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# Exploration Of Fuzzy Preemptive-Resume Priority Queuing System Using Robust Ranking Method

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***Abstract: The Preemptive-Resume priority of a fuzzy queuing model with Erlang service distribution is investigated in this work. The arrival rate and service rate are both triangular fuzzy numbers. The objective is to use a simple and conducive Robust ranking algorithm to turn the fuzzy arrival rate and service rate into crisp numbers. Furthermore, the model's efficacy is calculated for various number of phases.***

***Keywords: Fuzzy Queuing system, Triangular fuzzy number, Preemptive-Resume priority, Erlang service distribution, Characteristics Measures, Robust Ranking.***

## 1. INTRODUCTION:

Queuing Theory is a useful tool for analysing the common occurrence of standing in line. Real-life waiting line scenarios in today's hectic environment include priority considerations. Priorities are accepted favourably in certain cases, but in others, they take a long time to be accepted.

In various domains, such as call centers, telecommunications, mobile networks, manufacturing and production systems, priority queuing systems are essential. Here, Preemptive priority queuing systems are used when the arrival of a higher priority customer causes the service of a lower priority customer to be interrupted. In such cases, when the higher priority customer has been handled, service to the lower priority customer resumes.

Alan. C[2] was the first to propose a head-of-the line priority system with multiple levels of priority. Many scholars have designed and contributed priority queues in diverse contexts, including Harrison and Lee[15], Fumiaki[9], D. P. Gaver[4], S. Richa[13].

The absoluteness of fuzzy queuing models is higher than that of naturally occurring crisp queues. By expanding the queuing model to a fuzzy environment, the applications of the queuing model can be improved. Famous researchers such as R.J. Li and E.S. Lee [12], J.J. Buckley[8], R.S. Negi and E.S. Lee, S.P. Chen have discussed the queuing system in a fuzzy environment. Several researchers have looked into grading fuzzy numbers, including Jain, Cheng C. H[1], Wang Y.J and Lee H. S[16], R.R. Yager.

This paper is coordinated as follows: second segment gives some basic definitions of this research work. Third segment details a mathematical model which figures number of customers and waiting time. Fourth segment outlines the robust ranking technique to rank triangular fuzzy numbers. Fifth segment illustrates a numerical example. Sixth segment concludes the paper.

## 2. PRELIMINARIES:

**Definition: 2.1**

Let U be a classical set or a Universe. A fuzzy subset $\tilde{C}$ in U is defined by a membership function $\phi_{\tilde{C}}$: U $\rightarrow$ [0, 1], where $\phi_{\tilde{C}}(x)$ is the degree of x in $\tilde{C}$ and it is represented as $\tilde{C}=\{(x, \phi_{\tilde{C}}(x)) / x \in U\}$

**Definition: 2.2**

A fuzzy number $\tilde{C}$ is called triangular fuzzy number if its membership function is given by:

$$\phi_{\tilde{c}}(x)=\begin{cases} \dfrac{x-c_1}{c_2-c_1}, & c_1 \le x \le c_2 \\ 1, & x=c_2 \\ \dfrac{x-c_3}{c_2-c_3}, & c_2 \le x \le c_3 \\ 0, & otherwise \end{cases}$$

Denoted by $\tilde{c}=(c_1, c_2, c_3)$ or $\tilde{c}=(c_1 / c_2 / c_3)$

**Definition: 2.3**

The $\alpha$ - cut of a fuzzy number $\tilde{C}$ is defined as $\tilde{C}=\{x : \phi_{\tilde{C}}(x) \ge \alpha, \alpha \in [0,1]\}$

**Remark:**

**Operation of Triangular fuzzy number**

**(i)** Addition: $(c_1,c_2,c_3)+(d_1,d_2,d_3)=(c_1+d_1,c_2+d_2,c_3+d_3)$

**(ii)** Subtraction: $(c_1,c_2,c_3)-(d_1,d_2,d_3)=(c_1-d_3,c_2-d_2,c_3-d_1)$

## 3. FUZZY PREEMPTIVE-RESUME PRIORITY: MODEL DESCRIPTION

Preemptive priority queue systems are divided into two types: preemptive-resume and preemptive non-resume. When interruptions occur and are cleared, the proactive resume priority scenario is used. The service returns and continues from where it was interrupted. Preemptive non-resumption priority refers to a circumstance in which an interrupted service must restart from the beginning when it is restored.

We consider the single-server finite capacity fuzzy Queuing system with Preemptive-Resume priority. The first queue, represented by the random variable $X_1(t)$, is the $FM / FE_r / 1$ queuing system, which is unaffected by priority because customers in that category are served on a first come first served basis. The arrival rate $\tilde{\lambda}_1$ is Poisson, and the service rate $\tilde{\mu}_1$ is Erlang with Poisson Arrival See Time Average (PASTA) property. Customers in the first queue are referred to as having first priority. For the second queue $X_2(t)$, the arrival rate is Poisson, and the service duration is the sum of the customers' corresponding service durations and the busy period of interruptions. Customers in the second queue are known as second priority customers.

**Traffic Intensity for first and second priority in a Fuzzy Queuing system:**

$$\tilde{\rho}_1 = \frac{r\tilde{\lambda}_1}{\tilde{\mu}_1} \quad \text{and} \quad \tilde{\rho}_2 = \frac{r\tilde{\lambda}_2}{\tilde{\mu}_2(1-\tilde{\rho}_1)}$$

**In the first priority**,

The mean service time of the customers in $X_1(t)$ is $E(S_1)$ and the mean service time of the customer in service is $E(S)$. It is defined as follows:

$$E(S) = \frac{r+1}{2}.\frac{1}{\tilde{\mu}_1}$$

$$E(S_1) = \frac{r}{\tilde{\mu}_1}$$

Mean waiting time in the queue

$$E(w_q^{\,1}) = \frac{\tilde{\rho}_1}{(1-\tilde{\rho}_1)}.E(S)$$

$$= \frac{\tilde{\rho}_1}{(1-\tilde{\rho}_1)}\frac{r+1}{2\tilde{\mu}_1}$$

Average number of customers in the queue

$$E(L_q^{\,1}) = \tilde{\lambda}_1 E(W_q^{\,1})$$

$$= \frac{\tilde{\lambda}_1\tilde{\rho}_1}{(1-\tilde{\rho}_1)}\frac{r+1}{2\tilde{\mu}_1}$$

Mean waiting time in the system

$$E(W_s^{\,1}) = E(W_q^{\,1}) + E(s_1)$$

$$= \frac{\tilde{\rho}_1}{(1-\tilde{\rho}_1)}\frac{r+1}{2\tilde{\mu}_1} + \frac{r}{\tilde{\mu}_1}$$

Average number of customers in the system

$$E(L_s^{\,1}) = \tilde{\lambda}_1 E(W_s^{\,1})$$

$$= \frac{\tilde{\lambda}_1\tilde{\rho}_1}{(1-\tilde{\rho}_1)}\frac{r+1}{2\tilde{\mu}_1} + \frac{\tilde{\lambda}_1 r}{\tilde{\mu}_1}$$

**In the second priority,**

Let mean completion time of second priority is $E(C)$ and its second derivative is $E(C^2)$.

$$E(C) = E(S_2)\left[1+\tilde{\lambda}_1 E(B)\right]$$

$$= \frac{r}{\tilde{\mu}_2(1-\tilde{\rho}_1)}$$

Where $E(S_2)$ denotes mean service time and $E(B)$ denotes the mean busy period for the second priority queue.

$$E(C^2)=\frac{r(r+1)}{\tilde{\mu}_2^{\,2}}\cdot\frac{1}{(1-\tilde{\rho}_1)^2}+\frac{r(r+1)}{\tilde{\mu}_2}\cdot\frac{\tilde{\rho}_1}{\tilde{\mu}_1(1-\tilde{\rho}_1)^3}$$

Average number of customers in the system

$$E(L_s^{\,2})=\tilde{\rho}_2+\frac{\tilde{\rho}_2^{\,2}}{2(1-\tilde{\rho}_2)}\cdot\frac{E(C^2)}{(E(C))^2}$$

$$E(L_s^{\,2})=\tilde{\rho}_2+\frac{r(r+1)\tilde{\rho}_2^{\,2}}{2(1-\tilde{\rho}_2)}\left[1+\frac{\tilde{\mu}_2}{\tilde{\mu}_1}\cdot\frac{\tilde{\rho}_1}{(1-\tilde{\rho}_1)}\right]$$

Mean waiting time in the system

$$E(W_s^{\,2})=\frac{E(L_s^{\,2})}{\tilde{\lambda}_2}$$

$$=\frac{r}{\tilde{\mu}_2(1-\tilde{\rho}_1)}+\frac{r^3(r+1)\tilde{\lambda}_2}{2\tilde{\mu}_2^{\,2}(1-\tilde{\rho}_1)^2(1-\tilde{\rho}_2)}\left[1+\frac{\tilde{\mu}_2}{\tilde{\mu}_1}\cdot\frac{\tilde{\rho}_1}{(1-\tilde{\rho}_1)}\right]$$

Mean waiting time in the queue

$$E(W_q^{\,2})=E(W_s^{\,2})-E(C)$$

$$=\frac{r}{\tilde{\mu}_2}\left(1+\frac{r^2(r+1)\tilde{\rho}_2}{2\tilde{\mu}_2(1-\tilde{\rho}_2)}\left[1+\frac{\tilde{\mu}_2}{\tilde{\mu}_1}\cdot\frac{\tilde{\rho}_1}{(1-\tilde{\rho}_1)}\right]-\frac{1}{(1-\tilde{\rho}_1)}\right)$$

Average number of customers in the queue

$$E(L_q^{\,2})=\tilde{\lambda}_2E(W_q^{\,2})$$

$$=\tilde{\rho}_2\left(1+\frac{r^2(r+1)\tilde{\rho}_2}{2\tilde{\mu}_2(1-\tilde{\rho}_2)}\left[1+\frac{\tilde{\mu}_2}{\tilde{\mu}_1}\cdot\frac{\tilde{\rho}_1}{(1-\tilde{\rho}_1)}\right]-\frac{1}{(1-\tilde{\rho}_1)}\right)$$

## 4. ROBUST RANKING METHOD – ALGORITHM:

To determine the distinctive measures in terms of crisp values, we use a fuzzy number ranking algorithm to defuzzify the fuzzy numbers into crisp ones. A Robust ranking method that takes into account linearity and additive qualities while producing results that are consistent with human perception.

$$\text{R}\,(\tilde{c}) = \int_0^1 0.5(c_\alpha^{\,L}+c_\alpha^{\,U})d\alpha$$

Where $\left(c_\alpha^{\,L},\, c_\alpha^{\,U}\right)$ is the $\alpha$ - level cut of the fuzzy number $\tilde{c}$ .

## 5. NUMERICAL EXAMPLE:

Consider a two-class fuzzy Preemptive-resume priority queuing system with Erlang service distribution that runs on a single server with first come first served basis. The service rates are assumed to follow the Erlang-k ($E_k$) distribution, whereas the arrival rates follow the Poisson distribution. A robust ranking technique can be used to evaluate the efficiency of the fuzzy queuing model.

**Ranking of Triangular fuzzy Numbers**

Let us contemplate the arrival rates and service rates are triangular fuzzy numbers. Let $\tilde{\lambda}_1=[0.1, 0.2, 0.3]$ ; $\tilde{\lambda}_2=[0.4, 0.5, 0.6]$; $\tilde{\mu}_1=[4, 5, 6]$; $\tilde{\mu}_2=[7, 8, 9]$

The membership function of $\tilde{\lambda}_1=[0.1, 0.2, 0.3]$ is defined by

$$\phi_{\tilde{\lambda}_1}(x)=\begin{cases} \dfrac{(x-0.1)}{(0.2-0.1)} & , \ 0.1\le x\le 0.2 \\ 1 & , \ x=0.2 \\ \dfrac{(x-0.3)}{(0.2-0.3)} & , \ 0.2\le x\le 0.3 \\ 0 & , \ otherwise \end{cases}$$

Thereby, we write the membership functions of $\tilde{\lambda}_2$, $\tilde{\mu}_1$ *and* $\tilde{\mu}_2$ in the similar fashion. Subsequently, We apply robust ranking for the triangular fuzzy numbers $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}_1$ *and* $\tilde{\mu}_2$.

The $\alpha$ - cut of the fuzzy number $\tilde{\lambda}_1=[0.1, 0.2, 0.3]$ is $[\alpha+0.1, 0.3-\alpha]$

$$R(\tilde{\lambda}_1) = R(0.1, 0.2, 0.3) = \int_0^1 0.5(0.4)d\alpha = 0.2$$

Proceeding similarly, we get R ($\tilde{\lambda}_2$) = 0.5, R ($\tilde{\mu}_1$) = 5 and R ($\tilde{\mu}_2$) = 8.

We calculate the characteristic measures of described two class fuzzy Queuing system and tabulate the values with respect to 'r' (number of phases).

Table 1. Average number of customers vs. 'r'

| r | $\rho_1$ | $\rho_2$ | $E(L_s^1)$ | $E(L_s^2)$ | $E(L_q^1)$ | $E(L_q^2)$ | |
|---|---|---|---|---|---|---|---|
| 1 | 0.040 | 0.065 | 0.042 | 0.070 | 0.002 | 0.005 | |
| 2 | 0.080 | 0.136 | 0.085 | 0.209 | 0.005 | 0.073 | |
| 3 | 0.120 | 0.213 | 0.131 | 0.634 | 0.011 | 0.422 | |
| 4 | 0.160 | 0.298 | 0.179 | 1.949 | 0.019 | 1.647 | |
| 5 | 0.200 | 0.391 | 0.230 | 5.663 | 0.030 | 5.262 | |
| 6 | 0.240 | 0.493 | 0.284 | 15.647 | 0.044 | 15.180 | |

Table 2. Mean waiting time vs. 'r'

| r | $\rho_1$ | $\rho_2$ | $E(W_s^1)$ | $E(W_s^2)$ | $E(W_q^1)$ | $E(W_q^2)$ | |
|---|---|---|---|---|---|---|---|
| 1 | 0.040 | 0.065 | 0.208 | 0.140 | 0.008 | 0.010 | |
| 2 | 0.080 | 0.136 | 0.426 | 0.418 | 0.026 | 0.146 | |
| 3 | 0.120 | 0.213 | 0.655 | 1.269 | 0.055 | 0.843 | |
| 4 | 0.160 | 0.298 | 0.895 | 3.888 | 0.095 | 3.293 | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 5 | 0.200 | 0.391 | 1.150 | 11.305 | 0.150 | 10.524 | |
| 6 | 0.240 | 0.493 | 1.421 | 31.346 | 0.221 | 30.359 | |

With regard to the data in the preceding tables, it is clear that the service phase increases, as do the mean number of customers in the system, customers in the queue, system average waiting time, and queue average waiting time. As compared to the first priority system, the number of customers and waiting time increases in the second priority system because of the service phase.

## 6. CONCLUSION:

This paper work investigates the significant measures of two-class fuzzy preemptive-resume priority queuing systems and the use of a robust ranking algorithm to rank triangular fuzzy numbers. In practise, the predicted ranking algorithms may rank fuzzy numbers. This strategy not only yields crisp results, but it also does so with greater precision than most other approaches.

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