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Evaluation Of Performance Measures Of M/M/C Queue With Synchronous Vacation For Some Servers Using Pentagonal Fuzzy Number

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Abstract : This paper analyzes a multi-server (M/M/C) queue with synchronous vacation policy in fuzzy. Approximate method of extension namely DSW algorithm is used to define membership functions of the performance measures. It is based on the α-cut representation of fuzzy sets in a standard interval analysis.Arrivalrate,servicerate,vacationratearepentagonalfuzzynumbers.Performancemeasuressuch as fuzzy expected number of customers in queue, fuzzy expected waiting time of a customer in queue have been obtained. Numerical example is illustrated to test the feasibility of themodel.

Keywords: Fuzzy set theory,multiple vacation ,synchronous vacation , DSW Algorithm

1. **INTRODUCTION**

Queueing theory is a branch of applied probability theory. A queue is a waiting line of customers which demands service from a service counter and it is formed when service is not given immediately. Queueing theory was introduced by A.K Erlang [1]. The main purpose of the analysis of queueing systems is to understand the behavior of their underlying processes so that informed and intelligent decisions can be made in their organization.

Vacation queueing model was originated in the 1970's as an extension of the classical queueing theory. In a queueing system, the server is permitted to take vacations .Here vacations means maintenance, repairs, supplementary jobs etc. Also the queueing system will be functioning effectively in the long run whenever the servers are permitted to take vacations. In general, a server may go for vacation if there is nocustomerstoserveinqueue.Whenevertheserverreturnto the system after completing vacation, he start service ifthere isanywaitingcustomer.Otherwisehegoforanothervacation is known as multiple vacation. In multi-server queueing system, most of the vacation models were analyzed with all servers may be onvacation.

In practical life, the management of some organization wants to keep at least some servers always on duty (in either busy or idle status). In this paper, we consider a vacation model with some servers may be on vacation. Synchronousvacationmeansalleligibleserverstakevacationtogetherand return to the system at the sametime.

Levyand Leonard Kleinrock [2] investigated aqueue with starter and queue with a vacations. A single vacation Model G/M/1/K with N threshold policy is studied by J.C Keetal [3]. Choudhry [4] analyzed a batch arrival queue with a vacation time under single vacation policy. An M^x/G/1 queue with vacation time was discussed by Baba[5]. A short survey on recent developments in vacation queueingmodels was presented by Ke[6] .Batch Arrival Retrial Queue with General Vacation Time was presented by Senthil Kumar and Arumuganathan [7].Ayappanet.al[8] studied the impact of negative arrivals in single server fixed batch service queueing system with multiple vacations. A vacation queue with exceptional service for the customers is discussed by Kalyanaraman and Pazhani Bala Murugan[9].K.CMadanet.al[10] investigatedatwoserverqueuewithBernoullischedulesand a single vacationpolicy.

In practical,the input data such as arrival rate, service rate and vacation are uncertainly known. Uncertainty is resolved by using fuzzy set theory. The classical queuing model will have more application if it is expressed using fuzzy models. Fuzzy Logic was originated in 1965 by Zadeh[11]. Fuzzy queuing models have been developed by such researchers like Li and Lee [12], Buckley [13], Negi and Lee [14] are analyzed fuzzy queues using Zadeh'sextension principle . Application of fuzzy logic was discussed by Klir [15]. The theory of fuzzy subset is introduced by Kaufmann[16]. Zimmermann[17] developed fuzzy set theory and applications. Shanmugasundaram and Venkatesh [22] discussed the multiserver fuzzy queueing model using DSW algorithm.

2. DESCRIPTION OF THEMODEL

We consider M/M/c queue with synchronous vacation(SY) of some servers.Arrival follows poisson process withrateλ,servicetimefollowsexponentialdistributionwith parameterγ.Weassumethatthevacationtimefollowsanexponential distribution with parameter v. In this system, only asubsetofserversareallowedtotakevacationstogether.By introducing a control parameter d ($1 \leq d \leq c$) , the policy of the model is described as follows: at a service completion ≤ ≤ instant,ifthenumberofidleserversreachesd,thesedservers startvacationsimultaneously,andtheremainingc-dservers will be on duty(either busy or idle). Two cases raised when 'd'serversreturntothesystemaftercompletingavacation;

(i) if the number of customers does not exceed c - d, these d servers are allowed to take another vacation known as multiple vacation (MV).(ii) if number of customers exceedc-d,then these d servers start service for waiting customers(if j >c - d customers in the system), then two possibilitiesare

(i) if c - d <j c, then j - c + d returning server startservice andc-jserverbecomeidleand(ii)ifj>c,thenallreturning serverstartservingcustomersandj-ccustomersarewaiting in theline.

The above policy of the system is said to be semi-exhaustive.ThesystemisrepresentedbyM/M/C(SY,MV,d). The queue discipline is 'First come First Serve'. In this paper, we described M/M/C (SY,MV,d) queueing model under fuzzyenvironment.

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The crisp results for mean queue length E(L_q)and mean waiting time in the queue E(W_q) of the above model were presented by Tian and Zhang [24] .

$$E[L_q]=\frac{\rho}{1-\rho}+\frac{1}{\sigma}\frac{\upsilon r}{c\gamma(1-r)^3}\beta_{c_0}$$

$$E[W_q]=\frac{1}{c\gamma(1-\rho)}+\frac{1}{\sigma}\frac{\upsilon r^2}{c\gamma(1-r)^3}\beta_{c_0}\frac{1}{c\gamma}$$

Where

$$\sigma=\beta_{c1}+\frac{1}{\sigma}\frac{\upsilon r}{c\gamma(1-r)^2}\beta_{c_0}$$

$$\beta_{c_0}=\frac{1}{(c-d)!}(\frac{\lambda}{\gamma})^{c-d}r^d$$

$$\beta_{c_1}=\frac{1}{c!}(\frac{\lambda}{\gamma})^c\frac{\gamma r}{\lambda(1-r)}[1+\frac{1}{(c-d)!}\sum_{i=1}^{d-1}(c-d+i)(\frac{r\gamma}{\lambda})^i]$$

The stability condition for this model is

$$\rho=\frac{\lambda}{c\gamma}<1$$

Probability that all the servers are busy is $\frac{\sigma}{1-\rho}$

3. INTERVAL ANALYSIS ARITHMETIC

Let I_1 and I_2 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds. Define a general arithmetic property with the symbol * where $*=[+,-,\times,\div]$ symbolically the operation.

$I_1 *$ I_2=[a,b]*[c,d]representsanotherinterval.The interval calculation depends on the magnitudes and signs of the elements a, b, c and d.

$[a,b]+[c,d]=[a+c,b+d]$

$[a,b]-[c,d]=[a-d,b-c]$

$[a, b] \times [c, d] = [min(ac, ad, bc, bd), max(ac, ad, bc, bd)]$

$[a, b] \div [c, d] = [a, b] \times [\frac{1}{d},\frac{1}{c})$

where ac, ad, bc, bd are arithmetic products and

interval calculation depends on the magnitudes and signs of the elements a, b, c and d.

4. DSWALGORITHM

Any continuous membership function can be represented by a continuous sweep of α -cut interm from α = 0 to α = 1. It uses the full a-cut intervals in a standard interval analysis. The DSW algorithm [14] consists of the following steps:

(i) Select a α-cut value where $0 \le \alpha \le 1$.

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(ii) Findtheintervalsintheinputmembershipfunctions that correspond to this α.

(iii) Using standard binary interval operations, compute the interval for the output membership function for the se lectedα-cutlevel.

(iv) Repeat steps (i) to (iii) for different values of α to complete a α-cut representation of thesolution.

5. SOLUTIONPROCEDURE

The queueing model (M/M/C)(SY,MV,d) is described by fuzzy set theory. The arrival rate, service rate and vacation rate are uncertain parameters and so they aretrapezoidal fuzzynumbersrepresentedby $\tilde{\lambda}, \tilde{\upsilon}, \tilde{\gamma}$ .Wedefinethe following fuzzy sets.

$\tilde{\lambda} = \{(x, \mu_{\tilde{\lambda}}(x)),\ x \in X\}$

$\mu = \{(s, \mu_{\tilde{\mu}}(s)),\ s \in S\}$

$\upsilon = \{(v, \mu_{\upsilon}(v)),\ v \in V\}$

Where X,S,V are crisp universal set of arrival rate, service rate, vacation rate respectively.

Themembershipfunctionofarrivalrate, servicerate,vacation rate are given asfollows.

$$\mu_{\tilde{\lambda}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & if\ a_1 \le x \le a_2 \\ \frac{x-a_2}{a_3-a_2} & if\ a_2 \le x \le a_3 \\ 1 & if\ x = a_3 \\ \frac{a_4-x}{a_4-a_3} & if\ a_3 \le x \le a_4 \\ \frac{a_5-x}{a_5-a_4} & if\ a_5 \le x \le a_4 \end{cases}$$

$$\mu_{\gamma}(s) = \begin{cases} \frac{s-b_1}{b_2-b_1} & if\ b_1 \le s \le b_2 \\ \frac{s-b_2}{b_3-b_2} & if\ b_2 \le s \le b_3 \\ 1 & if\ s = b_3 \\ \frac{b_4-s}{b_4-b_3} & if\ b_3 \le s \le b_4 \\ \frac{b_5-s}{b_5-b_4} & if\ b_5 \le s \le b_4 \end{cases}$$

$$\mu_{\upsilon}(v) = \begin{cases} \frac{v-c_1}{c_2-c_1} & if\ c_1 \le v \le c_2 \\ \frac{v-c_2}{c_3-c_2} & if\ c_2 \le v \le c_3 \\ 1 & if\ v = c_3 \\ \frac{c_4-v}{c_4-c_3} & if\ c_3 \le v \le c_4 \\ \frac{c_5-v}{c_5-c_4} & if\ c_5 \le v \le c_4 \end{cases}$$

6. NUMERICAL EXAMPLE

Suppose arrival,service,and vacation rates are pentagonal fuzzy numbers represented by $\tilde{\lambda}$ =[5 6 7 8 9] , $\tilde{\gamma}$ =[21 22 23 24 25] and $\tilde{\upsilon}$ =[1 8 15 22 29]and we take the number of servers (c) is 6 . We fix d=4 i.e) whenever the number of idle servers reaches four, then all the four servers will take vacation together.

The interval of confidence at possibility α level as [5+2α ,9-2α], [21+2α , 25-2α],[1+14α , 29-14α] .

$$E[L_q] = \frac{\rho}{1-\rho} + \frac{1}{\sigma}\frac{\upsilon r}{cs\ (1-r)^3}\ \beta_{c_0}$$

$$E[W_q] = \frac{1}{cs(1-\rho)} + \frac{1}{\sigma}\frac{\upsilon r^2}{cs\ (1-r)^3}\ \beta_{c_0}\ \frac{1}{cs}$$

Where $\rho = \frac{x}{c\,s}$

$$\sigma = \beta_{c1} + \frac{\upsilon r}{c\gamma\,(1-r)^2}\ \beta_{c_0}$$

$$\beta_{c_0} = \frac{1}{(c-d)!}\left(\frac{x}{s}\right)^{c-d} r^d$$

$$\beta_{c_1} = \frac{1}{c!}\left(\frac{\lambda}{\gamma}\right)^c \frac{\gamma\, r}{\lambda\,(1-r)}\ [1+\frac{1}{(c-d)!}\sum_{i=1}^{d-1}(c-d+i)\left(\frac{r\,s}{x}\right)^i\ ]$$

The stability condition for this model is

$$\rho = \frac{\lambda}{c\,\gamma} < 1$$

Probability that all the servers are busy is $\frac{\sigma}{1-\rho}$

Table:1 The α-cuts of $E[L_q]$ and $E[W_q]$

α	$E[L_q]$	$E[W_q]$
0.0	[0.0351, 251.7212]	[0.0085 , 1.1841]
0.1	[0.0387 , 96.6235]	[0.0085 , 0.3865]
0.2	[0.0444 , 47.3816]	[0.0084 , 0.0913]
0.3	[0.0541 , 26.6568]	[0.0083 , 0.1097]
0.4	[0.0708 , 15.9898]	[0.0083 , 0.0681]
0.5	[0.1000, 9.9266]	[0.0083 , 0.0446]
0.6	[1.1504, 6.2586]	[0.0083 , 0.0306]
0.7	[0.2359 , 3.9677]	[0.0866 , 0.0219]
0.8	[0.3784 , 2.5112]	[0.0090 , 0.0165]
0.9	[0.6115 , 1.5799]	[0.0097 , 0.0131]
1.0	[0.9863 , 0.9863]	[0.0110 , 0.0110]

With the help of Matlab , we perform α - cuts of arrivalrate and service rate and vacation rate and fuzzy expected number customers in queue and fuzzy expected waitingtime ofacustomerinqueueatelevendistinctαvalues:0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9, 1.0. Crisp intervals for $E[L_q]$ and $E[W_q]$ at different possibilistic α levels are presented in table.

The fuzzy expected number of customers in queue$\widetilde{L_q}$has two characteristics to be noted. First the support of $\widetilde{L_q}$ ranges from 0.0351 to 251.7212 . Although the expected number of customers in queue is fuzzy, it's not possible for its values to fall below 0.0351 or exceed 251.7212. Then, the α-cut at α=1 contains the values from 0.9863 to 0.9863 which are the most possible values for the expected number of customers in the queue.
The fuzzy expected waiting time $\widetilde{W_q}$ has two characteristics to be noted. First the support of $\widetilde{W_q}$ranges from 0.0085 to 1.1841. Although the expected waiting time is fuzzy, it's not possible for its values to fall below 0.0085 or exceed 1.1841. Then, the α-cut at α=1 contains the values from 0.0110 to 0.0110 which are the most possible values for the expected waiting time in the queue.

8. CONCLUSION

Here we have investigated a multi-server queueing model with synchronous vacation of some servers(SY,MV,d) policy in fuzzy environment.Whenever the inter arrival time, service time and vacation time are fuzzy variables, the performance measures such as expected number of customers in queue , expected waiting time in queue will be fuzzy.In the numerical example, we observed that as α increases, the lower limit of average queue length increases, and the upper limit decreases, same things happened in average waiting time of queue.
Vacation models plays an important role in telecommunication network planning and design. Also it is used in the areas of flexible manufacturing, production and inventory control and call centers.

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