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Production Inventory Model – Panorama Harnessing Geometric Programming

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ABSTRACT: This paper focusses on production inventory model for deteriorating products with dependent selling price to balance the demand and shortage under inflationary environment. To represent cost parameters trapezoidal fuzzy numbers are expressed as ranking fuzzy numbers with best approximation interval. To derive the optimal decisions, the technique of Geometric programming is made relevant by solving a numerical problem.

KEYWORDS: Production inventory model, ranking of fuzzy numbers, best interval approximation, Geometric programming

1. INTRODUCTION:

Business investment can affect the economy's short term and long term growth. A business cycle expansion generates higher interest rate and a surplus of capital that prompts a decrease in investment and a business – cycle contractor. A business can reach a high pinnacle if and only if the inflation rate is greater than the return rate of investment. But in reality, with increase in time, inflation becomes a substantial function leading to the cost difference.

The effects of inflation, time value of money and deterioration of an inventory model was investigated by Wee and Law (2001). An inventory model was suggested by Chang (2004) for deteriorating products by introducing inflation's effect.

A developed model should focus on the shortages during stock out which should not be assumed as backlogged or completely lost. This model allows the shortages and backlogs partially the happening shortages.

This paper introduces a production inventory model for deteriorating items under inflationary environment with allowable shortages.

To apply this model, the concept of fuzzy Geometric programming technique is adopted. Considering that the coefficients of the problem are fuzzy which are taken in geometric form and solved using Geometric programming technique to form a fuzzy Geometric programing.

This model is constructed under fuzzy goal and fuzzy restrictions on budgetary cost. The inventory related costs and other parameters are assumed as fuzzy in nature and a numerical problem is solved using Geometric programming technique.

2. MATHEMATICAL MODEL:

2.1 ASSUMPTIONS:

1. The model promotes deteriorating products and finite time horizon.

2. The demand of the product is dependent on selling price which is given by:
   $D = \frac{\alpha}{P^{\beta}}$
3. Rate of production relies on demand rate.
4. This model contemplates inflation.
5. Shortages are permitted and it is partially backlogged.

2.2 NOTATIONS:

α, β - demand parameters
P - selling price of the product per unit
k - deterioration parameter
a - production coefficient, $a \succ 1$
C - unit production cost
r_1 - inflation rate
H - respective holding cost
D - deterioration cost per unit of time
t - timely production
v_1 - hitherto when inventory level becomes zero
S - shortage cost
T - time period of the cycle
L - lost sale cost
θ - backlog rate
O - ordering cost

3. CRISP MODEL:

This model is a production inventory model. When $t=0$, production starts and inventory level becomes maximum to satisfy the occurring demand and deterioration. When $t=t_1$, production ceases reducing the inventory level by reason of demand and deterioration for the period $[t_1, v]$. Finally upto $t=T$, shortages occur.

The unit production cost signifies $\frac{Ca\alpha}{r_1 P^{\beta}}(1-e^{-r_1 t})$

Holding cost is given by

$$H(a-1)\frac{\alpha}{P^{\beta}}\left(\frac{t^2}{2}-\frac{r_1 t^3}{3}-\frac{kt^4}{12}+\frac{kr_1 t^5}{15}\right)+\frac{H\alpha}{P^{\beta}}\left\{\frac{v_1{}^2}{2}+\frac{kv_1{}^4}{12}-r_1\left(\frac{tv_1{}^2}{2}+\frac{v_1{}^3}{6}\right)-\left(v_1 t-\frac{t^2}{2}\right)-\frac{k}{6}\left(v_1{}^3 t-\frac{t^4}{4}\right)+ r_1\left(\frac{3}{2}v_1 t^2-\frac{t^3}{6}\right)+\frac{k}{2}\left(\frac{v_1 t^3}{3}-\frac{t^4}{4}\right)\right\}$$

Deterioration cost becomes $\frac{D\alpha}{r_1 P^{\beta}}(at-v_1)(1-e^{-r_1 v_1})$

Shortage cost specifies $\frac{S\alpha}{r_1 P^{\beta}}(e^{-r_1 v_1}-e^{-r_1 T})$

Lost sale cost is denoted by $\frac{L\alpha(1-\theta)}{r_1P^\beta}(e^{-r_1v_1}-e^{-r_1T})$

Ordering cost is O

The total cost is given as

$$TC=\frac{1}{T}\left\{\frac{Ca\alpha(1-e^{-r_1t})}{r_1P^\beta}+\frac{H(a-1)\alpha}{P^\beta}\left(\frac{t^2}{2}-\frac{r_1t^3}{3}-\frac{kt^4}{12}+\frac{kr_1t^5}{15}\right)+\frac{H\alpha}{P^\beta}\left\{\frac{v_1^2}{2}+\frac{kv_1^4}{12}-r_1\left(\frac{tv_1^4}{2}+\frac{v_1^3}{6}\right)-\left(v_1t-\frac{t^2}{2}\right)-\right.\right.$$

$$\left.\frac{k}{6}\left(v_1^3-\frac{t^4}{4}\right)+r_1\left(\frac{3}{2}v_1t^2-\frac{t^3}{6}\right)+\frac{k}{2}\left(\frac{v_1t^3}{3}-\frac{t^4}{4}\right)\right\}+\frac{D\alpha}{r_1P^\beta}(at-v_1)(1-e^{-r_1v_1})+$$

$$\left.\frac{S\alpha}{r_1P^\beta}(e^{-r_1v_1}-e^{-r_1T})+\frac{L\alpha(1-\theta)}{r_1P^\beta}(e^{-r_1v_1}-e^{-r_1T})+O\right\}$$

4. FUZZY MODEL:

Here C,H,S,L,D are considered to be fuzzy numbers where C is the unit production cost, H is the holding cost, S is the shortage cost , D is the deterioration cost per unit and L is the lost sale cost.

$$\tilde{TC}=\frac{1}{T}\left\{\frac{\tilde{C}a\alpha(1-e^{-r_1t})}{r_1P^\beta}+\frac{\tilde{H}(a-1)\alpha}{P^\beta}\left(\frac{t^2}{2}-\frac{r_1t^3}{3}-\frac{kt^4}{12}+\frac{kr_1t^5}{15}\right)+\frac{\tilde{H}\alpha}{P^\beta}\left\{\frac{v_1^2}{2}+\frac{kv_1^4}{12}-r_1\left(\frac{tv_1^4}{2}+\frac{v_1^3}{6}\right)-\left(v_1t-\frac{t^2}{2}\right)-\right.\right.$$

$$\left.\frac{k}{6}\left(v_1^3-\frac{t^4}{4}\right)+r_1\left(\frac{3}{2}v_1t^2-\frac{t^3}{6}\right)+\frac{k}{2}\left(\frac{v_1t^3}{3}-\frac{t^4}{4}\right)\right\}+\frac{\tilde{D}\alpha}{r_1P^\beta}(at-v_1)(1-e^{-r_1v_1})+$$

$$\left.\frac{\tilde{S}\alpha}{r_1P^\beta}(e^{-r_1v_1}-e^{-r_1T})+\frac{\tilde{L}\alpha(1-\theta)}{r_1P^\beta}(e^{-r_1v_1}-e^{-r_1T})+O\right\}$$

5. NEAREST INTERVAL APPROXIMATION:

Assume, $\tilde{A}$ as a fuzzy number with α cut $[A_L(\alpha),A_R(\alpha)]$ then the interval $C_d(\tilde{A})=[\int_0^1 A_L(\alpha)d\alpha,\int_0^1 A_R(\alpha)d\alpha]$. Let $\tilde{A}=(a_1,a_2,a_3,a_4)$ be a trapezoidal fuzzy number. The α cut interval is defined as $[A_L(\alpha),A_R(\alpha)]$ where $A_L(\alpha)=a_1+(a_2-a_1)\alpha$ and $A_R(\alpha)=a_4-(a_2-a_3)\alpha$. Applying nearest interval approximation, the lower limit of the interval $C_L=\frac{a_1+a_2}{2}$ and the upper limit of the interval $C_R=\frac{a_3+a_4}{2}$.

6. RANKING FUZZY NUMBERS WITH BEST APPROXIMATION METHOD:

The ranking of fuzzy numbers with best approximation method can be defined as, "a convex combination of lower and upper boundary of the best approximation interval". If $\lambda \in [0,1]$ is the pre – assigned parameter called as the degree of optimism, then the ranking of fuzzy number $\tilde{A}$ is defined as

$$R_{\lambda,f}(\tilde{A}) = \lambda A_R(\alpha) + (1-\lambda) A_L(\alpha)$$

A bigger value of λ satisfies the higher degree of optimism. When $\lambda = 0$, $R(\tilde{A}) = A_L(\alpha)$ expresses that the decision maker's viewpoint is completely pessimistic. If $\lambda = 1$, $R(\tilde{A}) = A_R(\alpha)$ then the decision maker's viewpoint is completely optimistic. For $\lambda = \frac{1}{2}$, $R(\tilde{A}) = \frac{1}{2}[A_L(\alpha) + A_R(\alpha)]$ the decision maker's viewpoint is moderately optimistic or moderately pessimistic.

The ranking of fuzzy number $\tilde{A}$ is given as

$$R_{\lambda,f}(\tilde{A}) = \int_0^1 [\lambda A_R(\alpha) + (1-\lambda) A_L(\alpha)] f(\alpha) d\alpha \Big/ \int_0^1 f(\alpha) d\alpha$$

If $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number then $A_L(\alpha) = a_1 + \alpha(a_2 - a_1)$ and $A_R(\alpha) = a_4 - \alpha(a_2 - a_3)$. The lower limit of the interval $A_L(\alpha) = \frac{a_1 + 2a_2}{3}$ and the upper limit of the interval $A_R(\alpha) = \frac{2a_3 + a_4}{3}$. The corresponding ranking of fuzzy number is

$$R_{\lambda,f}(\tilde{A}) = 2\int_0^1 \left[\lambda\left(\frac{2a_3 + a_4}{3}\right) + (1-\lambda)\left(\frac{a_1 + 2a_2}{3}\right)\right] \alpha \, d\alpha$$

$$R_{\lambda,f}(\tilde{A}) = \frac{1}{3}[\lambda(2a_3 + a_4) + (1-\lambda)(a_1 + 2a_2)]$$

7. GEOMETRIC PROGRAMMING PROBLEM:

Primal program:

Primal Geometric programming problem is

$$\text{Minimize } g_0(x) = \sum_{k=1}^{T_o} C_{0k} \prod_{j=1}^{m} at_j^{\,b_{0kj}}$$

$$\text{subject to } \sum_{k=1}^{T_0} C_{rk} \prod_{j=1}^{m} at_j^{\,b_{rkj}} \le 1, \ r = 1,2,,,,I; \ j = 1,2,.....m; \ t_j \succ 0$$

$$C_{0k} \succ 0; \ k = 1,2,,,,,T_0$$

where C_{rk} and b_{rk} are real numbers. The above mentioned is a constrained posynomial problem in which the number of each term in the constrained function varies. It is denoted by

T for each $r=0,1,2,....$ Let $T=T_0+T_1+.....T_m$ be the total number of terms in the primal program then the degree of difficulty is $T-(m+1)$.

Dual program:

Maximize: $\prod_{r=0}^{l}\sum_{k=1}^{T_r}\left(\frac{C_{rk}}{\delta_{rk}}\right)^{\delta_{rk}}\left(\sum_{S=1+T_{r+1}}^{T}(\delta_{rs})\right)^{\delta_{rk}}$

subject to: $\sum_{k=1}^{T_0}\delta_{0k}=1$ (Normality condition)

$\sum_{r=o}^{l}\sum_{k=1}^{T_r}b_{rk_j}\delta_k=0$ (Orthogonality condition)

$\delta_{rk}\succ 0$ (Positive constant)

8. APPLYING GEOMETRIC PROGRAM TECHNIQUE TO SOLVE FUZZY INVENTORY MODEL:

The objective function is

$$\tilde{TC}=\frac{1}{T}\left\{\frac{\tilde{C}\,a\alpha(1-e^{-r_1t})}{r_1P^{\beta}}+\frac{\tilde{H}(a-1)\alpha}{P^{\beta}}\left(\frac{t^2}{2}-\frac{r_1t^3}{3}-\frac{kt^4}{12}+\frac{kr_1t^5}{15}\right)+\frac{\tilde{H}\,\alpha}{P^{\beta}}\left\{\frac{v_1^{\,2}}{2}+\frac{kv_1^{\,4}}{12}-r_1\left(\frac{tv_1^{\,4}}{2}+\frac{v_1^{\,3}}{6}\right)-\left(v_1t-\frac{t^2}{2}\right)-\frac{k}{6}\left(v_1^{\,3}-\frac{t^4}{4}\right)+r_1\left(\frac{3}{2}v_1t^2-\frac{t^3}{6}\right)+\frac{k}{2}\left(\frac{v_1t^3}{3}-\frac{t^4}{4}\right)\right\}+\frac{\tilde{D}\,\alpha}{r_1P^{\beta}}(at-v_1)(1-e^{-r_1v_1})+\frac{\tilde{S}\,\alpha}{r_1P^{\beta}}(e^{-r_1v_1}-e^{-r_1T})+\frac{\tilde{L}\,\alpha(1-\theta)}{r_1P^{\beta}}(e^{-r_1v_1}-e^{-r_1T})+O\right\}$$

Now using ranking of fuzzy numbers the objective function becomes

$$R_{\lambda,f}\,\tilde{TC}=\frac{1}{T}\left\{\frac{R_{\lambda,f}\,(\tilde{C})a\alpha(1-e^{-r_1t})}{r_1P^{\beta}}+\frac{R_{\lambda,f}\,(\tilde{H})(a-1)\alpha}{P^{\beta}}\left(\frac{t^2}{2}-\frac{r_1t^3}{3}-\frac{kt^4}{12}+\frac{kr_1t^5}{15}\right)+\frac{R_{\lambda,f}\,(\tilde{H})\,\alpha}{P^{\beta}}\left\{\frac{v_1^{\,2}}{2}+\frac{kv_1^{\,4}}{12}-r_1\left(\frac{tv_1^{\,4}}{2}+\frac{v_1^{\,3}}{6}\right)-\left(v_1t-\frac{t^2}{2}\right)-\frac{k}{6}\left(v_1^{\,3}-\frac{t^4}{4}\right)+r_1\left(\frac{3}{2}v_1t^2-\frac{t^3}{6}\right)+\frac{k}{2}\left(\frac{v_1t^3}{3}-\frac{t^4}{4}\right)\right\}+\frac{R_{\lambda,f}\,(\tilde{D})\alpha}{r_1P^{\beta}}(at-v_1)(1-e^{-r_1v_1})+\frac{R_{\lambda,f}\,(\tilde{S})\alpha}{r_1P^{\beta}}(e^{-r_1v_1}-e^{-r_1T})+\frac{R_{\lambda,f}\,(\tilde{L})\alpha(1-\theta)}{r_1P^{\beta}}(e^{-r_1v_1}-e^{-r_1T})+O\right\}$$

Using Geometric programming technique,

$$\prod_{i=1}^{n} = \frac{1}{T}\Bigg\{O + \frac{R_{\lambda,f}(\tilde{C})a\alpha(1-e^{-r_1 t})}{r_1 P^{\beta}} + \frac{R_{\lambda,f}(\tilde{H})(a-1)\alpha}{P^{\beta}}\left(\frac{t^2}{2} - \frac{r_1 t^3}{3} - \frac{kt^4}{12} + \frac{kr_1 t^5}{15}\right) + \frac{R_{\lambda,f}(\tilde{H})\,\alpha}{P^{\beta}}\Bigg\{\frac{v_1{}^2}{2} + \frac{kv_1{}^4}{12} - r_1\left(\frac{tv_1{}^4}{2} + \frac{v_1{}^3}{6}\right) -$$

$$\left(v_1 t - \frac{t^2}{2}\right) - \frac{k}{6}\left(v_1{}^3 - \frac{t^4}{4}\right) + r_1\left(\frac{3}{2}v_1 t^2 - \frac{t^3}{6}\right) + \frac{k}{2}\left(\frac{v_1 t^3}{3} - \frac{t^4}{4}\right)\Bigg\}\Bigg\} + \left(\frac{R_{\lambda,f}(\tilde{D})\alpha}{Tr_1 P^{\beta}}(at - v_1)(1-e^{-r_1 v_1})\right) \times \frac{\gamma_{1r}}{L\gamma_{1r}} +$$

$$\left(\frac{R_{\lambda,f}(\tilde{S})\alpha}{Tr_1 P^{\beta}}(e^{-r_1 v_1} - e^{-r_1 T})\right) \times \frac{\gamma_{2r}}{D\gamma_{2r}} + \left(\frac{R_{\lambda,f}(L)\alpha(\tilde{1}-\theta)}{Tr_1 P^{\beta}}(e^{-r_1 v_1} - e^{-r_1 T})\right) \times \frac{\gamma_{3r}}{\gamma_{3r}}$$

subject to the conditions

$\gamma_{1r} + \gamma_{2r} + \gamma_{3r} = 1$

$\gamma_{1r} - \gamma_{2r} = 0$

$-\gamma_{1r} + \gamma_{3r} = 0$

Solving these conditions we obtain

$$\gamma_{1r} = \gamma_{2r} = \gamma_{3r} = \frac{1}{3}$$

9. NUMERICAL EXAMPLE:

The following values are allowed in appropriate units.

Crisp Model:

$P =$ Rs. 24 / unit; $\beta = 1.5$; $S =$ Rs. 6 / unit; $r = 0.06$; $\alpha = 500$ units ; $d =$ Rs.15 / unit

$T = 50$; $H =$ Rs. 0.2 / unit; $k = 0.01$; $\theta = 0.5$; $L =$ Rs. 8 / unit; $C =$ Rs.12 / unit

The Total Cost is Rs.32.8825

Fuzzy Model:

$$\tilde{C} = (6,10,14,18); \tilde{S} = (3,5,7,9); \tilde{H} = (0.05,0.09,0.06,0.85); \tilde{L} = (5,7,9,11); \tilde{D} = (12,14,16,18)$$

when $\lambda = 0.5$ we have,

$R(\tilde{C}) = 12; R(\tilde{S}) = 6; R(\tilde{H}) = 0.2; R(\tilde{L}) = 8; R(\tilde{D}) = 15$

The Total Cost is Rs.30.6816

10. CONCLUSION:

This model focusses on deterioration, price sensitive demand, shortages and inflations that are realistic features. Here production rate contemplates as demand dependent which promotes the real – life situations and shortages during stock out is viewed as partial backlog. These realistic features are engulfed as a totality in this model. To enumerate the optimal value of production period and shortage period a solution procedure is put forth. As an analytical formulation, the costs are assumed to be trapezoidal fuzzy number. These fuzzy numbers sequentially defined by ranking fuzzy numbers with respect to the best approximation interval number. Hence, this model highly influences the reduction of total cost to balance the demand and shortage under inflationary environment.

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