

An Integrated Vendor Buyer Inventory Model With Imperfect Quality Items Considering The Screening Errors, And Budget Capacity Constraint

I.Francina Nishandhi

*Holy Cross College (Autonomous),
Trichy-620002*

Abstract: *This research paper is based on the production inventory model with the imperfect quality items; the investment cost is used to reduce the lead time variance. Under this integrated inventory model the buyer preceding the screen process for the products that is received from the vendor. But during the screening process unfortunately buyer makes the corrections of accepting the imperfect quality items and rejects the quality items with this assumption we develops the proposed model. This paper used a Budget Capacity constraint is caused by the limited purchasing cost. Using the Karush Khun Tucker approach we find an optimum solution for the proposed model. The main objective of the proposed work is to minimize the total cost of the supply Chain.*

Keywords: *Supply Chain, Inventory, Vendor- Buyer, Investment, Karush -Khun Tucker approach*

1. INTRODUCTION:

Supply chain is the key strategy for the every business and industrial organization. In today's competitive business world the main objective of the every industry is that to fulfill the customers demand and retain them for their product. Inventory management is an enlarged area for the academicians and the research people. Inventory is an enlarged area of the most academicians and research scholars. The first inventory model was established by Harris in the year 1915 followed him the next inventory model was that an Economic production quantity model which was developed by Taft in the year 1917.

The integrated inventory model is used to increase a mutual profit of the both buyer and vendor within the minimum total cost. Under this stream the vendor plays as a manufacturer and the buyer is the one who buys the product from the vendor and distribute them to the end customer. Inventory management with supply chain coordination was introduced by Goyal (1976). He introduced a vendor-buyer inventory model and adopted a lot-for-lot policy to deliver products from the vendor to the buyer. Banarjee (1986) established an integrated vendor-buyer inventory model with the objective is to reduce the total cost between the two players. Furthermore, many researchers have developed and extended their inventory model with the various assumptions along with the new innovative ideas it will be shown in the following inventory model papers Goyal (1988), Hoque (2011) and Sajadieh and Larsen

(2015).

The main assumption in the two echelon integrated inventory model is that all the items produced by the vendor are in a good conditions that is, all are in the good quality products. But in nature every production contains the fraction of defective items. Under this stream the Porteus (1986) was the first researcher to introduce the imperfect production process in an economic order quantity model. So the inventory models with imperfect production process were developed by Lin (2010a), Sana (2010) and Dey and Giri (2014). Some researcher further extended the imperfect inventory model with the assumption that the screening errors which means accepting the imperfect quality items and rejecting the perfect quality items during the inspection process, and developed the inventory model with repairing cost of the imperfect quality items. These concepts were seen in the following inventory model papers Khan et al., (2010), Jaber et.al. (2014) and Khan.et al., (2011).

This proposed work is the extension of the work of Jauhari et al., (2018) “A vendor-buyer inventory model with imperfect production considering investment to reduce lead time variability” We extends the above model with the assumption of the screening errors and Budget capacity constraint to limited the purchasing cost. The main objective of the proposed work is to reduce the total cost of the both the player and find the optimal value using the Karush Khun Tucker approach. Rest of the paper organized as follows, section 2 presents a notations and assumptions of the proposed model section 3 illustrates a mathematical model section 4 discusses the numerical example and the conclusion are given in section 5.

2. NOTATIONS AND ASSUMPTIONS:

The following notations are used to develop the model

D	: annual demand rate (unit/year)
P	: annual production rate (unit/year)
K	: production setup cost (\$/batch)
A	: buyer ordering cost (\$/order)
V	: vendor delivery cost (\$/shipment)
h_v	: vendor annual holding cost per unit product (\$/unit/ year)
h_b	: buyer annual holding cost per unit per product (\$/unit/year)
a_1	: production fixed cost (\$)
a_2	: production variable cost (\$/unit)
x	: buyer inspection rate (unit/year)
s	: buyer inspection cost (\$/unit)
C_a	: cost of accepting defective items
C_r	: cost of rejecting non defective items
m_1	: probability of committing type I error
m_2	: probability of committing type II error
γ	: defective item proportion
δ	: upper limit of the probability function of defective products
w	: warranty cost
$f_s(k)$: probability density function for k

- $F_s(k)$: cumulative density function for k
 π : back ordering cost (\$/unit)
 σ_o^2 : current lead time variance (days)
 σ_L^2 : minimum lead time variance (days)
 $\sigma_{(l)}^2$: target lead time variance (days)
 I : the amount of investment (\$)
 b : variance reduction coefficient
 θ : fractional opportunity cost (\$)
 q : optimal lot size (decision variable)
 n : number of shipments (/year)
 k : safety factor
 $\sigma_{(l)}^2$: lead time variance reduction target
 B : buyer's maximum available budget to purchase products
 p : buyer's purchasing price per unit item

Assumptions:

The following assumptions used for the proposed model

1. Production rate is always greater than the demand rate ($P > D$).
2. Demand (D) is deterministic.
3. Lead time is normally distributed
4. In each lot delivered to the buyer, there is always a portion of defective items with a probability of γ that follows a normal distribution.
5. Inspection process is imperfect. There are two types of possibilities, The first is type I of inspection error (when a non-defective item classified as defective) and second is type II of inspection error (when a defective item classified as non defective)
6. The purchasing cost for all the products is limited, mathematically

$$pq \leq B$$

3. MATHEMATICAL MODEL:

The proposed model has developed under two categories, the first category is to find the total cost without investment cost and second category is to find the total cost with investment cost. The results from both categories are then compared one to another in order to find out which one gives better outcome to the system.

Without Investment

Vendor cost function consists of the following cost function that is holding cost, production cost, production setup cost, warranty cost

$$\text{Holding cost} = \frac{h_v q}{2} \left\{ n \left(1 - \frac{D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) \right) + \frac{2D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) - 1 \right\}$$

$$\text{Vendor Production cost} = D \left(\frac{a_1}{P} + a_2 P \right) \left(E \left(\frac{1}{1-\gamma} \right) \right)$$

$$\text{Vendor setup Cost} = \frac{DK}{nq} \left(E \left(\frac{1}{1-\gamma} \right) \right)$$

$$\text{Warranty Cost} = wD \left(E \left(\frac{1}{1-\gamma} \right) \right)$$

Buyer Cost function consist of holding cost, ordering cost, delivery cost, inspection cost , type I and type II error, and Backorder cost

$$\text{Holding cost} = h_b \left(\frac{qD}{X} \left(E \left(\frac{1}{1-\gamma} \right) \right) \right) + \frac{q}{2} [1 - E(\gamma)] + kD\sigma_0$$

$$\text{Ordering cost} = \frac{DA}{nq} E \left(\frac{1}{1-\gamma} \right)$$

$$\text{Delivery Cost} = \frac{DV}{q} E \left(\frac{1}{1-\gamma} \right)$$

$$\text{Inspection cost} = sD \left(E \left(\frac{1}{1-\gamma} \right) \right) + C_a \gamma m_2 DE \left(\frac{1}{1-\gamma} \right) + C_r (1-\gamma) m_1 DE \left(\frac{1}{1-\gamma} \right)$$

$$\text{Backorder cost } BC_B = \frac{D^2}{nq} \pi \sigma_0 \psi(k) \left(E \left(\frac{1}{1-\gamma} \right) \right)$$

The expected total annual cost of the system is **ETC (q)**

$$\begin{aligned} & h_v \frac{q}{2} \left\{ n \left(1 - \frac{D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) \right) + \frac{2D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) - 1 \right\} + D \left(\frac{a_1}{P} + a_2 P \right) \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{DK}{nq} \left(E \left(\frac{1}{1-\gamma} \right) \right) \\ & + wD \left(E \left(\frac{1}{1-\gamma} \right) \right) + h_b q \left(\frac{D}{X} \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{1}{2} (1 - E(\gamma)) \right) + h_b k D \sigma_0 + \frac{DA}{nq} \left(E \left(\frac{1}{1-\gamma} \right) \right) + \\ & SD \left(E \left(\frac{1}{1-\gamma} \right) \right) + C_a \gamma m_2 DE \left(\frac{1}{1-\gamma} \right) + C_r (1-\gamma) m_1 DE \left(\frac{1}{1-\gamma} \right) + \frac{D^2}{nq} \pi \sigma_0 \psi(k) \left(E \left(\frac{1}{1-\gamma} \right) \right) \\ & + \frac{DV}{q} \left(E \left(\frac{1}{1-\gamma} \right) \right) \end{aligned}$$

The purchasing cost for all products is limited, mathematically $pq \leq B$. The vendor buyer inventory model with budget capacity constraint is
 Minimize $ETC (q) =$

$$\begin{aligned} & h_v \frac{q}{2} \left\{ n \left(1 - \frac{D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) \right) + \frac{2D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) - 1 \right\} + D \left(\frac{a_1}{P} + a_2 P \right) \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{DK}{nq} \left(E \left(\frac{1}{1-\gamma} \right) \right) \\ & + wD \left(E \left(\frac{1}{1-\gamma} \right) \right) + h_b q \left(\frac{D}{X} \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{1}{2} (1 - E(\gamma)) \right) + h_b k D \sigma_0 + \frac{DA}{nq} \left(E \left(\frac{1}{1-\gamma} \right) \right) + \end{aligned}$$

$$SD\left(E\left(\frac{1}{1-\gamma}\right)\right) + C_a\gamma m_2 DE\left(\frac{1}{1-\gamma}\right) + C_r(1-\gamma)m_1 DE\left(\frac{1}{1-\gamma}\right) + \frac{D^2}{nq}\pi\sigma\psi(k)\left(E\left(\frac{1}{1-\gamma}\right)\right) + \frac{DV}{q}\left(E\left(\frac{1}{1-\gamma}\right)\right)$$

Subject to $pq \leq B$(1)

Case 2: With investment

In this case vendor cost function includes the investment cost along with the already existing cost function that is holding cost, production cost, production setup cost, and warranty cost. Vendor used the investment cost to reduce the lead time variance. So the investment cost is formulated as,

$$IC_v = \alpha \frac{1}{b} \ln\left(\frac{\sigma^2(I) - \sigma_L^2}{\sigma_o^2 - \sigma_L^2}\right)$$

Buyer total inventory cost for the case (2) is similar to case (1) except the formulation of backorder cost and holding cost due to investment in the vendor.

$$HC_B = h_b \left(\frac{qD}{X} \left(E\left(\frac{1}{1-\gamma}\right) \right) + \frac{q}{2} [1 - E(\gamma)] + kD\sigma(I) \right)$$

$$BC_B = \frac{D^2}{nq} \pi\sigma[I]\psi(k) \left(E\left(\frac{1}{1-\gamma}\right) \right)$$

So the expected total cost for the system under case 2 is

Minimize $ETC(q) =$

$$h_v \frac{q}{2} \left\{ n \left(1 - \frac{D}{P} \left(E\left(\frac{1}{1-\gamma}\right) \right) \right) + \frac{2D}{P} \left(E\left(\frac{1}{1-\gamma}\right) \right) - 1 \right\} + D \left(\frac{a_1}{P} + a_2 P \right) \left(E\left(\frac{1}{1-\gamma}\right) \right) + \frac{DK}{nq} \left(E\left(\frac{1}{1-\gamma}\right) \right)$$

$$+ wD \left(E\left(\frac{1}{1-\gamma}\right) \right) - \alpha \frac{1}{b} \ln\left(\frac{\sigma^2(I) - \sigma_L^2}{\sigma_o^2 - \sigma_L^2}\right) + \frac{DA}{nq} \left(E\left(\frac{1}{1-\gamma}\right) \right) + \frac{DV}{q} \left(E\left(\frac{1}{1-\gamma}\right) \right)$$

$$+ h_b q \left(\frac{D}{X} \left(E\left(\frac{1}{1-\gamma}\right) \right) + \frac{1}{2} (1 - E(\gamma)) \right) + h_b k D \sigma [I] + SD \left(E\left(\frac{1}{1-\gamma}\right) \right) +$$

$$C_a \gamma m_2 DE \left(\frac{1}{1-\gamma} \right) + C_r (1-\gamma) m_1 DE \left(\frac{1}{1-\gamma} \right) + \frac{D^2}{nq} \pi\sigma[I]\psi(k) \left(E\left(\frac{1}{1-\gamma}\right) \right)$$

Subject to $pq \leq B$(2)

3.1 Solution Technique:

The development of Karush Khun Tucker conditions is based on the Lagrangian method.

Without Investment

The expected total cost of equation (1) can be written as follows,

Minimize $f(z) = ETC(q)$
 Subject to $g(z) = pq - B \leq 0$

A new function i.e. the Lagrangian function $ETC(q, \lambda)$ is formed by introducing Lagrangian multiplier λ then we have

$$ETC(z, \lambda) = f(z) - \lambda g(z)$$

$$\begin{aligned} & h_v \frac{q}{2} \left\{ n \left(1 - \frac{D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) \right) + \frac{2D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) - 1 \right\} + D \left(\frac{a_1}{P} + a_2 P \right) \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{DK}{nq} \left(E \left(\frac{1}{1-\gamma} \right) \right) \\ & + wD \left(E \left(\frac{1}{1-\gamma} \right) \right) + h_b q \left(\frac{D}{X} \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{1}{2} (1 - E(\gamma)) \right) + h_b k D \sigma_o + \frac{DA}{nq} \left(E \left(\frac{1}{1-\gamma} \right) \right) + \\ & SD \left(E \left(\frac{1}{1-\gamma} \right) \right) + C_a \gamma m_2 DE \left(\frac{1}{1-\gamma} \right) + C_r (1-\gamma) m_1 DE \left(\frac{1}{1-\gamma} \right) + \frac{D^2}{nq} \pi \sigma_o \psi(k) \left(E \left(\frac{1}{1-\gamma} \right) \right) \\ & + \frac{DV}{q} \left(E \left(\frac{1}{1-\gamma} \right) \right) - \lambda (pq - B) \end{aligned} \quad \dots \quad (3)$$

Similarly for the with investment case the expected total can be written as,

$$\begin{aligned} & h_v \frac{q}{2} \left\{ n \left(1 - \frac{D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) \right) + \frac{2D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) - 1 \right\} + D \left(\frac{a_1}{P} + a_2 P \right) \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{DK}{nq} \left(E \left(\frac{1}{1-\gamma} \right) \right) \\ & + wD \left(E \left(\frac{1}{1-\gamma} \right) \right) - \alpha \frac{1}{b} \ln \left(\frac{\sigma_{(I)}^2 - \sigma_L^2}{\sigma_o^2 - \sigma_L^2} \right) + \frac{DA}{nq} \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{DV}{q} \left(E \left(\frac{1}{1-\gamma} \right) \right) \\ & + h_b q \left(\frac{D}{X} \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{1}{2} (1 - E(\gamma)) \right) + h_b k D \sigma [I] + SD \left(E \left(\frac{1}{1-\gamma} \right) \right) + \\ & C_a \gamma m_2 DE \left(\frac{1}{1-\gamma} \right) + C_r (1-\gamma) m_1 DE \left(\frac{1}{1-\gamma} \right) + \frac{D^2}{nq} \pi \sigma [I] \psi(k) \left(E \left(\frac{1}{1-\gamma} \right) \right) - \lambda (pq - B) \end{aligned} \quad \dots \quad (4)$$

The Khun –Tucker conditions need z and λ to be stationary point of minimization problem which can be summarized as following:

$$\begin{cases} \nabla f(z) - \lambda g(z) = 0 \\ \lambda g(z) = 0 \\ g(z) \leq 0 \\ \lambda \geq 0 \end{cases}$$

By the method of Khun- Tucker conditions, consider the two cases $\lambda = 0$ and $\lambda \neq 0$

For $\lambda = 0$ the optimal order quantity of case (1) as follows

$$q^* = \sqrt{\frac{E\left(\frac{1}{1-\gamma}\right) D\left[\frac{K}{n} + \frac{A}{n} + V + \frac{D}{n} \pi \sigma_0 \psi(k)\right]}{\frac{h_v}{2} \left[n \left(1 - \frac{D}{P} \left(E\left(\frac{1}{1-\gamma}\right) \right) \right) \right] + \frac{2D}{P} \left(E\left(\frac{1}{1-\gamma}\right) \right) - 1 + h_b \left[\frac{D}{X} \left(E\left(\frac{1}{1-\gamma}\right) \right) + \frac{1}{2} (1 - E(\gamma)) \right]}}$$

$\lambda \neq 0$, the optimal value of q and λ expressed as follows,

$$\lambda^* = \frac{h_v}{2p} \left[n \left(1 - \frac{D}{P} \left(E\left(\frac{1}{1-\gamma}\right) \right) \right) \right] + \frac{2D}{P} \left(E\left(\frac{1}{1-\gamma}\right) \right) - 1 + \frac{DK}{npq^2} \left(E\left(\frac{1}{1-\gamma}\right) \right) + \frac{h_b}{p} \left[\frac{D}{X} \left(E\left(\frac{1}{1-\gamma}\right) \right) + \frac{1}{2} [1 - E(\gamma)] \right] + \frac{DA}{npq^2} \left(E\left(\frac{1}{1-\gamma}\right) \right) + \frac{DV}{pq^2} \left(E\left(\frac{1}{1-\gamma}\right) \right) + \frac{D^2}{npq} \pi \sigma_0 \psi(k) \left(E\left(\frac{1}{1-\gamma}\right) \right)$$

$$q^* = \sqrt{\frac{E\left(\frac{1}{1-\gamma}\right) D\left[\frac{K}{n} + \frac{A}{n} + V + \frac{D}{n} \pi \sigma_0 \psi(k)\right]}{\frac{h_v}{2} \left[n \left(1 - \frac{D}{P} \left(E\left(\frac{1}{1-\gamma}\right) \right) \right) \right] + \frac{2D}{P} \left(E\left(\frac{1}{1-\gamma}\right) \right) - 1 + h_b \left[\frac{D}{X} \left(E\left(\frac{1}{1-\gamma}\right) \right) + \frac{1}{2} (1 - E(\gamma)) \right] - \lambda p}}$$

For $\lambda = 0$ the optimal order quantity of case (2) as follows

$$q^* = \sqrt{\frac{E\left(\frac{1}{1-\gamma}\right) D\left[\frac{K}{n} + \frac{A}{n} + V + \frac{D}{n} \pi \sigma [I] \psi(k)\right]}{\frac{h_v}{2} \left[n \left(1 - \frac{D}{P} \left(E\left(\frac{1}{1-\gamma}\right) \right) \right) \right] + \frac{2D}{P} \left(E\left(\frac{1}{1-\gamma}\right) \right) - 1 + h_b \left[\frac{D}{X} \left(E\left(\frac{1}{1-\gamma}\right) \right) + \frac{1}{2} (1 - E(\gamma)) \right]}}$$

For $\lambda \neq 0$, the optimal value of q and λ expressed as follows,

$$\lambda^* = \frac{h_v}{2p} \left[n \left(1 - \frac{D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) \right) + \frac{2D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) - 1 \right] + \frac{DK}{npq^2} \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{h_b}{p} \left[\frac{D}{X} \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{1}{2} [1 - E(\gamma)] \right] + \frac{DA}{npq^2} \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{DV}{pq^2} \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{D^2}{npq} \pi \sigma [I] \psi(k) \left(E \left(\frac{1}{1-\gamma} \right) \right)$$

$$q^* = \sqrt{\frac{E \left(\frac{1}{1-\gamma} \right) D \left[\frac{K}{n} + \frac{A}{n} + V + \frac{D}{n} \pi \sigma [I] \psi(k) \right]}{\frac{h_v}{2} \left[n \left(1 - \frac{D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) \right) + \frac{2D}{P} \left(E \left(\frac{1}{1-\gamma} \right) \right) - 1 \right] + h_b \left[\frac{D}{X} \left(E \left(\frac{1}{1-\gamma} \right) \right) + \frac{1}{2} (1 - E(\gamma)) \right] - \lambda p}}$$

4. NUMERICAL EXAMPLE:

In this section, illustrates the application of the proposed model by giving the numerical value to the each parameter.

$D=1000$ units/year, $P=300$ units/year, $A=\$100$ /order, $K=\$400$ /setup, $V=\$50$ /delivery, $h_v = \$6$ /unit/year, $h_b=\$10$ /unit/year, $X= 7000$ unit/year, $s=\$0.5$ /unit, $w=\$2$ /unit, $a_1=\$2,500$ /unit, $a_2 =\$0.0004$ /unit, $\theta=\$0.1$ /year, $\sigma_o^2=0.01$, $\sigma_L^2=0.005$, $b=0.0035$, $\delta=0.04$, $B = \$500$ /year, $p =\$5$ /unit, $C_a=\$10$ /unit, $C_r=\$5$ /unit, $m_1 =0.01$, $m_2=0.02$, $n=2$ /year, $\pi=\$5$ /unit, $\sigma(I)=0.072$, $\lambda =0.19$, $k=1.56$

The optimal order quantity and the expected total cost of Case (1) and Case (2) is described as follows

Case 1: $\lambda = 0$, $q^* = \$226$, $ETC(q, \lambda) = \$5642$

$\lambda \neq 0$, $q^* = \$215$, $ETC(q, \lambda) = \$5532$

Case 2: $\lambda = 0$, $q^* = \$217$, $ETC(q, \lambda) = \$5601$

$\lambda \neq 0$, $q^* = \$210$, $ETC(q, \lambda) = \$5546$

5. CONCLUSION:

This proposed model discusses about the production inventory model with imperfect production processes along with the screening errors. The investment cost is used to reduce the lead time variability, the budget capacity constraint is used for buyer to purchase product with in the available budget, Budget capacity constraint and Karush –Khun –Tucker method both these strategies used in this proposed model to satisfies the objective of the system that is to reduce the total cost of the supply chain

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