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A Stochastic Model on Time to Recruitment in a Single Grade Manpower System with Cluster of Exits

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Abstract* — In this paper, a single grade marketing organization is considered in which exits of personnel occur in clusters at random epochs. A suitable univariate recruitment policy based on cumulative damage process and shock model approach in reliability theory is used. Analytical result for variance of time to recruitment and expected total number of exits up to time to recruitment are obtained when (i) the number of clusters of exits forms a homogeneous Poisson process (ii) number of exits in each cluster forms a sequence of independent and identically distributed geometric random variables and (iii) the mandatory breakdown threshold is a positive integer valued random variable. The influence of nodal parameters on the performance measures is studied and relevant conclusions are presented.

***Keywords*—Single grade marketing organization, Cluster process, breakdown threshold, univariate recruitment policy, variance of time to recruitment and expected total number of exits up to time to recruitment.**

AMS Subject Classification: 60H30, 60K05, 90B70, 91D35

1. INTRODUCTION

In [2] and [3], the author, a pioneer in the study of manpower planning, has used appropriate statistical techniques and studied renewal theoretical models in manpower planning. Many researchers have studied the problem of time to recruitment for a single grade marketing organization, where exits occur as the effect of policy decisions such as revision of pay, targets etc. taken by this organization, by considering different assumptions on loss of manpower [[6], [11]], mandatory threshold for loss of manpower [[8], [13]] and inter-policy decision times [[5], [7]] using univariate policy of recruitment. In [12] the authors have obtained long run average cost for single grade marketing organization by assuming the threshold as a constant and survival time process as a geometric process using univariate policy of recruitment. In the above cited work involving policy decisions, separate epochs for policy decisions and exits are not considered, in spite of the fact that such separate epochs are more realistic. In this context, the author in [4] has studied the problem of time to recruitment for a single grade manpower system with different epochs for decisions and exits when the system has a mandatory breakdown threshold using a method different from the conventional method which uses Laplace transform. In [10] the author has studied the work in [4] when the system has optional and mandatory breakdown thresholds for different wastages. In [1] the author has obtained the variance of time to recruitment for a single grade manpower system, where depletion of manpower occurs due to (i) voluntary and involuntary exits of personnel from the organization and (ii) breaks taken by personnel working in the organization, under different assumptions on inter-exit times and inter-breaking decision times using univariate policy of recruitment. Recently, in [9] the authors have studied the problem of time to recruitment for a three graded marketing organization by considering two different cluster processes due to exits and transfers with new extended threshold for loss of manpower. In the present paper, the problem of time to recruitment for a single grade marketing organization in which exits of personnel occur in clusters at random epochs is discussed. A stochastic model is constructed with suitable assumptions on the cluster process, number of exits in any cluster and the mandatory breakdown threshold. A univariate recruitment policy is used and variance of time to recruitment, average number of clusters of exits up to time to recruitment and the total number of exits up to time to recruitment are determined for the present model. A numerical illustration with findings and conclusion are presented for a better understanding.

2. MODEL DESCRIPTION AND ANALYSIS

Consider a marketing organization consisting of one grade (referred as a system) in which clusters of exits of personnel takes place at random epochs in $(0, \infty)$. Let $B(t)$ be the number of clusters of exits in $(0\ t]$. It is assumed that $\{B(t);\ t \geq 0\}$ is a Poisson process with rate $b, b > 0$. Let X_i be the number of exits in the i^{th} cluster, $i = 1,2,3, \ldots$. It is assumed that $\{X_i\}_{i=1}^{\infty}$ form a sequence of independent random variables following decaptivated geometric distribution with parameter $p,\ 0 < p < 1$.

Let $C(t)$ be the total number of exits in $B(t)$ clusters in $(0\ t]$. Note that $C(t) = \sum_{i=1}^{B(t)} X_i$. Let Y be a positive integer valued random variable representing the mandatory random breakdown threshold level for the cumulative number of exits in the system with mean $E(Y)$ and variance $V(Y)$. It is assumed that Y is independent of $B(t)$, for all $t \geq 0$ and $X_i,\ i = 1,2,3, \ldots$. The univariate recruitment policy states that recruitment is done when the total number of exits exceeds the mandatory breakdown threshold. Let T be the random variable denoting the time to recruitment with mean $E(T)$ and variance $V(T)$.

3. MAIN RESULT

From the recruitment policy, the tail distribution of time to recruitment is given by

$$P[T > t] = P[C(t) \leq Y]. \quad (1)$$

Conditioning upon the event [Y=m] and noting that C(t) and Y are independent, we get

$$P[T > t] = \sum_{m=1}^{\infty} P[Y = m]\{P[C(t) = 0] + \sum_{n=1}^{m} P[C(t) = n)]\} \qquad (2)$$

Since $C(t)$ is a randomly indexed partial sum indexed by the Poisson process $\{B(t);\ t \geq 0\}$ with independent and identically distributed decaptivated geometric random variables as summands, we note that

$$P[C(t) = 0] = e^{-bt}$$

and

$$P[C(t) = n] = e^{-bt} \sum_{r=1}^{n} \frac{1}{r!} \binom{n-1}{r-1} (btp)^r (1-p)^{n-r},\ n = 1,2,3, \dots \qquad (3)$$

From (2) and (3), it can be shown that

$$P[T > t] = e^{-bt} + \sum_{m=1}^{\infty} P[Y = m] \sum_{n=1}^{m} (1-p)^n \sum_{r=1}^{n} \frac{1}{r!} \binom{n-1}{r-1} \left(\frac{bp}{1-p}\right)^r t^r e^{-bt}. \qquad (4)$$

We now determine $E(T)$ and $V(T)$ from (4).
We know that

$$E(T^r) = r \int_0^{\infty} t^{r-1} P[T > t] dt, r = 1,2,3, \dots \qquad (5)$$

From (4), (5) and taking $r - 1 = l$, it can be shown that

$$E(T) = \frac{1}{b} + \frac{1}{b} \sum_{m=1}^{\infty} P[Y = m] \sum_{n=1}^{m} (1-p)^n \left(\frac{p}{1-p}\right) \sum_{l=0}^{n-1} \binom{n-1}{l} \left(\frac{p}{1-p}\right)^l.$$

$$= \frac{1}{b} + \frac{1}{b} \sum_{m=1}^{\infty} P[Y = m] \sum_{n=1}^{m} (1-p)^n \left(\frac{p}{1-p}\right) \left(1 + \frac{p}{1-p}\right)^{n-1}.$$

$$E(T) = \frac{1}{b} + \frac{p}{b} \sum_{m=1}^{\infty} m\, P[Y = m]$$

i.e $E(T) = \frac{1}{b}\{1 + pE[Y]\}$. (6)

We next determine $E(T^2)$.
From (4) and (5), it can be shown that

$$E(T^2) = \frac{2}{b^2} + \frac{2}{b^2} \sum_{m=1}^{\infty} P[Y = m] \sum_{n=1}^{m} (1-p)^n \sum_{r=1}^{n} (r+1) \binom{n-1}{r-1} \left(\frac{p}{1-p}\right)^r. \qquad (7)$$

Consider $\sum_{r=1}^{n} (r+1) \binom{n-1}{r-1} \left(\frac{p}{1-p}\right)^r$.

Since $(r+1) = (r-1) + 2$ and $\binom{n}{r} = \frac{n}{r} \binom{n-1}{r-1}$, we get

$$\sum_{r=1}^{n} (r+1) \binom{n-1}{r-1} \left(\frac{p}{1-p}\right)^r = \sum_{r=1}^{n} (n-1) \binom{n-2}{r-2} \left(\frac{p}{1-p}\right)^r + 2\left(\frac{p}{1-p}\right) \sum_{r=1}^{n} \binom{n-1}{r-1} \left(\frac{p}{1-p}\right)^{r-1}. \qquad (8)$$

Consider $\sum_{r=1}^{n} (n-1) \binom{n-2}{r-2} \left(\frac{p}{1-p}\right)^r$.

Taking $r - 2 = j$, it can be shown that

$$\sum_{r=1}^{n} (n-1) \binom{n-2}{r-2} \left(\frac{p}{1-p}\right)^r = (n-1) \left(\frac{p}{1-p}\right)^2 \sum_{j=0}^{n-2} \binom{n-2}{j} \left(\frac{p}{1-p}\right)^j$$

i.e $\sum_{r=1}^{n} (n-1) \binom{n-2}{r-2} \left(\frac{p}{1-p}\right)^r = \frac{(n-1)p^2}{(1-p)^n}$. (9)

Since $\sum_{r=1}^{n}\binom{n-1}{r-1}\left(\frac{p}{1-p}\right)^{r-1} = \left[1+\frac{p}{1-p}\right]^{n-1}$,

$$2\left(\frac{p}{1-p}\right)\sum_{r=1}^{n}\binom{n-1}{r-1}\left(\frac{p}{1-p}\right)^{r-1} = \frac{2p}{(1-p)^{n}}. \qquad (10)$$

From (8), (9) and (10), we get

$$\sum_{r=1}^{n}(r+1)\binom{n-1}{r-1}\left(\frac{p}{1-p}\right)^{r} = \frac{(n-1)p^2+2p}{(1-p)^{n}}. \qquad (11)$$

From (7), (11) and on simplification, we get

$$E(T^2) = \frac{2}{b^2}+\frac{p^2}{b^2}\sum_{m=1}^{\infty}(m-1)m\,P[Y=m]+\frac{4p}{b^2}\sum_{m=1}^{\infty}m\,P[Y=m]$$

i.e $E(T^2) = \frac{1}{b^2}\{2+p^2[E(Y^2)-E(Y)]+4pE(Y)\}$. (12)

We now find $V(T)$.

From (6) and (12), it can be shown that

$$V(T) = \frac{1}{b^2}+\frac{p^2}{b^2}\{E(Y^2)-[E(Y)]^2\}++\left\{\frac{2p-p^2}{b^2}\right\}E(Y)$$

i.e $V(T) = \frac{1}{b^2}\{1+p^2V(Y)+p(2-p)E(Y)\}$. (13)

Equations (6) and (13) give the mean and variance of time to recruitment for the present model.

We next determine $E[C(T)]$, expected total number of exits up to time to recruitment. By Wald's Equation, expected total number of exits up to time to recruitment is given by

$$E[C(T)] = E(X)E[B(T)]. \qquad (14)$$

where $E(X) = \frac{1}{p}$ is the common mean of decaptivated geometric random variables X_i, $i = 1,2,3,\ldots$.

Since $\{B(t);\ t \geq 0\}$ is a Poisson process with rate b, we know that $E[B(t)] = bt$.

By conditioning upon T and using law of total probability we find that

$$E[B(T)] = b\,E[T]. \qquad (15)$$

From (6), (14) and (15), we get

$$E[C(T)] = \frac{1}{p}+E[Y]. \qquad (16)$$

Remark:

When the mandatory breakdown threshold Y is a constant threshold, analogous results for the mean and variance of time to recruitment can be obtained.

In fact, if $Y = c, c > 0$, it can be shown that

$$P[T>t] = e^{-bt}+\sum_{m=1}^{c}(1-p)^{m}\sum_{r=1}^{m}\frac{1}{r!}\binom{m-1}{r-1}\left(\frac{bp}{1-p}\right)^{r}t^{r}e^{-bt}.$$

$$E(T) = \frac{1}{b}\{1+pc\}.$$

$$E(T^2) = \frac{1}{b^2}\{2+4pc+c(c-1)p^2\}.$$

and

$$V(T) = \frac{1}{b^2}\{1+2pc-p^2c\}.$$

4. NUMERICAL ILLUSTRATION AND FINDING

'p' and 'b' are the nodal parameters in the performance measures $E(T)$ and $V(T)$. 'p' is the only nodal parameter in the performance measure $E[C(T)]$. It is palpable that $E(T)$ and $V(T)$ increase with 'p', keeping 'b', $E(Y)$ and $V(Y)$ fixed. But $E[C(T)]$ decreases when p increases, keeping $E(Y)$ fixed. These are also true logically. In fact, if 'p' increases, then 1/p, the average number of exits in each cluster decreases, which implies, the average time taken for exceeding the threshold level increases, but the average total number of exits decreases.

It is also clear that $E(T)$ and $V(T)$ decrease when 'b' increases, keeping 'p', $E(Y)$ and $V(Y)$ fixed. In fact, if 'b', the average number of clusters per unit time increases, then the average time taken for exceeding the threshold level decreases and hence the mean time to recruitment decreases.

The following table gives the effect of simultaneous variation of 'p' and 'b' on the performance measures $E(T)$ and $V(T)$ when $E(Y) = 150$ and $V(Y) = 250$.

Tab. 1

$\boldsymbol{b}$	$\boldsymbol{p}$	$\boldsymbol{E(T)}$	$\boldsymbol{V(T)}$
5	0.1	3.2000	1.2800
6	0.3	7.6667	2.7778
7	0.5	10.8571	3.5918
8	0.7	13.2500	4.0625
9	0.9	15.1111	4.3457

Since $E[C(T)]$ is independent of 'b', the following table gives effect of variation of 'p' on the performance measure $E[C(T)]$ when $E(Y) = 150$.

Tab. 2

$\boldsymbol{p}$	$\boldsymbol{E[C(T)]}$
0.1	160.0000
0.3	153.3333
0.5	152.0000
0.7	151.4286
0.9	151.1111

FINDING:

Tables (1) and (2) reflect the logical conclusions on the monotonicity of $E(T)$, $V(T)$ and $E[C(T)]$ when the respective nodal parameters increase.

5. CONCLUSION

The present work contributes to the existing literature in the sense that the model discussed in this paper is new in the context of considering (i) the number of clusters of exits forms a homogeneous Poisson process (ii) the number of exits in each cluster forms a sequence of independent and identically distributed geometric random variables and (iii) the mandatory random breakdown threshold as a positive integer valued random variable.

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6. REFERENCES

[1] P. Arokkia Saibe, "A study on time to recruitment in a single grade manpower system with two types of attrition generated by exits and breaks", Ph.D. thesis, Bharathidasan University, Tiruchirappalli, Tamil Nadu, India, 2018.

[2] D. J. Bartholomew, "The statistical approach to manpower planning", Statistician, 20, 3–26, 1971.

[3] D. J. Bartholomew, "Renewal theory models in manpower planning", Symposium Proceedings Series No. 8, The Institute of Mathematics and its Applications, 57–73, 1976a.

[4] A. Devi, "A probabilistic analysis on time to recruitment for a single grade manpower system with different epochs for decisions and exits", Ph.D. thesis, Bharathidasan University, Tiruchirappalli, Tamil Nadu, India, 2019.

[5] K. Elangovan, B. Esther Clara and A. Srinivasan, "A stochastic model for expected time to recruitment in a single grade manpower system with discrete loss of manpower and inter-decision times as order statistics", International Journal of Science Technology and Management, 5(6), 290–297, 2016.

[6] K. Elangovan, B. Esther Clara and A. Srinivasan, "Expected time to recruitment in single grade manpower system under correlated wastages", International Journal of Innovative Science, Engineering & Technology, 2(7), 79–85, 2015.

[7] R. Lalitha, A. Devi and A. Srinivasan, "A stochastic model on the time to recruitment for a single grade manpower system with attrition generated by a geometric process of inter-decision times", Journal of Engineering Computer and Applied Sciences, 3(7), 12–15, 2014.

[8] S. Parthasarathy and R. Vinoth, "Determination of expected time to recruitment in manpower planning", Recent Research in Science and Technology, 1(3), 147–150, 2009.

[9] S. Poornima and B. Esther Clara, "Recruitment in a three grade marketing organization with an extended threshold and loss of manpower forming cluster process", The International journal of analytical and experimental modal analysis, 11(9), 3960–3965, 2019.

[10] G. Ravichandran, "A study on time to recruitment in a single grade manpower system with different epochs for decisions and exits having optional and mandatory thresholds for different wastages", Ph.D. thesis, Bharathidasan University, Tiruchirappalli, Tamil Nadu, India, 2020.

[11] R. Sathiyamoorthi and R. Elangovan, "Shock model approach to determine the expected time for recruitment", Journal of Decision and Mathematika Sciences, India, 3(1-3), 67–78, 1998.

[12] V. Saavithri and A. Srinivasan, "Cost Analysis on Univariate Polices of Recruitment", International Journal of Management and Systems, 18(3), 249–264, 2002.

[13] P. Sudharani, K. Kalidass and K. P. Uma, "Stochastic model for a single grade system with three components of threshold and correlated inter-decision times", The International journal of Mathematics and Computer applications research, 6(1), 7–16, 2016.