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Cost Interpretation Of Fuzzy Queuing Model Using Ranking Based On Statistical Beta Distribution

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Abstract:
This paper intends to explore the cost analysis of Multi server fuzzy queuing model with finite capacity. Ranking methods play a remarkable role in the fuzzy numbers system for defuzzification. The prime idea is to convert the fuzzy arrival rate and service rate into crisp values by applying ranking based on the mean value of statistical beta distribution. It facilitates an algorithm for ranking fuzzy numbers considering the crisp ranking system on R. The significant measures and the expected total cost of a given fuzzy queuing model are computed for different number of servers.

Keywords:Trapezoidal fuzzy number, M/M/s/K Queuing model, Performance Measures, Service cost, Waiting cost, Total Expected cost, Statistical Beta Distribution, Fuzzy Ranking.

1. INTRODUCTION

Queuing Theory is a mathematical analysis of queues and waiting times in a stochastic systems. It is extremely useful in predicting and evaluating systems. There are service costs and waiting costs. Both of the costs conflict with each other.
The service costs increase in several servers while the waiting cost decreases with an increase in servers. The optimization of the queuing system is to minimize the total cost for balancing service and waiting for the cost to run a system effectively.
Fuzzy Queuing Models have been analyzed by notable researchers like R. J. Li and E. S. Lee, S. P. Chen and J.J.Buckley. Many authors have investigated fuzzy numbers and different ranking techniques to evaluate the performance measures of fuzzy queues. Fuzzy Ranking plays an significant in optimization technique, Control Theory, Decision analysis, Artificial Intelligence etc.,
This paper discussesthe cost interpretation of M/M/s/K Queuing Model in a fuzzy environment. The inter-arrival rate and service rate are trapezoidal fuzzy numbers. The mean value of the statistical beta distribution is utilized for ranking fuzzy numbers. The significant measures and total expected cost computed for the given fuzzy queuing system.

2. PRELIMINARIES

Definition: 2.1
Let U be a classical set or a Universe. A fuzzy subset $\tilde{C}$ in U is defined by a membership function$\eta_{\tilde{C}}$: U→ [0,1], where $\eta_{\tilde{C}}(x)$ is the degree of x in $\tilde{C}$ and it is represented as $\tilde{C}$= {(x, $\eta_{\tilde{C}}(x)$): $x \in U$ }.

Definition: 2.2
A fuzzy number $\tilde{c}$ is said to be triangular fuzzy number if and only if there exists three real numbers $c_1 \le c_2 \le c_3$, such that:

$$\eta_{\tilde{c}}(x) = \begin{cases} \dfrac{x-c_1}{c_2-c_1}, & c_1 \le x \le c_2 \\ 1, & x = c_2 \\ \dfrac{x-c_3}{c_2-c_3}, & c_2 \le x \le c_3 \\ 0, & otherwise \end{cases}$$

Denoted by $\tilde{c} = (c_1, c_2, c_3)$ or $\tilde{c} = (c_1/c_2/c_3)$

Definition: 2.3
A fuzzy number $\tilde{c}$ is said to be trapezoidal fuzzy number if and only if there exists real numbers, $c_1 \le c_2 \le c_3 \le c_4$, such that:

$$\eta_{\tilde{c}}(x) = \begin{cases} \dfrac{x-c_1}{c_2-c_1}, & c_1 \le x \le c_2 \\ 1, & c_2 \le x \le c_3 \\ \dfrac{x-c_4}{c_3-c_4}, & c_3 \le x \le c_4 \\ 0, & otherwise \end{cases}$$

Denoted by $\tilde{c} = (c_1, c_2, c_3, c_4)$ or $\tilde{c} = (c_1/c_2/c_3/c_4)$.

3. MODEL DESCRIPTION:

We consider the Multi-server finite capacity fuzzy Queuing Model. The following Considerations are made in the given fuzzy queuing system.

a) Arrivals of customers befall successively in line with Poisson probability distribution with fuzzy inter arrival rate $\tilde{\lambda}$.
b) There is no balking or reneging in the queuing system.
c) Service time adheres to an exponential distribution with fuzzy service rate $\tilde{\mu}$.
d) Queue Discipline is First Come First Served.
e) Service provided through a Multichannel.
f) The capacity of the system will be finite, noted as ‘k’.
g) The service rate is independent of queue length.
h) The average arrival rate is smaller than the average service rate.

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3.1 Significant Measures:

For the (M/M/s):(K/FCFS) Queuing model,

$$\lambda_n = \begin{cases} \lambda & for \;\; n = 0,1,2,3,4....k-1 \\ 0 & for \;\; n = k, k+1, k+2, ... \end{cases}$$

and $\mu_n = \begin{cases} n\mu & for \;\; n = 0,1,2,3,4....s-1 \\ s\mu & for \;\; n = s, s+1, s+2, ... \end{cases}$

For 1 < s < k,

(i) The probability of having zero customers in the system

$$\pi_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \sum_{n=s}^{k} \left(\frac{\lambda}{\mu} \right)^n \frac{1}{s!s^{n-s}} \right]^{-1}$$

(ii) The probability of having k customers in the system

$$\pi_n = \begin{cases} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n \pi_0 & , \; 0 \le n \le s \\ \left(\frac{\lambda}{\mu} \right)^n \frac{1}{s!s^{n-s}} \pi_0 & , \; s < n \le k \\ 0 & , \; otherwise \end{cases}$$

(iii) Expected Number of customers in the System

$$N_s = \sum_{n=0}^{k} n \pi_n$$

(iv) Expected Number of customers in the Queue

$$N_q = \sum_{n=s}^{k} (n-s) \pi_n$$

(v) Expected waiting time of customers in the system

$$T_s = \frac{N_s}{\lambda}$$

(vi) Expected waiting time of customers in the Queue

$$T_q = \frac{N_q}{\tilde{\lambda}}$$

Defuzzifying and Ranking Fuzzy Numbers Using Beta Distribution:

The proposed distribution is the only reason for the equality of left spread, right spread, and mode of beta distribution with their corresponding values in fuzzy numbers with in (0,1) interval. Consider a fuzzy number $\tilde{A}$, the mean value of its corresponding Beta distribution in its domain is considered as the crisp real number corresponding to $\tilde{A}$, which based on crisp ranking system on R.

For Triangular Fuzzy Number

Let $\tilde{A} = (l, m, u)$, the ranking function and corresponding crisp real number $\mu_{\tilde{A}}$ is defined by

$$R(\tilde{A}) = \mu_{\tilde{A}} = \frac{l+m+u}{3}$$

For Trapezoidal Fuzzy Number

Let $\tilde{A} = (l, m_1, m_2, u)$, the ranking function and corresponding crisp real number $\mu_{\tilde{A}}$ is defined by

$$R(\tilde{A}) = \mu_{\tilde{A}} = \frac{2l + 7m_1 + 7m_2 + 2u}{18}$$

Numerical Illustration:

Here our consideration is Muli-server finite capacity fuzzy queuing model. The customers are entered by a fuzzy arrival rate $\tilde{\lambda}$ and be in service by a fuzzy service rate $\tilde{\mu}$ with k = 10.
Let us contemplate the arrival rate and service rate are Trapezoidal fuzzy numbers.
Let $\tilde{\lambda}$ = [4,5,6,7] and $\tilde{\mu}$ = [10,11,12,13] per unit time respectively.
The membership function of $\tilde{\lambda}$ is constructed by

$$\eta_{\tilde{\lambda}}(x) = \begin{cases} \frac{(x-4)}{(5-4)}, & 4 \le x \le 5 \\ 1, & 5 \le x \le 6 \\ \frac{(x-7)}{(6-7)}, & 6 \le x \le 7 \\ 0, & otherwise \end{cases}$$

$\tilde{\mu}$

Also, We write the membership function of in the similar fashion. Subsequently, we applySatistical beta distribution for ranking trapezoidal fuzzy numbers $\tilde{\lambda}$ & $\tilde{\mu}$, we get

$$R(\tilde{\lambda}) = R(4,5,6,7) = \frac{2(4)+7(5)+7(6)+2(7)}{18} = 5.5$$

$$R(\tilde{\mu}) = R(10,11,12,13) = \frac{2(10)+7(11)+7(12)+2(13)}{18} = 11.5$$

Recourse of this fuzzy ranking, We compute the performance measures of described queuing model and tabulate the values with respect to 's'(the number of servers).

Performance MeasuresWith respect to 's'

s	π_0	N_s	N_q	T_s	T_q
2	0.614035	0.507266	0.029005	0.092230	0.005274
3	0.619452	0.480806	0.002545	0.087419	0.000463
4	0.619834	0.478469	0.000208	0.086994	0.000038
5	0.619859	0.478277	0.000016	0.086959	0.000003
6	0.619860	0.478263	0.000002	0.0869569	0.0000004
7	0.619861	0.478262	0.000001	0.0869567	0.0000002

Expected Total cost:

Putting good queue management systems helps to automate the queuing process whilst improving service, safety, and gaining customer loyalty. It is difficult to measure the true cost of walkaways to a business, but putting good queuing systems in place can help reduce the possibility of people leaving the queue.

E(TC) = E(SC)+E(WC)

= $s.C_S + C_w(\tilde{\lambda}.T_s)$

Where

E(TC) : Expected total cost

E(SC) : Cost of providing service

E(WC) : Expected cost of waiting

C_s : Service cost of each server

C_w : Waiting cost of each customer

T_s : Average time an arrival spends in the system

$\tilde{\lambda}$: Fuzzy arrival rate

s : Number of servers

Consider the waiting cost per unit time of each customer C_w is Rs.100 and the service cost per unit time of each server is Rs.100. The total expected cost, waiting cost and service cost computed and tabulated below.

Total Expected cost

S	N_s	**E(SC)**	**E(WC)**	**E(TC)**
2	0.507266	200	50.7266	250.7266
3	0.480806	300	48.0806	348.0806
4	0.478469	400	47.8469	447.8469
5	0.478277	500	47.8277	547.8277
6	0.478263	600	47.8263	647.8263
7	0.478262	700	47.8262	747.8262

Case(i):

Fixed Waiting cost V_s Variant Service cost

s	N_s	**E(WC)**	**s*** C_S	**s*** C_S	**s*** C_S	**s*** C_S	**s*** C_S
2	0.507266	50.7266	60	100	140	180	200
3	0.480806	48.0806	90	150	210	270	300
4	0.478469	47.8469	120	200	280	360	400
5	0.478277	47.8277	150	250	350	450	500
6	0.478263	47.8263	180	300	420	540	600
7	0.478262	47.8262	210	350	490	630	700

With increase in service capacity, there is a reduction in the number of customers in the line and their waiting times, which thereby decreases the queuing cost. With a gradual increase in the service cost of each server the expected service cost increases gradually. When the expected service cost is less than the expected waiting cost of the customer is also less but more number of servers are needed.

Case(ii):

Fixed Service cost V_sVariant Waiting cost

s	N_s	**E(SC)**	N_s ***C**$_w$	N_s ***C**$_w$	N_s ***C**$_w$	N_s ***C**$_w$
2	0.507266	200	71.0172	91.3079	111.5985	131.8892

3	0.480806	300	67.3128	86.5451	105.7773	125.0096
4	0.478469	400	66.9857	86.1244	105.2632	124.4019
5	0.478277	500	66.9588	86.0899	105.2209	124.3520
6	0.478263	600	66.9568	86.0873	105.2179	124.3484
7	0.478262	700	66.9567	86.0872	105.2176	124.3481

If the expected waiting cost of the system increases gradually, then the service cost also increases gradually. Hence, service provider has to provide less number of servers to optimize the system.The optimal total cost is found at the intersection between the service capacity and waiting line curves.

4. CONCLUSION

Thus this paper highlights the significant measures, the total expected cost of M/M/s/K fuzzy queuing model and ranking trapezoidal fuzzy numbers on the basis of statistical beta distribution. The projected ranking methods can rank fuzzy numbers in practical reality. This method not only produces crisp results, but it also does so with more precision than most other methods.

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