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Decomposition of various graphs in to sum divisor cordial graphs

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Abstract: A sum divisor cordial labeling of a graph G with vertex set V is a bijection $f: V \rightarrow \{1,2,3,...|V(G)|\}$ and the edge labeling $f^*: E \rightarrow \{0,1\}$ is defined by $f^*(uv) = 1$, if 2 divides $f(u) + f(v)$ and 0 otherwise. The function f is called a sum divisor cordial labeling if $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$. That is the number of edges labeled with 0 and the number of edges labeled with 1 differs by at most 1. A graph with a sum divisor cordial labeling is called a sum divisor cordial graph. A decomposition of G is a collection $\psi_S = \{H_1, H_2, ...H_r\}$ such that H_i are edge disjoint and every edges in H_i belongs to G. If each H_i is a sum divisor cordial graphs, then ψ_S is called a sum divisor cordial decomposition of G. The minimum cardinality of a sum divisor cordial decomposition of G is called the sum divisor cordial decomposition number of G and it is denoted by $\pi_S(G)$. In this paper we define sum divisor cordial decomposition and sum divisor cordial decomposition number $\pi_S(G)$ of a graphs. Also investigate some bounds of $\pi_S(G)$ in product graphs like Cartesian product, composition etc.

Keywords: Sum divisor cordial, sum divisor cordial decomposition and sum divisor cordial decomposition number.

1. INTRODUCTION:

For all further usual terms and notations we follow Harary [1]. A labeling of a graph is a mapping that transfers the graph components to the set of numbers, typically to the set of non-negative or positive integers. If the domain is the set of vertices the labeling is called vertex labeling. If the domain is the set of edges then the labeling is called edge labeling. If the labels are allocated to both vertices and edges then the labeling is called total labeling. A. Lourdusamy and F. Patrick introduced the concept of sum divisor cordial labeling in [2,3].

A graph is an well-ordered pair $G = (V, E)$, where V is a non-empty finite set, called the set of vertices or nodes of G, and E is a set of unordered pairs (2-element subsets) of V, called the edges of G. If $xy \in E$, x and y are called adjacent and they are incident with the edge xy.

The complete graph on n vertices, denoted by K_n, is a graph on n vertices such that every pair of vertices is connected by an edge. The empty graph on n vertices, denoted by E_n, is a graph on n vertices with no edges.A graph $G' = (V', E')$ is a sub graph of $G = (V, E)$ if and only if $V' \subseteq V$ and $E' \subseteq E$.The order of a graph $G = (V, E)$ is $|V|$, the number of its vertices. The size of G is $|E|$, the quantity of its edges. The degree of a node $x \in V$, represented by $d(x)$, is the quantity of edges incident with it.

A sub graph H of G is a graph such that $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. For a graph $G(V, E)$ and a sub set $W \subseteq V$, the sub graph of G induced by W, denoted as $G[W]$, is the graph $H(W, F)$ such that, for all $u, v \in W$, if $uv \in E$, then $uv \in F$. We say H is an induced sub graph of G.

A graph $G(V, E)$ is said to be connected if every pair of vertices is connected by a path. If there is exactly one path connecting each pair of vertices, we say G is a tree. Equivalently, a tree is a connected graph with n − 1 edges. A path graph P_n is a connected graph on n vertices such that each vertex has degree at most 2. A cycle graph C_n is a connected graph on n vertices such that every vertex has degree 2.

A complete graph P_n is a graph with n vertices such that every vertex is adjacent to all the others. On the other hand, an independent set is a set of vertices of a graph in which no two vertices are adjacent. We denote In for an independent set with n vertices.

A bipartite graph $G(V, E)$ is a graph such that there exists a partition $P(A, B)$ of V such that every edge of G connects a vertex in A to one in B. Equivalently, G is said to be bipartite if A and B are independent sets. The bipartite graph is also denoted as $G(A, B, E)$

The Brush graph B_n , $(n \geq 2)$ can be constructed by path graph P_n , $(n \geq 2)$ by joining the star graph $K_{1,1}$ at each vertex of the path. i.e., $B_n = P_n + nK_{1,1}$.

In this paper we define sum divisor cordial decomposition and sum divisor cordial decomposition number $\pi_S(G)$ of a graphs. Also investigate some bounds of $\pi_S(G)$ in product graphs like Cartesian product, composition etc.

1. Sum divisor cordial decomposition

In this section we define sum divisor cordial decomposition of a graph $G(V, E)$ and investigate some bounds of sum divisor cordial decomposition number in various graphs $G(V, E)$.

Definition 2.1.[2]: A sum divisor cordial labeling of a graph G with vertex set V is a bijection $f : V \to \{1,2,3,...|V(G)|\}$ and the edge labeling $f^* : E \to \{0,1\}$ is defined by $f^*(uv) = 1$, if 2 divides $f(u) + f(v)$ and 0 otherwise. The function f is called a sum divisor cordial labeling if $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$.That is the number of edges labeled with 0 and the number of edges labeled with 1 differs by at most 1.

A graph with a sum divisor cordial labeling is called a sum divisor cordial graph.

Definition 2.2: A decomposition of G is a collection $\psi_S = \{H_1, H_2, ...H_r\}$ such that H_i are edge disjoint and every edges in H_i belongs to G. If each H_i is a sum divisor cordial graphs, then ψ_S is called a sum divisor cordial decomposition of G. The minimum cardinality of a sum divisor cordial decomposition of G is called the sum divisor cordial decomposition number of G and it is denoted by $\pi_S(G)$.

Definition 2.3: Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple graphs. The join $G_1 + G_2$ of G_1 *and* G_2 with disjoint vertex set $V_1 \& V_2$ and the edge set E of $G_1 + G_2$ is defined by the two vertices (u_i, v_j) if one of the following conditions are satisfied

i) $u_i v_j \in E_1$.

ii) $u_i v_j \in E_2$.

iii) $u_i \in V_1 \& v_j \in V_2$, $u_i v_j \in E$

Theorem 2.1: A graph $(P_m + P_n)$ is a join of two path sum divisor cordial graphs with $(m < n)$ .The bounds of sum divisor cordial decomposition number of the graph $(P_m + P_n)$ is, $3 \le \pi_S(P_m + P_n) \le (m(n+1) + (n-1))$. .

Proof: Let P_m and P_n be two path sum divisor cordial graphs of order m and n $(m > n)$ respectively and $(P_m + P_n)$ is a join of P_m and P_n with edge set E. The graph $(P_m + P_n)$ contains $(m+n)$ vertices and the edge set is $E = E_1 \cup E_2 \cup S(K_{m,n})$, Here $S(K_{m,n})$ is a size of a complete bipartite graph $K_{m,n}$. In the graph $(P_m + P_n)$ there are graphs P_m , P_n and the complete bipartite graphs $K_{m,n}$ .Note that P_m and P_n be two sum divisor cordial graphs and complete bipartite graphs $K_{m,n}$ also sum divisor cordial graph. This implies $\psi_S \supseteq \{P_m \cup P_n \cup K_{mn}\}$ and $|\psi_S| \ge |\{P_m \cup P_n \cup K_{mn}\}|$. Note that the graphs P_m , P_n and $K_{m,n}$ are sum divisor cordial graphs. Hence $\pi_S(P_m + P_n) \ge (3)$.

Illustration 2.1: The Join of two sum divisor cordial graphs $P_2 \& P_3$ is given in figure.2.1

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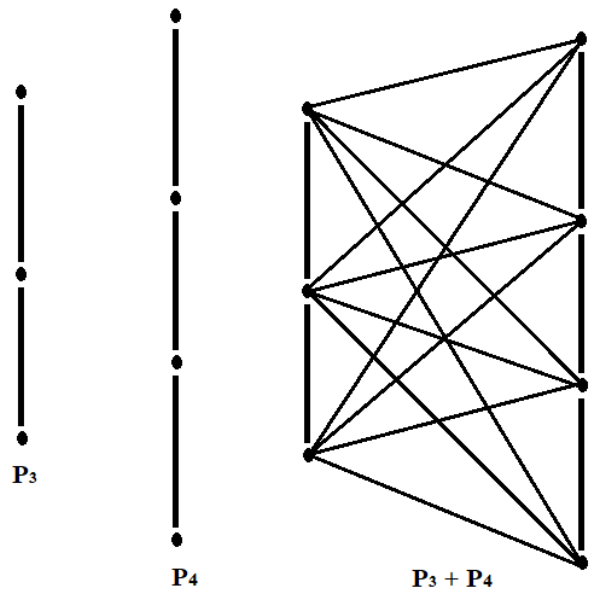
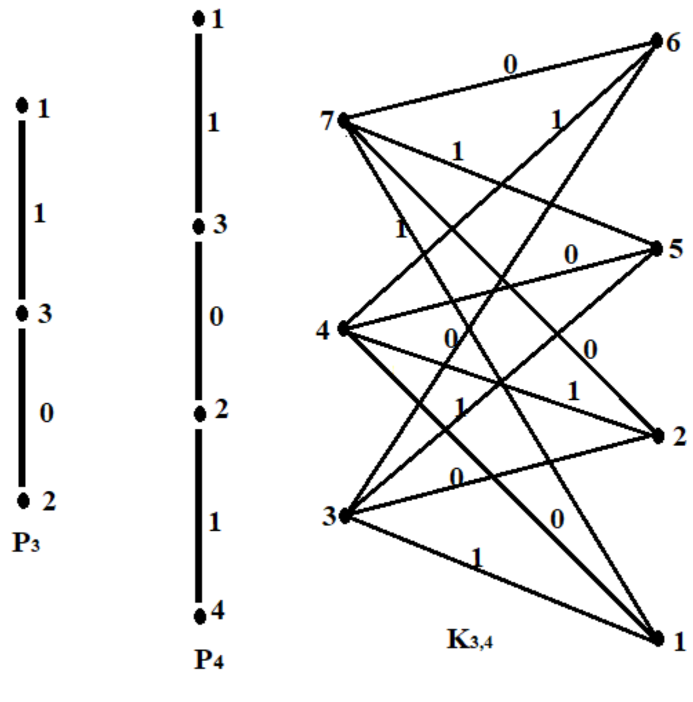


Decomposition of the graph $(P_2 + P_3)$ in to minimum copies of sum divisor cordial graph. This implies the lower bound of $\pi_S(P_3 + P_4)$ is $3 \leq \pi_S(P_2 + P_3)$.

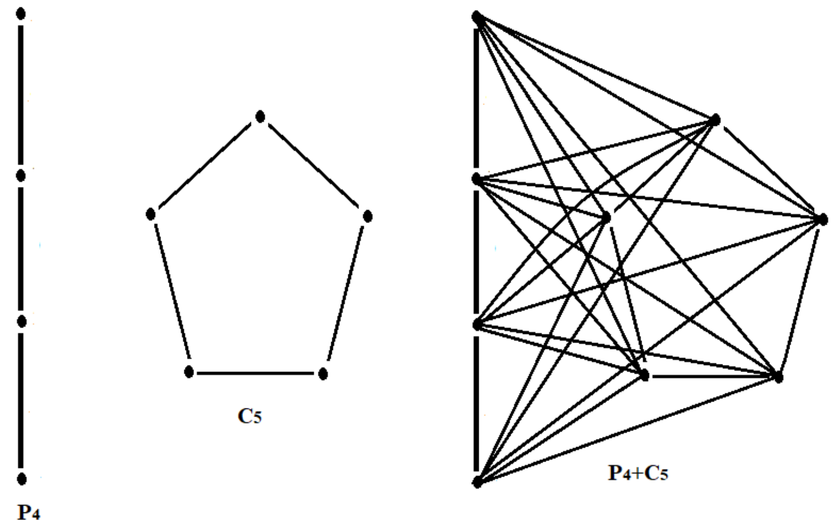


Theorem 2.2. A graph $(P_m + C_n)$ is a join of sum divisor cordial graphs P_m and cycle C_n with $(m < n)$. The bounds of sum divisor cordial decomposition number of the graph $(P_m + C_n)$ is, $3 \leq \pi_p(P_m + C_n)$..

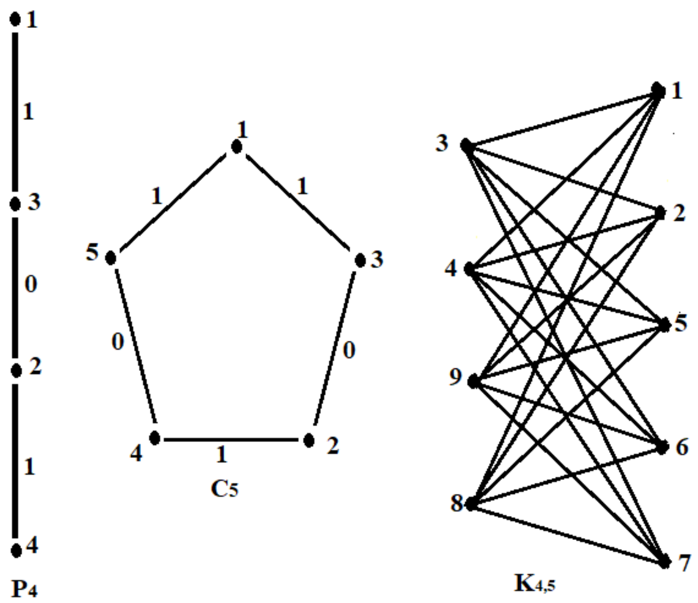
Proof: Let P_m and C_n be two path sum divisor cordial graphs of order m and n $(m > n)$ respectively and $(P_m + C_n)$ is a join of P_m and C_n with edge set E. The graph

$(P_m + C_n)$ contains $(m+n)$ vertices and the edge set is $E = E_1 \cup E_2 \cup S(K_{m,n})$, Here $S(K_{m,n})$ is a size of a complete bipartite graph $K_{m,n}$. In the graph $(P_m + C_n)$ there are graphs P_m , P_n and the complete bipartite graphs $K_{m,n}$ .Note that P_m and C_n be two sum divisor cordial graphs and complete bipartite graphs $K_{m,n}$ also sum divisor cordial graph. This implies $\psi_p \supseteq \{P_m \cup C_n \cup K_{mn}\}$ and $|\psi_p| \geq |\{P_m \cup C_n \cup K_{mn}\}|$. Note that the graphs P_m , C_n and $K_{m,n}$ are sum divisor cordial graphs. Hence $\pi_p(P_m + C_n) \geq (3)$.

Illustration 2.2: The Join of two sum divisor cordial graphs $P_2 \& P_3$ is given in figure.2.2



Decomposition of the graph $(P_4 + C_5)$ in to minimum copies of sum divisor cordial graph this implies the bound of $\pi_S(P_4 + C_5)$ is $3 \leq \pi_S(P_4 + C_5)$



Definition 2.4: Let $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ be two simple graphs. The Cartesian product $G_1 \times G_2$ of G_1 *and* G_2, is a graph with vertex set $V=V_1 \times V_2$ and the edge set of $G_1 \times G_2$ is defined by the two vertices (u_i,v_j) & (u_k,v_l) if one of the following conditions are satisfied

i) $u_1=v_1$ and u_2,v_2 are adjacent vertices in $G_2=(V_2,E_2)$.
ii) $u_2=v_2$ and u_1,v_1 are adjacent vertices in $G_1=(V_1,E_1)$.

Theorem 2.3: A graph $(P_m \times P_n)$is a Cartesian product of two sum divisor cordial graphs $(P_m \times P_n)$ with order m and n. Then bounds of sum divisor cordial decomposition number of the graph $(P_m \times P_n)$ is, $m+n \le \pi_S(P_m \times P_n)$.

Proof: Let P_m and P_n be two path sum divisor cordial graphs of order m and n respectively and $(P_m \times P_n)$ is a Cartesian product of P_n & P_m with edge set E. An edge $((x_1x_2)(y_1y_2)) \in E$ satisfies one of the following conditions

i) $x_1=y_1$ and x_2,y_2 are adjacent vertices in $G_2=(V_2,E_2)$.
ii) $x_2=y_2$ and x_1,y_1 are adjacent vertices in $G_1=(V_1,E_1)$.

Case (i): If $x_1=y_1$ and x_2,y_2 are adjacent vertices in $G_2=(V_2,E_2)$

If $x_1=y_1$ and x_2,y_2 are adjacent vertices in P_n . Let the sub graph H_i is isomorphic to the graph P_n . In $(P_m \times P_n)$ there are 'm' copies of graph P_n and it is sum divisor cordial graph .This implies H_i is also a sum divisor cordial graph. This implies $H_i \subset \psi$

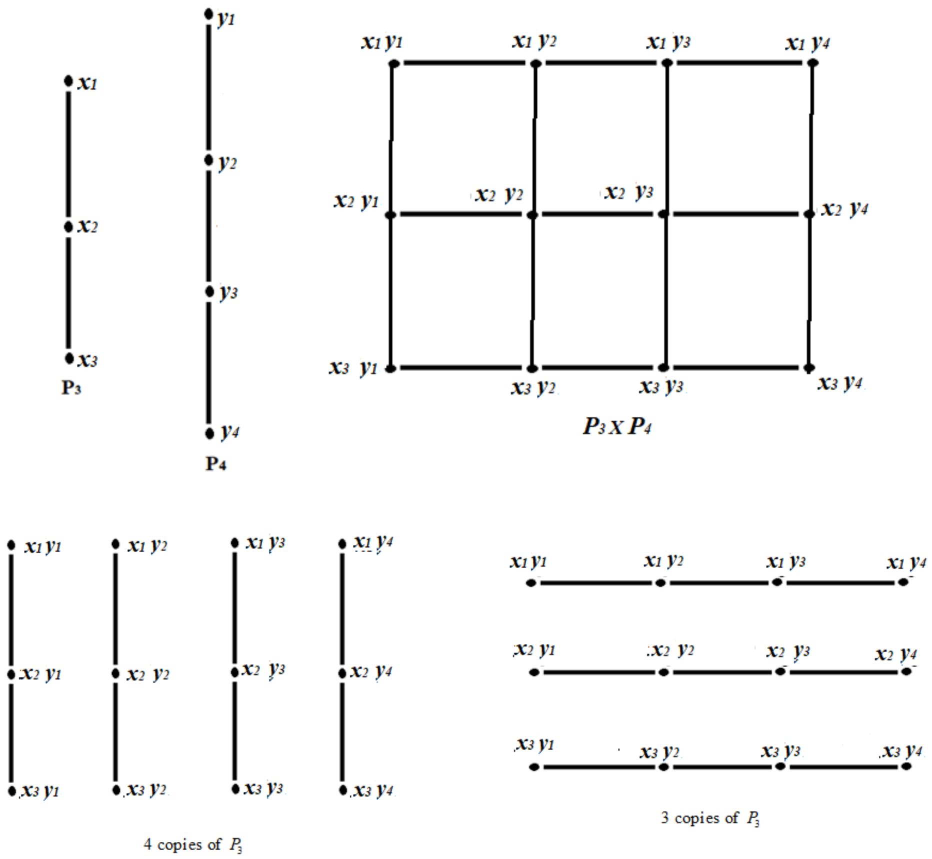
Case (ii): If $x_2=y_2$ and x_1,y_1 are adjacent vertices in P_m

If $x_2=y_2$ and x_1,y_1 are adjacent vertices in P_m . Note that the sub graph H_j is isomorphic to the graph P_m . In $(P_m \times P_n)$ there are 'n' copies of graph P_m and it is sum divisor cordial graph This implies H_j is also a sum divisor cordial graph. This implies $H_j \subset \psi$.

From case (i) and (ii), we get $\psi=\left\{\left(\bigcup_{i=1}^{m} H_i\right) \cup \left(\bigcup_{j=1}^{n} H_j\right)\right\}$ this implies $|\psi|=\sum_{i=1}^{m} H_i+\sum_{j=1}^{n} H_j=m+n$.

In $(P_m \times P_n)$ there are $n(m-1)+m(n-1) \Rightarrow$ $2(nm)-(m+n)$ edges. Note that every edge is a sum divisor cordial graph. This implies $\pi_p(P_m+P_n) \le 2(nm-1)$. Hence we get $m+n \le \pi_p(P_m+P_n) \le 2(nm)-(m+n)$.

Illustration 2.3: The Cartesian product of two sum divisor cordial $P_3 \& P_4$ is given in Figure.2.3



Note that the lower bound of $\pi_S(P_3+P_4)$ is $7 \le \pi_S(P_3+P_4)$.

Theorem 2.4: A graph $(P_m \times C_n)$ is a Cartesian product of two sum divisor cordial graphs $(P_m \times C_n)$ with order m and n. Then bounds of sum divisor cordial decomposition number of the graph $(P_m \times C_n)$ is, $m+n \le \pi_p(P_m \times C_n) \le 2(nm-1)$.

Proof: Let P_m and C_n be two path sum divisor cordial graphs of order m and n respectively and $(P_m \times C_n)$ is a Cartesian product of $P_m \& C_n$ with edge set E. An edge $((x_1x_2)(y_1y_2)) \in E$ satisfies one of the following conditions

i) $x_1 = y_1$ and x_2, y_2 are adjacent vertices in $G_2 = (V_2, E_2)$.

ii) $x_2 = y_2$ and x_1, y_1 are adjacent vertices in $G_1 = (V_1, E_1)$.

Case (i): If $x_1 = y_1$ and x_2, y_2 are adjacent vertices in C_n.

If $x_1 = y_1$ and x_2, y_2 are adjacent vertices in C_n . Let the sub graph H_i is isomorphic to the graph C_n. In $(P_m \times C_n)$ there are 'm' copies of graph C_n and it is sum divisor cordial graph .This implies H_i is also a prime graph. This implies $H_i \subset \psi$

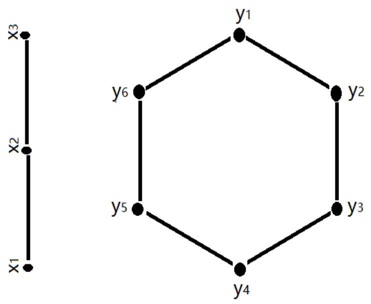
Case (ii): If $x_2 = y_2$ and x_1, y_1 are adjacent vertices in P_m

If $x_2 = y_2$ and x_1, y_1 are adjacent vertices in P_m . Note that the sub graph H_j is isomorphic to the graph P_m . In $(P_m \times C_n)$ there are 'n' copies of graph P_m and it is sum divisor cordial graph This implies H_j is also a sum divisor cordial graph. This implies $H_j \subset \psi$.

From case (i) and (ii), we get $\psi = \left\{ \left(\bigcup_{i=1}^{m} H_i \right) \cup \left(\bigcup_{j=1}^{n} H_j \right) \right\}$ this implies $|\psi| = \sum_{i=1}^{m} H_i + \sum_{j=1}^{n} H_j = m + n$.

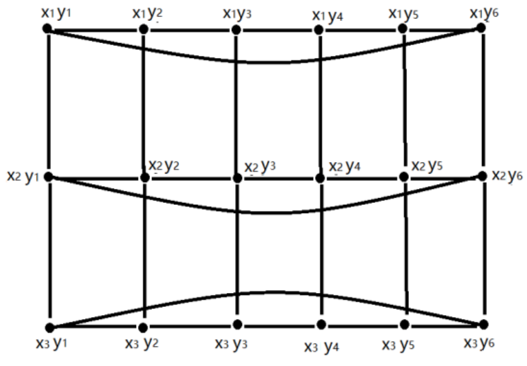
In $(P_m \times C_n)$ there are $n(m-1) + m(n) \Rightarrow 2(nm) - 1$ edges. Note that every edge is a sum divisor cordial graph. This implies $\pi_p(P_m \times C_n) \le 2(nm) - 1$. Hence we get $m + n \le \pi_p(P_m \times C_n) \le 2(nm) - 1$.

Illustration 2.4: The Cartesian product of two sum divisor cordial graphs $P_3 \& C_6$ is given in Figure.2.3. The bound of $\pi_p(P_3 \times C_6)$ is $9 \le \pi_p(P_3 \times C_6) \le 35$.

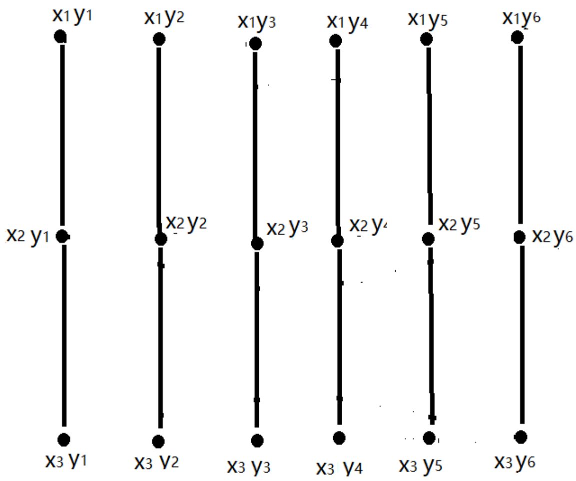


P3

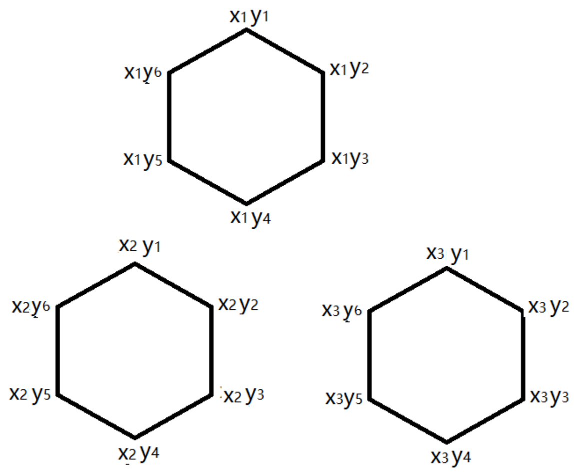
C6



(P3 × C6)



6 copies of P_3



3 copies of C_6

Definition 2.4: Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple graphs. The Composition $G_1 \circ G_2$ of G_1 *and* G_2, is a graph with vertex set $V = V_1 \times V_2$ and the edges in $G_1 \circ G_2$ is defined by the two vertices (u_1, u_2) & (v_1, v_2) if one of the following conditions are satisfied

i) $u_1 = v_1$ and u_2, v_2 are adjacent vertices in $G_2 = (V_2, E_2)$.
ii) $u_2 = v_2$ and u_1, v_1 are adjacent vertices in $G_1 = (V_1, E_1)$.
iii) u_1, v_1 are adjacent vertices in $G_1 = (V_1, E_1)$.

Theorem 2.5: A graph $G_1 \circ G_2$ is a Composition of two sum divisor cordial graphs $G_1 \& G_2$ with order m and n, can be decomposed in to at least $(mn + m + n)$ sum divisor cordial graphs (i.e $\pi_g(G_1 \circ G_2) \geq (mn + m + n)$).

Proof: Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two sum divisor cordial graphs of order m and n respectively and $G_1 \circ G_2$ is a Composition of G_1 *and* G_2 with edge set E the one of the following conditions are satisfied

i) $u_1 = v_1$ and u_2, v_2 are adjacent vertices in $G_2 = (V_2, E_2)$.
ii) $u_2 = v_2$ and u_1, v_1 are adjacent vertices in $G_1 = (V_1, E_1)$.
iii) u_1, v_1 are adjacent vertices in $G_1 = (V_1, E_1)$.

Case (i): If $u_1 = v_1$ and u_2, v_2 are adjacent vertices in $G_2 = (V_2, E_2)$

If $u_1 = v_1$ and u_2, v_2 are adjacent vertices in $G_2 = (V_2, E_2)$. Let the sub graph H_i is isomorphic to the graph $G_2 = (V_2, E_2)$. The graph $G_2 = (V_2, E_2)$ be a sum divisor cordial graph this implies H_i is also a sum divisor cordial graph. This implies $H_i \subset \psi$

Case (ii): If $u_2 = v_2$ u_1, v_1 are adjacent vertices in $G_1 = (V_1, E_1)$

If $u_2 = v_2$ u_1, v_1 are adjacent vertices in $G_1 = (V_1, E_1)$. Let the sub graph H_j is isomorphic to the graph $G_1 = (V_1, E_1)$. The graph $G_1 = (V_1, E_1)$ be a sum divisor cordial graph this implies H_j is also a sum divisor cordial graph. This implies $H_j \subset \psi$.

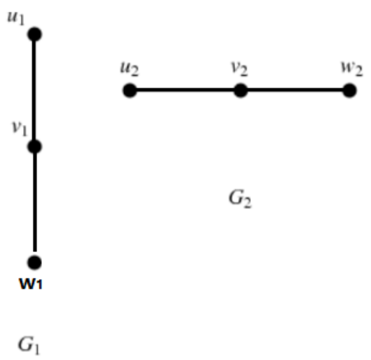
Case (iii): If u_1, v_1 are adjacent vertices in $G_1 = (V_1, E_1)$.

If u_1, v_1 are adjacent vertices in $G_1 = (V_1, E_1)$. The graph $G_1 = (V_1, E_1)$ be a sum divisor cordial graph therefore we get mn number sum divisor cordial graph isomorphic to $G_1 = (V_1, E_1)$. Hence we get mn times of $G_1 = (V_1, E_1)$.

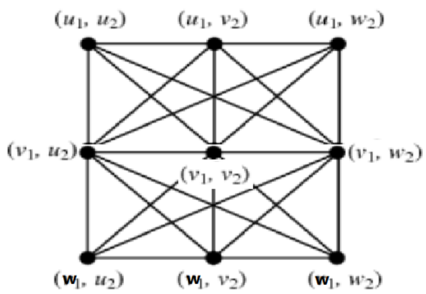
From case (i) and (ii), we get $\psi = \left\{ \left(\bigcup_{i=1}^{m} H_i \right) \cup \left(\bigcup_{j=1}^{n} H_j \right) \cup \left(\bigcup_{j=1}^{n} \left(H_{1j}, H_{2j}, \ldots H_{mj} \right) \right) \right\}$ this implies $|\psi| = \sum_{i=1}^{m} H_i + \sum_{j=1}^{n} H_j + \sum_{j=1}^{n} \sum_{i=1}^{m} H_{ij} = m + n + mn$. Hence we get

$$\pi_g (G_1 \circ G_2) \geq (m + n + mn).$$

Illustration 2.3: The Cartesian product of two sum divisor cordial graphs $P_3 \& P_3$ is given in Figure.2.3

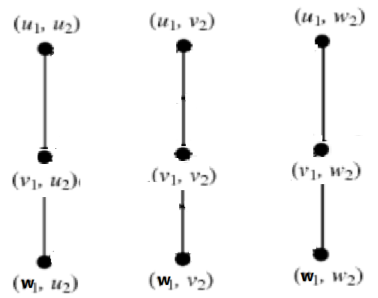


Decomposition of $G_1 \circ G_2$

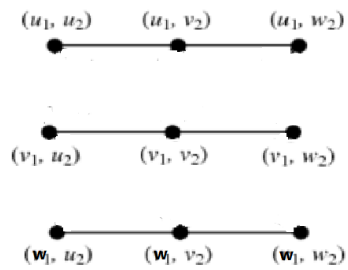


$G_1 \circ G_2$

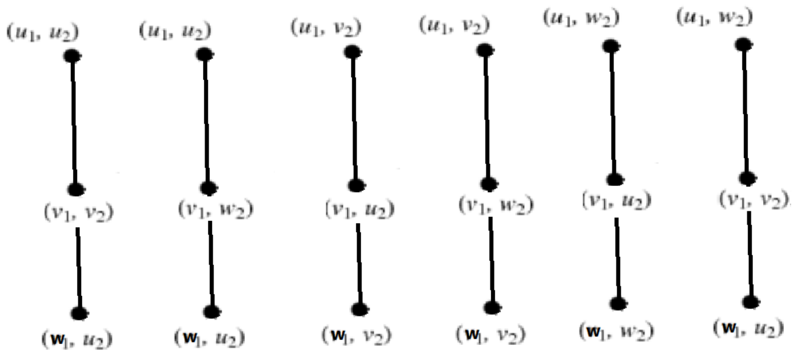
Isomorphic to G_1



Isomorphic to G_2



Isomorphic to 'mn' times of G_1



2. CONCLUSION:

In this paper we define prime decomposition and prime decomposition number $\pi_S(G)$ of graphs. Also investigate some bounds of $\pi_S(G)$ in product graphs like Cartesian product, composition etc. In future we will investigate the decomposition number various labeling in graphs.

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