z(x)$ belongs to the interval
[0, 1], called the *membership function*.

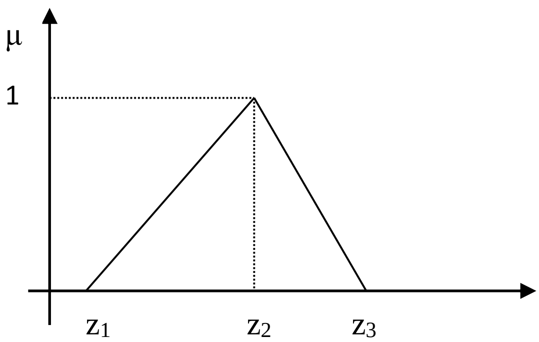
A fuzzy set can also be indicated by $\tilde{\mathbf{z}}=\{\mu_z(x)/x: x\in z, \mu_z(x)\in[0,1]\}$. Here the symbol '/' does not represent the division sign, $\mu_z(x)$ is the membership value of the element x on the bottom.

2.2 FUZZY NUMBER

2.2.1. Triangular Fuzzy Number

The fuzzy set $\tilde{\boldsymbol{z}}=(\boldsymbol{z_1},\boldsymbol{z_2},\boldsymbol{z_3})$ where $\boldsymbol{z_1}\le\boldsymbol{z_2}\le\boldsymbol{z_3}$ and defined on R, is called the triangular fuzzy number if the membership function of $\tilde{\boldsymbol{z}}$ is given by

$$\mu_{\tilde{z}}(x)=\begin{cases}\dfrac{x-z_1}{z_2-z_1}, & z_1\le x\le z_2\\ \dfrac{z_3-x}{z_3-z_2}, & z_2\le x\le z_3\\ 0, & \text{otherwise}\end{cases}$$



Triangular fuzzy number $\tilde{z}$ = (z_1, z_2, z_3)

2.3 Fuzzy Linear Programming Problem

Consider the following fuzzy linear programming problem:

Maximum (or Minimum) $\tilde{Z}=\tilde{f}\tilde{x}$

Constraints of the form

$M_i\tilde{x}(\le,=,\ge)\widetilde{N_j},\ j=1,2,\dots,m$

and the nonnegative conditions of the fuzzy variables $\tilde{x}\ge(0,0,0)$ where $\tilde{f}^T=(\tilde{f}_1,\dots,\tilde{f}_n)$ is an n-dimensional constant vector, $M\in R^{m\times n}$, $\tilde{x}=(\tilde{x}_i)$, i=1,2,..,n and $\widetilde{N}_j$ are non-negative fuzzy variable vectors such that $\tilde{x}_i$ and $\widetilde{N}_j\in F(R)$ for all $1\le i\le n$, $1\le j\le m$, is called a fuzzy linear programming (FLP) problem. f

2.4. Feasible solution: The vector $\tilde{x}\in F(R)^n$is a feasible solution if and only if $\tilde{x}$ satisfies the constraints of the problem.

2.5. Optimal Solution: A feasible solution $\tilde{x}^*$ is an optimal solution if for all feasible solution $\tilde{x}$, $\tilde{f}\tilde{x}^*\ge\tilde{f}\tilde{x}$.

2.6. Fuzzy basic feasible solution:

Consider the system $M\tilde{x} = \tilde{N}$ and $\tilde{x} \geq \tilde{0}$. Let $M=[m_{ij}]_{m\times n}$. Assume that rank(B)=m. Partition M as [B n] where B is nonsingular. It is obvious that rank ($\tilde{B}$)=m. Let y_j be the solution to $B_y= m_j$. It is apparent that the basic solution $\tilde{x}_N = \left(\tilde{x}_{B_1}, \tilde{x}_{B_2}, \dots, \tilde{x}_{B_m}\right)^T = B^{-1}\tilde{N}, \tilde{x}_n = 0$ is a solution of $M\tilde{x} = \tilde{N}$. If $\tilde{x}$ is accordingly partitioned as $(\tilde{x}_B^T \tilde{x}_n^T)^T$, a fuzzy basic solution corresponding to the basis B. If $\tilde{x}_B \geq \tilde{0}$, then the fuzzy basic solution is feasible, and the corresponding fuzzy objective value is $\tilde{z} = \tilde{f}_B\tilde{x}_B$, where $\tilde{f}_B = (\tilde{f}_{B_1}, \dots, \tilde{f}_{B_m})$. Now corresponding to every fuzzy non- basic variable $\tilde{x}_j, 1 \leq j \leq n, j \neq B_i$, and i=1,… ,m, define $z_j = f_B y_j = f_B B^{-1} m_j$). If $\tilde{x}_B > \tilde{0}$, then $\tilde{x}$ is called non- degenerate fuzzy basic feasible solution.

3. FUZZY POLYNOMIAL BARRIER METHOD

Consider the primal Fuzzy linear programming problem

Maximize $\tilde{Z} = \tilde{N}_\iota(\tilde{x})$

$M_i(\tilde{x}) \leq \tilde{f}(\tilde{x}), i = 1,2.$ (1)

Where M_i and $\tilde{N}_i$ the i[th] row of fuzzy matrices M and N respectively.

Consider the dual fuzzy linear programming problem

Minimize $\tilde{Z} = \tilde{f}(\tilde{y})$

Subject to $M_j\tilde{y} \geq \tilde{N}_j, j = 1,2.$

where $M \in R^{m\times n}, \tilde{f}, x \in R^n, \tilde{N} \in R^m, \tilde{f} = (f_1, f_2, f_3), \tilde{N} = \left(n_{1,} n_2, n_3\right)$

Where M_j and $\tilde{N}_j$ the j[th] transpose of fuzzy matrices M and N respectively.

Without loss of generality, assume that M has full rank m. The notation ~ denotes the fuzzy quantity described a triangular fuzzy number.

Assume that the problem (1) has at least one feasible solution of the fuzzy linear programming problem of the fuzzy polynomial barrier method.

For any scalar, $\gamma > 0$ we define the Fuzzy polynomial barrier function $\tilde{B}(x, \eta)$ for the fuzzy linear programming problem.

Define $\tilde{B}(\tilde{x}, \gamma): R^n \to R$ by the Fuzzy polynomial barrier function

$\tilde{B}(\tilde{x}) = \sum_{i=1}^{2}(M_i x - \tilde{N}_i)^\beta$, where β is an even number. (2)

The Fuzzy polynomial barrier function (2) is fuzzy concave. Hence $\tilde{B}(x, \gamma)$ is a global minimum and γ is a positive increasing value of the Fuzzy polynomial barrier parameter.

Define the function $\tilde{B}: R^n \to (-\infty, \infty)$ by

$\tilde{B}(\tilde{x}) = M_i\tilde{x} - \tilde{N}_\iota \leq 0, if\ M_i\tilde{x} - \tilde{N}_\iota = 0$ for all i,

$\tilde{B}(\tilde{x}) = M_i\tilde{x} - \tilde{N}_\iota > 0, if\ M_i\tilde{x} - \tilde{N}_\iota \neq 0$ for all i.

Convert the fuzzy equation into two weak fuzzy inequalities of the fuzzy polynomial barrier method.

A Fuzzy polynomial barrier function defined the interior of the boundary region R, such that

(i) Fuzzy polynomial barrier function $\tilde{B}$ is continuous.

(ii) Fuzzy polynomial barrier function is always positive, $\tilde{B}(\tilde{x}) \geq 0$.

(iii) A fuzzy polynomial barrier function $\tilde{B}(\tilde{x}) \to \infty$ where x approaches the boundary of the fuzzy set $\tilde{Z}$.

The Fuzzy polynomial barrier function with a fuzzy linear programming problem is given by $\tilde{B}(\tilde{x}, \gamma) = \tilde{f}(\tilde{x}) - \gamma \sum_{j=1}^{2}(M_j x - \tilde{N}_j)^\beta$.

Fuzzy polynomial barrier methods are also they called as fuzzy interior methods.

3.1: Fuzzy polynomial barrier Lemma

(i).$\tilde{B}(\tilde{x}^k,\gamma^k) \geq \tilde{B}(\tilde{x}^{k+1},\gamma^{k+1})$

(ii).$\tilde{B}(\tilde{x}^k) \leq \tilde{B}(\tilde{x}^{k+1})$

(iii).$\tilde{f}(\tilde{x}^k) \geq \tilde{f}(\tilde{x}^{k+1})$

(iv).$\tilde{f}(\tilde{x}^*) \leq \tilde{f}(\tilde{x}^k) \leq \tilde{B}(\tilde{x}^k,\gamma^k)$.

Theorem:2

3.2: A fuzzy polynomial barrier convergence theorem

A fuzzy linear programming problem an increasing sequence of positive Fuzzy polynomial barrier parameter $\{\gamma^k\}$ such that $\gamma^k \geq 1, \gamma^k \to \infty, k \to \infty$. Suppose$\tilde{f}(x), M_i\tilde{x} - \widetilde{N_\iota}$& $\tilde{B}(\tilde{x})$ is a continuous fuzzy function a fuzzy optimal solution exists x^* of $\tilde{Z}. N(\in, x^*) \cap \{x: M_i\tilde{x} - \widetilde{N_\iota} < 0\} \neq \emptyset$, for every $\in > 0$. We get any limit point of $\tilde{x}$ of $\{\tilde{x}^k\}$

Proof:

Let $\tilde{x}$ be any limit point of $\{\tilde{x}^k\}$.

$$\tilde{B}(\tilde{x}^k,\gamma^k) = \tilde{f}(\tilde{x}) - \gamma \sum_{j=1}^{2}(M_j x - \widetilde{N_j})^\beta$$

From the fuzzy continuity of $\tilde{f}(\tilde{x}), M_i\tilde{x} - \widetilde{N_\iota}$, we get,

$$\lim_{k\to\infty} \tilde{f}(\tilde{x}^k) = f(\bar{x}), \lim_{k\to\infty} M_i\tilde{x} - \widetilde{N_\iota}^k = 0.$$

$\tilde{x}$ isan optimal solution of the fuzzy polynomial method for fuzzy linear programming problem..

3.3:ALGORITHM:

1. Identify the fuzzy objective function, constraints of the problem and rewrite in standard forms. Write Max $\tilde{Z} = \tilde{f}(\tilde{x})$. Subject to $\left(M_i\tilde{x}^k - \widetilde{N_\iota}\right) \geq 0$.
2. Convert the Lagrangian fuzzy polynomial barrier function of the form $\tilde{B}(\tilde{x}) = \gamma \sum_{j=1}^{2}(M_j x - \widetilde{N_j})^\beta$, Where β is an even number.
3. Given problem can be transformed with the inequality constraints to Maximize
 Max $\tilde{B}(\tilde{x},\gamma) = \tilde{f}(\tilde{x}) - \gamma \sum_{j=1}^{2}(M_j x - \widetilde{N_j})^\beta$
4. Applying the first-order necessary condition for optimality, taking the limit $\gamma \to \infty$ we get the optimal value of the given fuzzy linear programming problem for the fuzzy polynomial barrier method.
5. Compute $\tilde{B}(\tilde{x}^k,\gamma^k) = \underset{x\geq 0}{\text{Max}}\, \tilde{B}(\tilde{x},\gamma^k)$, then fuzzy maximize x^k & $\gamma = 10, k = 1,2,$ ...,*k*=*I* then stop.

Otherwise, go to step 5.

4. NUMERICAL EXAMPLE

Problem:1

Consider the primal fuzzy linear programming problem

$Max\ \tilde{z} = (3.75,4,4.25)\widetilde{x_1} + (2.75,3,3.25)\widetilde{x_2}$

$2\widetilde{x_1} + 3\widetilde{x_2} \geq (5.75,6,6.25)$,

$4\widetilde{x_1} + \widetilde{x_2} \geq (3.75,4,4.25)$.

Solution:

Using the above algorithm we get,

$$\widetilde{x_1} = \frac{1}{\gamma}(0.025,0.05,0.075) + (0.35,0.60,0.85), \widetilde{x_2} = \frac{1}{\gamma}(0.04,0.10,0.16) + (0.95,1.60,2.15).$$

Table:1

No.	γ^k	$\widetilde{x_1}$	$\widetilde{x_2}$
1	10	(0.35,0.61,0.86)	(0.95,1.61,2.17)
2	10^2	(0.35,0.60,0.85)	(0.95,1.60,2.15)

The optimal value of the given primal fuzzy linear programming problem for fuzzy polynomial barrier method Max $\tilde{z} = (3.93,7.2,10.6)$.
Dual of the fuzzy linear programming problem of the fuzzy polynomial barrier method solutions are

$$y_1 = (0.45,0.80,1.15) + \frac{1}{\gamma}(0.26,0.29,0.33), y_2 = (0.30.6,0.9) + \frac{1}{\gamma}(-0.10,-0.07,-0.04).$$

When we look at the few iterations of the fuzzy optimal of the problems are given as follows: The table below shows the value of $y_1(\gamma)$ and $y_2(\gamma)$for a sequence of the fuzzy polynomial barrier parameters. We see that $Y = (y_1(\gamma), y_2(\gamma))$ displays a linear rate of convergence to the fuzzy optimal solution.

Table 2:

No.	γ^k	y_1	y_2
1	10	(0.48,0.83,1.18)	(0.29,0.59,0.90)
2	10^2	(0.45,0.80,1.15)	(0.30,0.60,0.90)

The optimal value of the given fuzzy linear programming problem with the fuzzy polynomial barrier method Min $\tilde{z} = (3.9,7.2,10.5)$

5. CONCLUSION:

In this paper, the Fuzzy polynomial barrier function with the Fuzzy polynomial barrier parameter for solving the primal-dual fuzzy linear programming problem by using the suggested algorithm to yield a better optimal solution. The table for the problems considered above displays that the computational procedure for the primal-dual algorithm developed by us when γ is the fuzzy polynomial barrier parameter gives a better the rate of convergence to the fuzzy optimal solution.

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