

# Analyzing Various Graph Theory Applications Using Mathematical And Computational Intelligence Approach

<sup>1</sup>Dr.R.Vijaya, <sup>2</sup>Dr.K.Maheswari, <sup>3</sup>Utpal Saikia, <sup>4</sup>Dr.Shaik.Shakeer Basha, <sup>5</sup>Dr.Syed Khasim, <sup>6</sup>Dr. R. Sreeparimala,

<sup>1</sup>Assistant Professor, PG & Research Dept. Computer Science, Arignar Anna Govt. Arts College for Women , Walajapet - 632 513 , Ranipet Dist. , Tamil Nadu , India.

<sup>2</sup>Associate Professor, Department of Computer Science and Engineering, CMR Technical Campus, Kandlakoya (V), Medchal Road, Hyderabad 501401, Telangana,India.

<sup>3</sup>Assistant Professor, Department of Mathematics, Silapathar College, Dhemaji Assam-787059.

<sup>4</sup>Assistant Professor, Computer Science & Engineering, Avanthi Institute of Engineering and Technology, Gunthapally, Abdullahpurmet Mandal, Telangana-501512.

<sup>5</sup>Professor, Department of Computer Science & Engineering, Dr.Samuel George Institute of Engineering & Technology, Markapur, Prakasam Dt, Andhra Pradesh, 523316.

<sup>6</sup>Associate professor, Mathematics, Sri Eshwar College of Engineering, Coimbatore-641202.

## **Abstract**

*Graph theory is a part of mathematical analysis which studies the relationships between fundamental results in several fields with pure mathematics. The goal of this research is two - fold: first, to grasp the fundamental concepts of graph theory, second, to emphasise the importance of graph theory thru a practical case which was used as a framework investigation as well as character development of the structural brain system, similar to how machine learning can be used to apply models based on factors spatial information. Data pre - processing, associations, attributes, and techniques are some of the approaches used in this approach. The pictures from the Magnetic Resonance Imaging (MRI) device are used to demonstrate an automatic tool for performing a typical process. Pre-processing, graph creation for every area with various associations, mapping, essential extraction of features based on literature review, and lastly offering a collection of machine learning models which can give interpretable findings for clinicians or experts are all part of a process. This research will examine the most viable method of graph theory in numerous domains to emphasize the impact of graph theory. A summary of graph theory issues pertinent to their ideas and tactics is also included in this study.*

**Keywords:** Graph Theory, Applications, Computational Intelligence, Set Theory, Representations

## **1. INTRODUCTION**

When a theory is used in actual life, it will be more significant. Arithmetic modelling is the use of statistical methods or instruments to depict or simulate real-world problems. One such technique for representing real-world objects and activities called graph theory. Graphs have some of the most used patterns with both environmental & man-made structures. A graph is indeed a geometrical formal expression of vertex that connect pairings of vertex which is used to depict the connection amongst items. Graphs could be used to represent a variety of real concerns. In economic, industrial, ecological, & computer programming domains, they will be used to depict a variety of relationships underlying operation dynamics.

Along with its experience in a variety domain including such knowledge discovery and picture processing, communications & code technique, grouping & sequencing, and optimization techniques & operations, the graph idea has really become a core of engineering and innovation. Using graph theory to solve a fundamental condition is the same as estimating solutions to source of actual scenario. Graph theory is indeed a subfield of mathematics education that studies the properties and characteristics of graphs [1]. It shows the elements' interconnections. A few of the advantages of graph theory is that it provides a common framework for a range of issues. It just gives you graph techniques to solve this issue. The vertex or node indicates the objects throughout all domains wherein graph are employed for modelling, while the edge indicates the relationships among the objects. The Konignberg bridging challenge is where graph theory begins. The answer to some well conundrum gave rise to the concept of Eulerian graph.

Euler examined this Konignberg bridges challenge & discovered a workable approach in 1736, when he published Euler's resolution to the Konigsberg bridging challenge, now known as the Eulerian graph [2]. Mobius proposed the full graphs with bipartite graph in 1840, and Kuratowski used leisure puzzles to show that they have been plane. Kirchhoff invented the concept of trees (a linked graphs having no loops) in 1845, and he is using graph concepts to estimate voltages and power within electronic systems. Guthrie created the well-known four-color dilemma in 1852. Later, in 1856, Hamilton studied polyhydra loops & came up with the concept of the Hamiltonian graphs via looking at journeys which visits specific places precisely only one time.

## 2. GRAPH THEORETIC NOTATIONS

It is required to be knowledgeable with all elementary concepts throughout the graph to get a strong understanding about graph theory. A graph is indeed an ordered pair  $G = (V, E)$  that contains a subset  $V$  comprising node vertex and a set  $E$  of edges that connect the node in  $V$ . Graphs get their name from the fact that they're being represented graphically, and this graphical depiction helps us grasp many of their characteristics. In graphic representations of graphs, nodes are represented by spots or tiny spheres. A graph's edge is composed of 2 node (e.g.,  $n_1, n_2$ ). Edges are usually represented graphically as curving or vertical/horizontal lines connecting the spots associated with the corresponding nodes. Points which sharing edges are mostly referred to as neighboring or neighbors. Occurrence to every one of the pair of nodes refers to such an edge which connects 2 node. Adjacent edge would be those who intersect at a specific layer. The vertex in Fig.1 were  $V_e = a, b, c, d, e$  as well as the edges were  $(a, b), (a, c), (a, d), (b, e), (c, d), (d, e)$ .

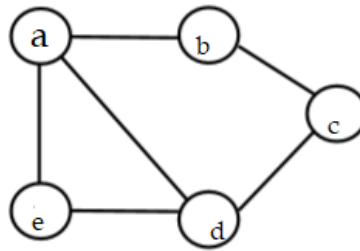


Figure 1: Graph

**Definition 1:** A bipartite graph is one in which the vertex set  $V_e(T)$  is made up of bipartitions X and Y, with the intersections of A and B being the empty set as well as the intersection with A or B being  $V_e(T)$ . A bipartite graph's corner subset is made up entirely of lines of one endpoint in A and the other in B. The nodes of a network shown in Figure 5 could be split into 2 groups:  $A = D,C$  and  $B = E,F$ . Set A nodes only connect with set A nodes, & conversely. Entities in same subset will not link together. As a result, it was a bipartite graph.

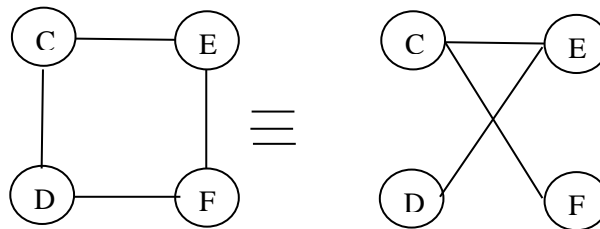


Figure 2: bipartite graph

**Definition 2:** A full bipartite graph was defined as a network in which each point of group A is connected to every point of group B, as shown in Figure 3

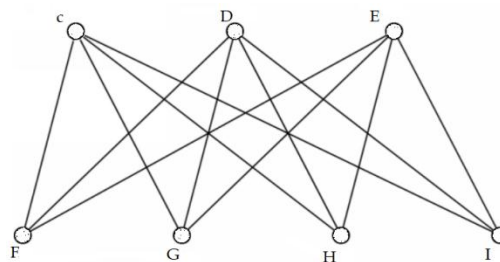


Figure 3: A complete bipartite

**Definition 3:** A sub-graph  $T_0$  of T, often known as  $T_0 T$ , is a graph where almost every edges & vertex within  $T_0$  is indeed present in T. Figure 4 shows how this works.

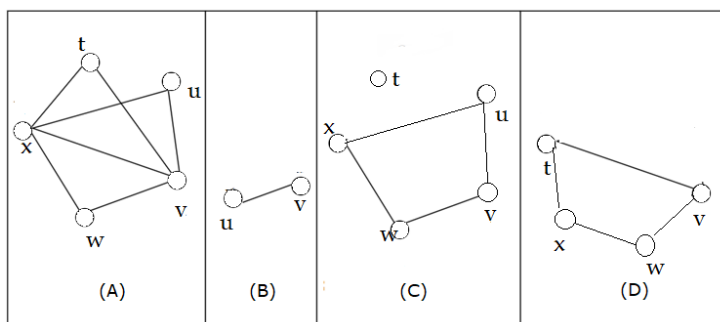


Figure 4: Graphs (B), (C) and (D) are subgraphs of the graph (A)

**Definition 4:** Assume that  $D \subseteq E$  is a sub-set of T's nodes group. The generated sub-graph  $T_0 = T[U]$  then is made up of node within D as well as solely these edges from T that have all these endpoints in D.

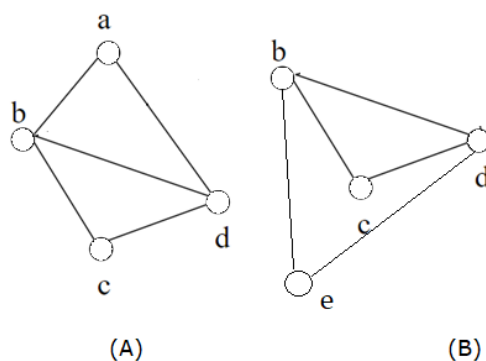


Figure 5: Graphs (B) is induced subgraphs of the graph (A)

**Definition 5:** A graph walk is also an alternate ordered set of nodes, with links displayed near to vertices acting as incidence edges to certain nodes. The number of edges in the array refers to the length of a path. If the path draws to a close in which it began, it is considered completed.

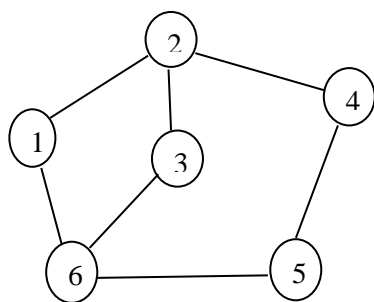


Figure 6: Example for walks in graph 1-2-3-6-5-4

**Definition 6:** A route inside a graph G is a sub-graph of T with  $V(\text{Path}) = i_0, i_1, i_2, \dots, i_k$  and  $E(\text{Path}) = i_0i_1, i_1i_2, \dots, i_{k-1}i_k$ , wherein  $i_0, i_1, \dots, i_k$  are unique graph vertices. The vertices  $i_0$  and  $i_k$  are known as Path's endpoints. The number of vertices throughout the pathway determines its length, as well as a shorthand method for denoting pathways has become an ordered set with vertices (e.g. Path =  $i_0i_1 \dots i_k$ ). Since no node were duplicated throughout the path, it is thus a route.

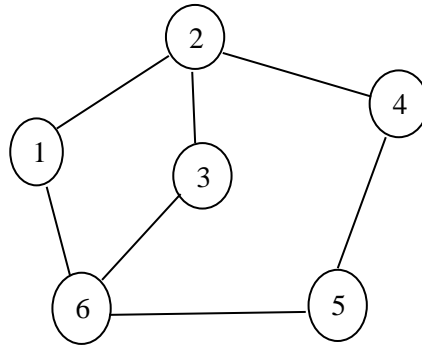


Figure 7: An example path in the graph 1-2-4-5-6

### 3. MATHEMATICAL REPRESENTATION OF THE GRAPH

The adjacency matrix is an arithmetical description for a graph. The adjacency matrix seems to be a 2D array which each square represents whether or not 2 nodes are connected. Whenever there is a link among the two nodes, cell include '1', & because there's not, cells contain '0.' Whenever self-edges really aren't permitted, diagonal cells have '0.' And for graph shown in Figure 1, Figure 8 illustrate the adjacency cell matrix.

Vertex ID	a	b	c	d	e
a	0	1	1	1	0
b	1	0	0	0	1
c	1	0	0	1	0
d	1	0	1	0	1
e	0	1	0	1	0

Figure 8: Adjacency Matrix for the Graph

Controlling Sets (CS) is a word that is used frequently in graph theory . A CS for a graph  $T=(V_e, E)$  is indeed a collection  $V_e'$  of  $V_e$  in which every vertex which isn't in  $V_e'$  is linked with at least single component of  $V_e'$  by an edge [4]. A controlling set of size 3 is shown in Figure 9, with the red node p, q, and r forming the controlling sets.

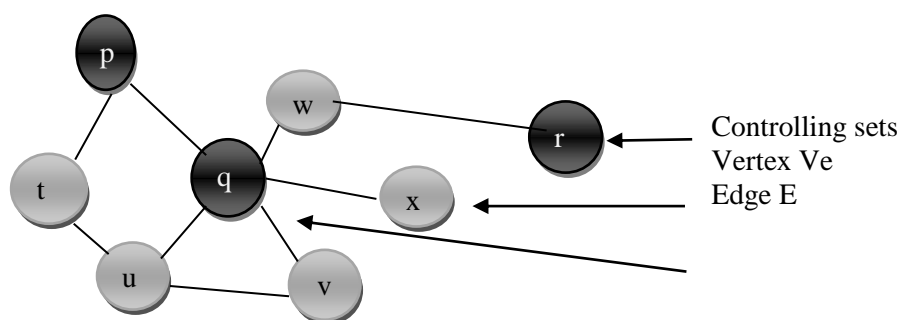


Figure 9: Dominating Set

A Minimal Dominant Set (MDS) is a Controlling Sets that has the shortest cardinality between all the CS of T. MDS of size 2 is depicted in Figure 17, with the dark lines forming MDS.

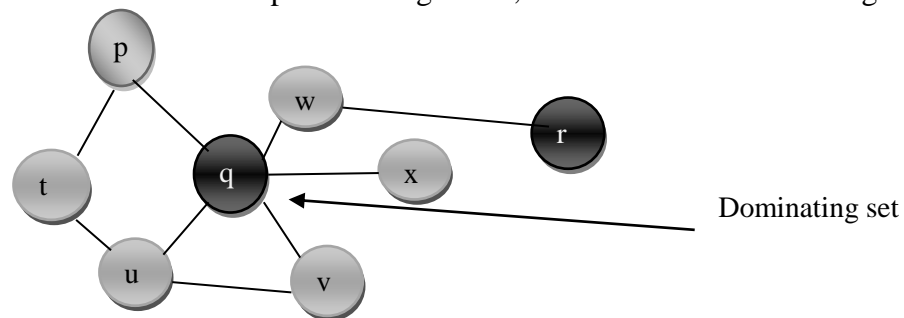


Figure 10: Minimum Dominating Set

Remember that even a node covering C is really a sub-set of the vertices in something like a simplified given Graph T that has at minimum 1 endpoint in C in each edge. As a result, in the dispute graph T, the goal is to find a min node overlap (it is an NP-complete problem). Lets take a glance at a particular instance of a Snps assembling dilemma from [8] and show how the nodes covers approach can help us solve it. A single system alteration within DNA is called a Single Nucleotide Polymerase reaction (SNPR, called "snip"). The most prevalent form of genomic variations in human chromosome is considered to be SNPs ( 91 percent of all human DNA polymorphisms).

This is how the SNPR Assembly Challenge is described. An SNPR assembly is indeed a trio (F, G, H), where  $F = f_1, \dots, f_n$  is a collection of n SNPRs,  $G = g_1, \dots, g_m$  is a subset of m segments, and H is a connection  $G: FG \rightarrow \{0, A, B\}$  that specifies if an SNPR  $f_i$  F does not appear on a fragmentation  $g_j$  G (marked by 0) and if it does, the non-zero number of  $f_i$  (A or B). 2 SNPs  $f_i$  and  $f_j$  are said to be in conflict if there are two fragments  $G_k$  and  $G_l$  with the same non-zero value in  $H(f_i, g_k), H(f_i, g_l), H(f_j, g_k), H(f_j, g_l)$  and the opposite non-zero value in  $H(f_j, g_l)$ . The objective is to end as few SNPs as feasible in order to remove any disputes. Figure 10 depicts the simple guidelines from [7]. It's worth noting because H is only specified for such a sub-set of FG derived from experimental data. For example, since  $H(f_1, g_2) = B, H(f_1, g_5) = B, H(f_5, g_2) = B, H(f_5, g_5) = A$ ,  $f_1$  and  $f_5$  are in dispute.  $(f_4, g_1) = A, H(f_4, g_3) = A, H(f_6, g_1) = B, H(f_6, g_3) = A$ , hence  $f_4$  &  $f_6$  are in dispute once more. Similarly, the table makes it simple to compute all pairings of opposing SNPRs. Figure 11 depicts the conflicts graph relating to this SNPR assembling difficulty.

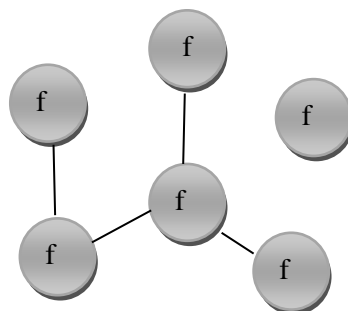


Figure 11: The conflict graph for SNP assembly problem

The minimum node coverage throughout the dispute graph are now determined using the nodes covering methodology. The no of nodes 6 is provided as an input, accompanied by adjacency matrix including its graph shown in Figure 12. If another nodes  $f_i$  &  $f_j$  use an edge throughout the dispute graph, the item in column  $j$  and row  $i$  of the adjacency matrix is one, otherwise it is zero.

0	0	0	0	1	0
0	0	0	1	0	0
0	0	0	0	0	0
0	1	0	0	1	1
1	0	0	1	0	0
0	0	0	1	0	0

Figure 12: The input for the vertex cover algorithm

Two unique minimum vertex coverings are discovered by the vertex software.

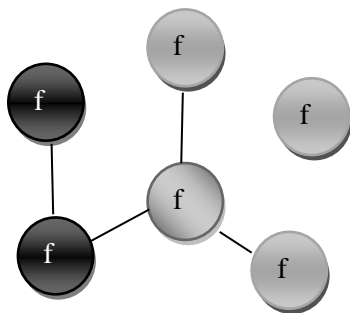


Figure 13: Minimum Vertex Cover:  $f_1, f_2$

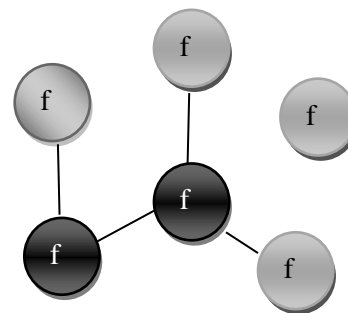


Figure 14: Minimum Vertex Cover:  $f_2, f_3$

As a result, whether removing  $f_1, f_2$  or removing  $f_2, f_3$  addresses the SNP assembling challenge. Figure15 illustrates an image of a graph demonstrate the html page. The title, images, & phrases are used to mark the borders.

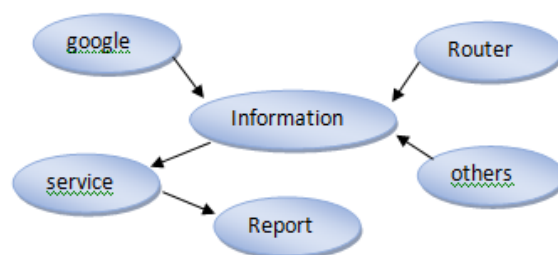


Figure 15: Web document – Graph representation

Whenever entities pass the border from one detector, i.e. the sensing area of one detector, then join the sensing zone of yet another detector, the preceding detector must correctly communicate this to the adjoining detector. The detecting strength is determined by the incidence rates among two adjacent detectors. The system is described as just an undirected weighted network  $T(DeT, ET, WT)$  wherein  $v$  corresponds to DeT and edge  $(u,v)$  belongs to ET, assuming that perhaps the device's transmit power is broad enough so the two neighbours can interact directly with one another. The detectors are represented by  $D$ , whereas the neighbours are represented by  $u,v$ .  $WT(u,v)$  is the EG's weighed edge of  $(u,v)$ . The idea of wraps was employed by the scholars.

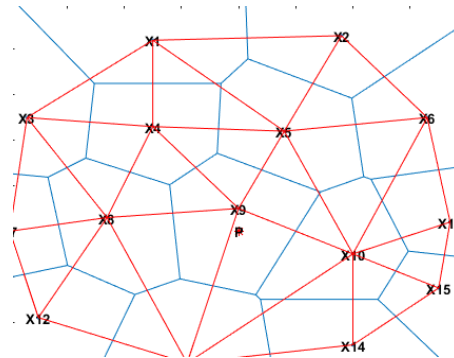


Figure 16: Voronoi diagram with regions

#### 4. RESULTS

Whenever the methodology was put into practice, a testing based on photos from either a prior migraines project was conducted.

Table 1 Graph theory results.

Char	Area	Controls N(M/SL)	SD(M/SL)	Sporadic Migraine N(M/SL)	SD(m/SL)	Medication abuse N(M/SL)	SD(M/SL)
X	91	1.0678/1.0602	0.234/0.342	1.09/1.098	0.013/0.023	1.079/1.054	0.014/0.04
	118	1.0655/1.093	0.032/0.477	1.08/1.089	0.013/0.003	1.066/1.075	0.012/0.031
M	91	1.004/1.045	0.003/0.008	1.095/0.323	0.008/0.002	1.006/1.045	0.001/0.007
	118	1.003/1.05	0.003/0.006	1.098/0.008	0.004/1.02	1.005/1.031	0.002/0.005



N	91	0.993/0.895	0.005/0.283	0.997/0.987	0.001/0.9	0.994/0.05	0.003/0.015
	118	0.994/0.865	0.003/0.012	0.8976/0.98	0.993/0.84	0.9953/0.884	0.003/0.014

From the data in Table 1, a classification with different classifiers, areas, and correlations was To accomplish so, a research containing 91 & 118 segments of AAL areas, necessarily coincide, and SL with either the following criteria:  $X=1$ ,  $M=1$ ,  $N=5$ , and  $Pr=0.06$  is incorporated into the technique. After 45 random iterations of a dataset, most values are standardized. The outcomes of a graph theory computations are shown in Table 1. With every one of the groups. The 3 characteristics average score & standard deviation were investigated. The findings are supplied for all of the parts (118), as well as 91 explanatory segments.

A categorization using several classifier, regions, and relationships were carried out using the data from Table 1. Table 2 indicates the results. The results of accuracy and precision are listed. The sensitivities of a classification determines its capacity to identify diseases in sick patients, whereas the specific determines its ability to recognize diseases with in lack of sickness. The new framework can handle the entire procedure, including acquiring fMRI pictures to delivering complete details that doctors or experts can understand. It is an effective algorithm in which the client merely inputs fMRI data then determine the best cartography and connections. To test this strategy, researchers looked at people who had migraines and were also drug addicts. The method does a thorough study and suggests various classifiers, some of which achieve 92.86 percent accuracy (Nn) and some others 86 percent (SVM). Different research using comparable machine learning algorithms in all the other diseases found chances of success of 76 to 88%, indicating that the suggested methodology has yielded satisfactory outcomes. The current discrepancies in classification outcomes can be attributed to a variety of factors, along with the kind of classification (supervised, uncontrolled, or partial-supervised) or the variation among classifier using same information that may achieve regional or global effectiveness.

Due to the random learning framework, some few classifier, such as NN, might produce diverse outputs. Increasing the amount of respondents inside each participating organization would allow for a more thorough investigation. These method is challenging for migraine sufferers since the noise produced by the MRI scanner causes individuals discomfort. Furthermore, one of the study results present limitations is the inability to employ automated classifier throughout conjunction with the entire map or a personal association. New atlases and relationships must be introduced in the order to improve the outcomes by allowing experts to study pathologies with a larger variety of factors.

Table 2 Classifiers.

Classifier		Success percentage	Connection percentage	AB %	Specification	Sensitivity
SVM	91	65.09/45.03	66.87/56.97	80/45	0.59/0.86	0.99/0.54

	118	67.98/68.09	78.99/56.00	60/30	0.094/0.65	0.65/0.64
K-means	91	89.00/66.98	87.99/67.95	40/70	0.87/0.77	1/1
	118	87.09/56.98	56.98/59.98	60/50	0.56/0.45	1/0.98
Knn	91	56.96/47.99	89.00/90.76	60/30	0.53/0.66	0.59/0.87
	118	57.99/52.87	48.98/65.98	0/80	0.76/0.66	0.65/0.73
AdaBoost	91	64.55/57.95	55.67/66.94	80/30	0.79/0.44	0.94/0.34
	118	64.44/46.96	63..75/66.56	60/40	0.93/0.56	0.44/0.64
Nn( 3 layers)	91	86.09/45.03	84.87/56.97	100/100	0.80/0.86	0.49/0.84
	118	83.98/68.09	74.99/56.00	100/20	0.93/0.65	0.85/0.74
LDA	91	90.43/67.94	87.44/95.99	100/40	0.64/0.334	0.77/0.97
	118	51.55/21.93	45.77/86.44	60/100	0.77/0.83	0.86/0.22

Graph theory-based numerical methods are simple to develop using common graphs methods, as well as the predictions were simple to identify thanks to the graph's links and routes. Nevertheless, because graph technologies primarily analyse comparatively home network knowledge, predictive accuracy is usually poor. Graph connection estimations are frequently biased in favour of connected dominating nodes in the cluster, resulting in poor rankings for novel medications and far less genomes. As a result, graph connectedness measurements are hardly used to estimate.

## 5. CONCLUSION

This study looked at several aspects of graph theory, like computer-assisted graph representations as well as graph-theoretic database systems like lists & matrices hierarchies. This study provides a better approach in representing and characterisation of a brain connection network, as well as machine learning in categorizing clusters based on factors retrieved from photographs, to emphasise the importance of graph theory. Data pre - processing, correlates, attributes, and techniques are some of the approaches used by this program. This research shows how an automated tool can be used to automate a systematic pattern utilizing MRI templates. Pre-processing, graph creation per topic using various connections, mapping, important extraction of features found in the literature, and lastly

offering a set of machine learning techniques that really can give interpretable findings for doctors or experts are all component of the method. This paper also discusses a most typical advantages of graph theory in numerous domains to emphasize the highlights of graph theory. A summary of graph theory difficulties pertinent to their ideas and tactics is also included in this study.

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