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Analysis Of $M^X/G/1$ Queueing System With Priority Queue And Multiple Vacations: A Simulation Approach

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Abstract: In this paper, $M^X/G/1$ queueing system with priority queue, non-priority queue and multiple vacations is considered. The server provides service one by one to the arriving customers in the priority queue which has finite capacity 'N' and the non-priority customers which has infinite capacity. On service completion epoch of priority or non-priority customers, the server checks the priority queue and continues service if customers are available in priority queue. Otherwise, it checks the non-priority queue and if customers are available, then it provides service for the customers in non-priority queue. If both the queues are empty, the server goes for multiple vacations until customers arrive to any of the two queues.Since the analytical results are tedious to obtain performance measures, the proposed model is analyzed using simulation software Flexsim 2017 and various metrics are obtained. Numerical illustration is also provided to validate the results.

Keywords:Multiple vacations, Priority Queue, Simulation, Flexsim.Mathematics Subject Classification: 60K25, 60K20, 90B22, 68M20.

1. INTRODUCTION

Priority queueing models are more realistic in nature. Qing and Chakravarthy (2012) have modelled multi-server queue with Markovian arrivals and priority services and have obtained an analytical solution for exponential services and simulated results for non-exponential services. Krishnamoorthy *et al.* (2010) have focused on the impact of priority generations in a multi-priority queueing system. They have simulated the proposed queueing model using ARENA software and discussed some numerical examples.

In this paper, $M^X/G/1$ queueing system with priority queue, non-priority queue and multiple vacations is considered. The server provides service one by one to the arriving customers in the priority queue which has finite capacity 'N' and the non-priority customers which has infinite capacity. On service completion epoch of priority or non-priority customers, the server checks the priority queue and continues service if customers are available in priority queue. Otherwise, it checks the non-priority queue and if customers are available, then it provides service for the customers in non-priority queue. If both the queues are empty, the server goes for multiple vacations until customers arrive to any of the two queues. Figure 1 depicts the schematic representation of the model considered. The

Probability Generating Function for the proposed model is derived using the supplementary variable technique and simulation is also performed to justify it.

An example of the proposed queueing model is the automatic oil press machine which is used in oil production line. The oil press machine is used to extract oil from seeds like coconut, walnut, peanut, sesame, etc. by pressing process (**service**). A small machine has the capacity of pressing 20kg of seeds/hr. Among those edible seeds, copra, the chief commercial product from coconut cannot be stored for long time since it may decay and lead to deterioration very soon. Therefore copra (**priority customer**) iskeptin the priorityqueue which has finite capacity (to avoid prolonged preservation of other items)while other seeds such as sesame, peanut, walnut (**non-priority customers**) are

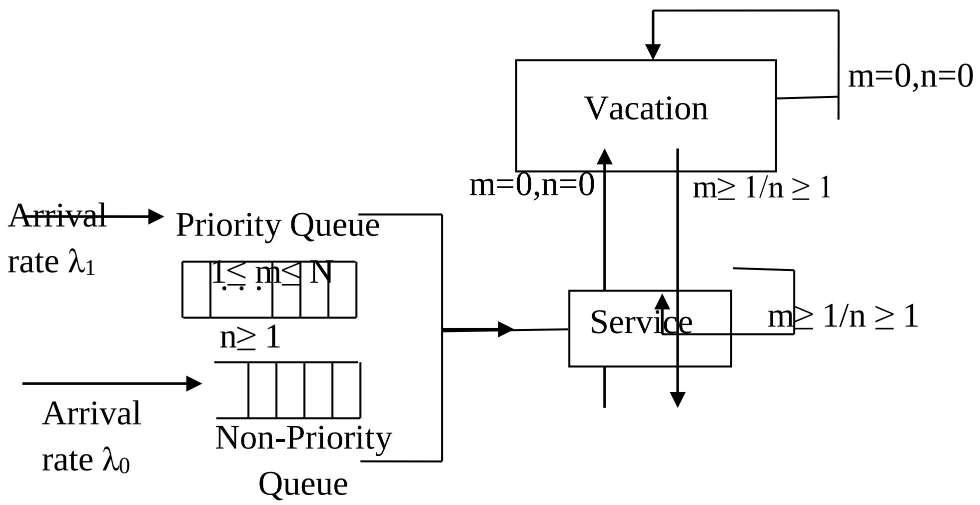

Figure1 Schematic representation

kept in non-priority queue. The smaller particles of the edible seeds (say, one unit of a particular raw material because of one by one service) are loaded into the pressing machine for pressing and oil is extracted. After pressing or squeezing, the oil is filtered and discharged from the pressing machine. On process completion epoch of priority or non-priority queue materials, the server checks the priority queue and continues service if materials are available in priority queue. Otherwise the server begins process for the materials in non-priority queue if available.

When there are no raw materials for pressing process, the server does other works like refining, roasting or cutting raw materials etc., which can be considered as the secondary process **(vacation)** until materials arrive to any of the two queues. The situation mentioned above can be modeled as bulk queueing system with multiple vacations, priority and non-priority queue.

2. LITERATURE SURVEY

Simulation technique is one of the trending fields used to reduce time and cost in production system. Labzovski *et al.* (2000) have discussed a priori vacation probability in the single server queueing models with single vacation. Van Dijk and Der Sluis (2006) have investigated a check-in problem in airport and found the optimum value using simulation. Using a queueing model approach, simulation of urban traffic control has been discussed by Grether *et al.* (2012).

Using simulation, Lin and Chen (2012) have established an analytical procedure to evaluate the behaviour of pull production systems. Beaverstock *et al.* (2012) have discussed simulation modeling and analysis using Flexsim. Using Flexsim, research on AS/RS simulation has been carried out by Tang *et al.* (2013). Semanco and Marton (2013) have investigated M/G/1 manufacturing queueing model using simulation and observed some of the performance measures. They have used Arena, Simul8 and Witness software to compare the results. Ahsan *et al.* (2014) have carried out queueing analysis of a busy restaurant and suggested a new model to reduce the waiting time of the customers by incorporating shifting server. Kumar *et al.* (2015) have modelled the Flexible Manufacturing System (FMS) using the simulation software, Flexsim and analyzed its performance measures.

A preemptive priority queue has been analyzed by Horvath (2012). Sharif *et al.* (2014) have discussed a multi server accumulating priority queue along with health care application. Wang *et al.* (2015) have analyzed M/M/c queueing system with two priority classes. Krishnamoorthy and Manjunath (2018) have considered two priority queueing systems with feedback and have derived the waiting time distribution. Mojalal *et al.* (2019) have discussed the delayed accumulating priority queue and waiting time distribution for the lower-class of customers in accumulating priority queue. Using integrated computer simulation model, Azadeh *et al.* (2015) have presented an M/G/C retrial queueing system with linear retrial policy, feedback and geometric loss. Their objective has been to minimise the total cost and to find the optimal solution. Galankashi *et al.* (2016) have developed a simulation model for a petrol station queuing system to optimize the sales rate. They have used Witness 2014 simulation software for performance evaluation. Greasley and Owen (2018) have provided a literature review on modelling behavioural responses of people in Operations Management (OM) using discrete-event simulation. Chew (2019) has analyzed the continuous-service M/M/1 queueing system using simulation. He has obtained the important performance measures and compared them with the results of standard M/M/1 queueing system. A Non-Markovian bulk queueing system with state dependent arrivals and multiple vacations has been studied by Ramaswami and Jeyakumar (2014). They used ARENA software to model the system and derived some of the performance measures. Moazzami*et al.* (2013) have focussed on modeling the behaviour of a petrol station and they have used WITNESS 2004 simulation software to model and analyze it.

Therefore, research on queueing models with simulation has been gaining importance in recent years. In the literature, only very few authors have given attention on analyzing queueing models using simulation. The primary focus of this paper will be on modelling and analysis of the proposed queueing model using simulation technique.

3. MATHEMATICAL MODEL

Let 'X' be the group size random variable of the arrival, 'g_k' be the probability that 'k' customers arrive in a batch and X(z) be its probability generating function (PGF) and 'λ_1' and 'λ_0' be the Poisson arrival rate of priority and non-priority customers respectively. Let $S_1(x)$ ($s_1(x)$) $\{\tilde{S}_1(\theta)\}$ $[S^0(x)]$ be the cumulative distribution function (probability density function) {Laplace-Stieltjes transform} [remaining service time] of service for priority customer. Let $S_0(x)$ ($s_0(x)$) $\{\tilde{S}_0(\theta)\}$ $[S^0(x)]$ be the cumulative distribution function (probability density function) {Laplace-Stieltjes transform} [remaining service time] of service for non-priority customer. Let V(x) (v(x)) $\{\tilde{V}(\theta)\}$ $[V^0(x)]$ be the cumulative distribution function (probability density function) {Laplace-Stieltjes transform} [remaining vacation time] of vacation. $N_1(t)$

and $N_2(t)$ denote the number of customers in priority and non-priority queue at time 't' respectively. The different states of the server at time 't' are defined as follows:

$$c(t) = \begin{cases} 0, \text{when the server is busy} \\ 1, \text{when the server is on vacation} \end{cases}$$

To obtain system equations, the state probabilities are defined as follows:

$$P_{m,n}(x,t)\,dt = \Pr\begin{Bmatrix} N_1(t) = m; N_2(t) = n; \\ x \le S^0(t) \le x + dt; c(t) = 0 \end{Bmatrix}, \quad n \ge 0, 0 \le m \le N \qquad (1)$$

$$Q_{m,n,l}(x,t)\,dt = \Pr\begin{Bmatrix} N_1(t) = m; N_2(t) = n; \\ x \le V^0(t) \le x + dt; c(t) = 1 \end{Bmatrix}, \quad n \ge 0, 0 \le m \le N, l \ge 1 \qquad (2)$$

STEADY- STATE ANALYSIS

IN THIS SECTION, THE STEADY STATE QUEUE SIZE EQUATIONS ARE OBTAINED AS FOLLOWS:

$$-\frac{d}{dx} P_{0,0}(x) = -(\lambda_1 + \lambda_0) P_{0,0}(x) + P_{1,0}(0)\, s_1(x) + \sum_{j=1}^{\infty} Q_{1,0,j}(0) s_1(x) + \sum_{j=1}^{\infty} Q_{0,1,j}(0) s_0(x) + P_{0,1}(0)\, s_0(x) \qquad (3)$$

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$$-\frac{d}{dx}P_{m,0}(x) = -(\lambda_1+\lambda_0)P_{m,0}(x) + \sum_{k=1}^{m} P_{m-k,0}(x)\,\lambda_1 g_k + P_{m+1,0}(0)s_1(x) + \sum_{j=1}^{\infty} Q_{m+1,0,j}(0)s_1(x)\,; \quad 1 \le m \le N-1 \quad (4)$$

$$-\frac{d}{dx}P_{0,n}(x) = -(\lambda_1+\lambda_0)P_{0,n}(x) + \sum_{j=1}^{\infty} Q_{1,n,j}(0)s_1(x) + \sum_{k=1}^{n} P_{0,n-k}(x)\,\lambda_0 g_k + P_{1,n}(0)\,s_1(x) + P_{0,n+1}(0)\,s_0(x) + \sum_{j=1}^{\infty} Q_{0,n+1,j}(0)s_0(x)\quad ; n \ge 1 \quad (6)$$

$$-\frac{d}{dx}P_{N,0}(x) = -(\lambda_1+\lambda_0)P_{N,0}(x) + \sum_{k=1}^{N} P_{N-k,0}(x)\,\lambda_1 g_k \quad (5)$$

$$-\frac{d}{dx}P_{m,n}(x) = -(\lambda_1+\lambda_0)P_{m,n}(x) + P_{m+1,n}(0)\,s_1(x) + \sum_{j=1}^{\infty} Q_{m+1,n,j}(0)s_1(x) + \sum_{k_2=1}^{n}\sum_{k_1=1}^{m} P_{m-k_1,n-k_2}(x)\lambda_1\lambda_0 g_{k_1}g_{k_2} + \sum_{k_1=1}^{m} P_{m-k_1,n}(x)\lambda_1 g_{k_1} + \sum_{k_2=1}^{n} P_{m,n-k_2}(x)\lambda_0 g_{k_2}\,;$$

$$1 \le m \le N-1, n \ge 1 \quad (7)$$

$$-\frac{d}{dx}P_{N,n}(x) = -(\lambda_1+\lambda_0)P_{N,n}(x) + \sum_{k_2=1}^{n}\sum_{k_1=1}^{N} P_{N-k_1,n-k_2}(x)\lambda_1\lambda_0 g_{k_1}g_{k_2} + \sum_{k_1=1}^{N} P_{N-k_1,n}(x)\lambda_1 g_{k_1} + \sum_{k_2=1}^{n} P_{N,n-k_2}(x)\lambda_0 g_{k_2}\,;$$

$$n \ge 1 \quad (8)$$

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$$-\frac{d}{dx}Q_{0,0,1}(x)=-(\lambda_1+\lambda_0)Q_{0,0,1}(x)+P_{0,0}(0)\,v(x) \quad (9)$$

$$-\frac{d}{dx}Q_{0,0,j}(x)=-(\lambda_1+\lambda_0)Q_{0,0,j}(x)+Q_{0,0,j-1}(0)\,v(x);\quad j\geq 2 \quad (10)$$

$$-\frac{d}{dx}Q_{m,0,j}(x)=-(\lambda_1+\lambda_0)\,Q_{m,0,j}(x)+\sum_{k=1}^{m}Q_{m-k,0,j}(x)\lambda_1 g_k\,; \quad (11)$$
$$j\geq 1,\ 1\leq m\leq N$$

$$-\frac{d}{dx}Q_{0,n,j}(x)=-(\lambda_1+\lambda_0)Q_{0,n,j}(x)+\sum_{k=1}^{n}Q_{0,n-k,j}(x)\lambda_0 g_k\,; \quad (12)$$
$$j\geq 1,\ n\geq 1$$

$$-\frac{d}{dx}Q_{m,n,j}(x)=-(\lambda_1+\lambda_0)Q_{m,n,j}(x)+\sum_{k_2=1}^{n}\sum_{k_1=1}^{m}Q_{m-k_1,n-k_2}(x)\lambda_1\lambda_0 g_{k_1}g_{k_2}+\sum_{k=1}^{m}Q_{m-k,n,j}(x)\lambda_1 g_k+\sum_{k=1}^{n}Q_{m,n-k,j}(x)\lambda_0 g_k\,;$$
$$1\leq m\leq N\ ,n\geq 1,\ j\geq 1 \quad (13)$$

PROBABILITY GENERATING FUNCTION

At an arbitrary timeepoch, the
Probability Generating function (PGF) of the queue size is derived as

$$P(z)=\tilde{P}\left(z_1,z_2,0\right)+\tilde{Q}_1\left(z_1,z_2,0\right)+\sum_{j=2}^{\infty}\tilde{Q}_j\left(z_1,z_2,0\right)$$

Since it is tedious to obtain the closed form of PGF, analytical expressions as well as numerical justifications of the performance measures of the proposed model, simulation is performed.

SIMULATION MODELLING

Figure 2 depicts the simulation model developed for the proposed queueing system using Flexsim 2019. The objects used for modelling are source, queue, processor (server) and sink (exit). There are two queues in the model, one is the priority queue whose capacity is 'N' and the other one is non-priority queue which has infinite capacity. Source1 and source2 generate entities (customers) for

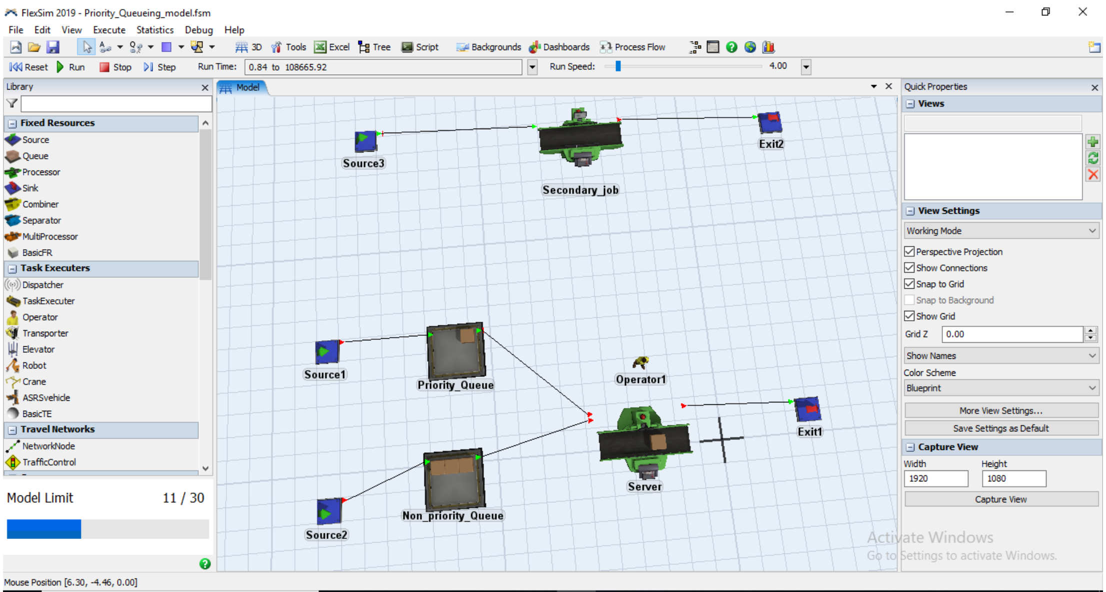

Figure 2 Simulation modelling of the bulk queueing system with priority queue and multiple vacations

priority queue and non-priority queue respectively. The inter-arrival time of the customers in both the queues follow exponential distribution. At first, the server provides service for the customers in priority queue and it serves non-priority customers after completing the service for all priority customers. After service completion, the customers leave the system through exit1.

On the other hand, when there are no customers in both queues, then the operator goes for vacation/secondary job. Source 3 in the model generates entities for secondary job and after completion of secondary job, the entities leave the system through exit2.

4. NUMERICAL ILLUSTRATION

The performance measures are evaluated in order to predict the behaviour of the queueing system. Across industries and disciplines, simulation modelling provides valuable solutions by giving clear insights into complex systems. Simulation is often utilized when conducting experiments on a real system is impossible or impractical, often because of cost and time. In general, to validate the queueing model, numerical illustration needs to be provided. But for the proposed model, since the analytical solution is too complex, it is tedious to justify numerically. Thus simulation for the proposed queueing model is performed. The performance metrics such as total number of inputs, outputs, average content, average waiting time, processing time and idle time, etc., which are difficult to find theoretically, are easily be obtained using simulation. To be precise, a simulation model can capture many more details than an analytical model such as number of inputs, outputs, etc., in every object/state. The simulation software, Flexsim 2019, is used for simulation of the model and justified through numerical illustration.

The simulation model developed for the proposed queueing system is justified in this section. In the oil production plant, the arrival of raw material copra **(priority customer)** follows Poisson process with arrival rateλ_1' and the arrival of other raw materials **(priority customer)** like walnut, peanut and sesame etc., follow Poisson process with arrival rate'λ_0'. It is noted that the pressing process **(service)** takes same time for all the raw materials (i.e. once the process starts, the operation takes particular time for the process to complete). The server provides service one by one (i.e. one unit of a particular raw material, say 'm' kg) to the arriving customers in the priority queue which has finite capacity 'N' and the non-priority customers which has infinite capacity. On service completion epoch of priority or non-priority customers, the server checks the priority queue and continues service if customers are available in priority queue. Otherwise, it provides service for the customers in non-priority queue if available. If both the queues are empty, the server goes for other processes like refining, roasting or cutting **(multiple vacations)** until customers arrive to any of the two queues.

The results are obtained using theassumptions and notations: Service time distribution is 2-Erlang random variable with parameter μ ($E(S)= 1/\mu$, $E(S^2)=3/2\mu^2$). Vacation time is exponential random variable with parameter ξ ($E(V)= 1/\xi$, $E(V^2)= 2/\xi^2$). Batch size distribution of the arrival is geometric random variable with mean 2 ($E(X)=2$).

The simulated results are noted for run time: 108665.92 minutes or 75.46 days approx. The simulation process has not taken 75.46 days to run. The operator can fix the running time and increase the running speed through the run speed option available in the software where it doesn't take longer time to produce the results. In order to obtain the steady state result, the run time is fixed as 75 days approximately and validated through 3 replications. It is possible to generate results within short period (few time units) through simulation whereas it is tedious to obtain the results analytically.

4.1 Arrival Rate Vs Performance Metrics

The effects of different parameters like average number of components, average staytime, idle time, processing time, inputs and outputs for different objects/states at an arbitrary time are summarized in the Tables1 and 2. These performance metrics are found for various arrival rates of priority and non-priority customers. Source1 and Source2 generate entities/customers for priority and non-priority customers respectively.

- From Table 1 and Figures 3 and 4, when the arrival rate (λ_1) of the priority customers increases
    - **source1**: the total number of components (which is used to represent the arrival of priority customers) from source1 increases.
    - **source2**: the total number of components (which is used to represent the arrival of non-priority customers) from source2 has no significant variation (has only negligible difference).
    - **queue1 (priority queue)**: the average number of components increases in the priority queue size. Average staytime in the queue has slight variation which is negligible. Total number of inputs and outputs of priority queue increases.
    - **Queue2 (non-priority queue)**: the average number of components and average staytime in the non-priority queue increases. Total number of inputs and outputs of non-priority queue have no significant variation.
    - **server**: the average number of components, processing time, total number of inputs and outputs increase whereas idle time decreases.

- **exit1**: the average number of components in exit1 has no significant difference and the total number of outputs increases.

o From Table 2 and Figures 5 and 6, when the arrival rate (λ_0) of the non-priority customers increases
  - **source1**: the total number of components(which is used to represent the arrival of priority customers) from source1 has slight variation which is negligible.
  - **source2**: the total number of components (which is used to represent the arrival of non-priority customers) from source2 increases.
  - **queue1 (priority queue)**: the average number of components, average staytime in the queue decreases. Total number of inputs and outputs of priority queue has no change.
  - **queue2 (non-priority queue)**: the average number of components, average staytime, total number of inputs and outputs of the non-priority queue increases.
  - **server**: the average number of components, processing time, total number of inputs and outputs increase whereas idle time decreases. Average staytime has no variation.
  - **exit1**: the average number of components in exit1 has no significant difference and the total number of output increases.

4.2 Service Rate Vs Performance Measures

The parameters like average number of components, staytime, etc., are found for different service rates in the Table 3.

o From Table 3 and Figures 7 and 8, when the service rate (μ) of the server increases, the following effects are seen:

- **source1**: the total number of componentsfrom source1 has negligible difference.
- **source2**: the total number of componentsfrom source2 has negligible difference.
- **queue1 (priority queue)**: the average number of components and average staytime in the queue decreases. Total number of inputs and outputs of priority queue has slight (negligible) variations.
- **queue2 (non-priority queue)**: the average number of components and average staytime in the queue decreases. Total number of inputs and outputs of the non-priority has slight (negligible) variations.
- **server**: the average number of components, average staytime processing time, decrease whereas idle time increases. Total number of inputs and outputs has no significant effect.
- **exit1**: the average number of components and the total number of outputs have no significantdifference.

Flexsim summary report
Time in minutes and components in units (1 unit = m kg)

Table 1. Arrival rate of Priority queue Vs Performance metrics
(For μ=30, N=10, ξ= 0.5)

λ	Object	Average Components / E(Q)	Average Staytime	Idle Time	Processing Time	Total no. of Input components	Total no. of Output components
λ_1=1, λ_0= 10	Source1	-	-	-	-	-	108619
	Source2	-	-	-	-	-	1087855
	Queue1	0.1776	0.1777	-	-	108619	108619
	Queue2	7.1766	0.7169	-	-	1087855	1087852
	Server	0.7706	0.0699	24922.71	79621.12	1196471	1196470
	Exit1	0.9999	-	-	-	1196470	-
λ_1=2, λ_0= 10	Source1	-	-	-	-	-	217172
	Source2	-	-	-	-	-	1086359
	Queue1	0.3234	0.1618	-	-	217172	217170
	Queue2	8.3879	0.8390	-	-	1086359	1086346
	Server	0.8277	0.0690	18727.37	86822.98	1303516	1303515
	Exit1	0.9999	-	-	-	1303515	-
λ_1=3, λ_0=10	Source1	-	-	-	-	-	326033
	Source2	-	-	-	-	-	1087198
	Queue1	0.4125	0.1375	-	-	326033	326033
	Queue2	10.5941	1.0588	-	-	1087198	1087195
	Server	0.8855	0.0681	12443.74	94125.02	1413228	1413227
	Exit1	0.9999	-	-	-	1413227	-
λ_1=4, λ_0=10	Source1	-	-	-	-	-	434340
	Source2	-	-	-	-	-	1086910
	Queue1	0.4366	0.1092	-	-	434340	434340
	Queue2	16.3652	1.6361	-	-	1086910	1086903
	Server	0.9420	0.0673	6299.63	101290.41	1521243	1521242
	Exit1	0.9999	-	-	-	1521242	-

Source1-Arrival of components for priority queue; Source2-Arrival of components for non-priority queue; Queue1-Priority queue; Queue2-Non-priority queue; Exit1-Number of components exit from the system after service completion; Input-Total input to the object at an arbitrary time; Output-Total output from the object at an arbitrary time; Nil value in a row represents Not Applicable/No value

Table 2 Arrival rate of Non-Priority queue Vs Performance metrics
(For μ=30, N=10, ξ= 0.5)

λ	Object	Average Components	Average Staytime	Idle Time	Processing Time	Total no. of Input components	Total no. of Output components
$\lambda_1=2\ \lambda_0=11$	Source1	-	-	-	-	-	217514
	Source2	-	-	-	-	-	1195402
	Queue1	0.2562	0.1280	-	-	217514	217514
	Queue2	10.9339	0.9939	-	-	1195402	1195386
	Server	0.8857	0.0681	12416.54	94162.81	1412900	1412899
	Exit1	0.9999	-	-	-	1412899	-
$\lambda_1=2,\ \lambda_0=11.5$	Source1	-	-	-	-	-	216987
	Source2	-	-	-	-	-	1249300
	Queue1	0.2174	0.1089	-	-	216987	216987
	Queue2	12.8484	1.1176	-	-	1249300	1249279
	Server	0.9135	0.0677	9403.59	97672.40	1466266	1466265
	Exit1	0.9999	-	-	-	1466265	-
$\lambda_1=2,\ \lambda_0=12$	Source1	-	-	-	-	-	217106
	Source2	-	-	-	-	-	1306762
	Queue1	0.1832	0.0917	-	-	217106	217106
	Queue2	17.2163	1.4317	-	-	1306762	1306752
	Server	0.9435	0.0673	6136.21	101497.19	1523858	1523857
	Exit1	0.9999	-	-	-	1523857	-
$\lambda_1=2,\ \lambda_0=12.5$	Source1	-	-	-	-	-	216788
	Source2	-	-	-	-	-	1358317
	Queue1	0.1505	0.0754	-	-	216788	216788
	Queue2	27.3692	2.1895	-	-	1358317	1358284
	Server	0.9703	0.0669	3228.74	104893.62	1575072	1575071
	Exit1	0.9999	-	-	-	1575071	-

Source1-Arrival of components for priority queue; Source2-Arrival of components for non-priority queue; Queue1-Priority queue; Queue2-Non-priority queue; Exit1-Number of components exit from the system after service completion; Input-Total input to the object at an arbitrary time; Output-Total output from the object at an arbitrary time; Nil value in a row represents Not Applicable/No value

Table 3 Service rate (μ) Vs Performance metrics
(For λ_1=0.5, λ_0=1.5, N=10, ξ= 0.5)

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μ	Object	Average Components	Average Staytime	Idle Time	Processing Time	Total no. of Input components	Total no. of Output components
μ=4.0	Source1	-	-	-	-	-	54455
	Source2	-	-	-	-	-	162602
	Queue1	0.2509	0.5007	-	-	54455	54454
	Queue2	175.1122	117.0735	-	-	162602	162522
	Server	0.9984	0.50000	170.3044	108478.19	216976	216975
	Exit1	0.9999	-	-	-	216975	-
μ=4.5	Source1	-	-	-	-	-	54387
	Source2	-	-	-	-	-	163129
	Queue1	0.2260	0.4516	-	-	54387	54386
	Queue2	6.1032	4.0656	-	-	163129	163119
	Server	0.8984	0.4488	11041.36	96474.78	217505	217504
	Exit1	0.9999	-	-	-	217504	-
μ=5.0	Source1	-	-	-	-	-	54272
	Source2	-	-	-	-	-	162882
	Queue1	0.2120	0.4244	-	-	54272	54272
	Queue2	3.1784	2.1204	-	-	162882	162875
	Server	0.8177	0.4092	19806.09	86778.09	217147	217146
	Exit1	0.9999	-	-	-	217146	-
μ=5.5	Source1	-	-	-	-	-	54465
	Source2	-	-	-	-	-	163029
	Queue1	0.2061	0.4112	-	-	54465	54465
	Queue2	2.3296	1.5528	-	-	163029	163029
	Server	0.7530	0.3762	26837.50	79053.63	217494	217494
	Exit1	0.9999	-	-	-	217494	-

Source1-Arrival of components for priority queue; Source2-Arrival of components for non-priority queue; Queue1-Priority queue; Queue2-Non-priority queue; Exit1-Number of components exit from the system after service completion; Input-Total input to the object at an arbitrary time; Output-Total output from the object at an arbitrary time; Nil value in a row represents Not Applicable/No value

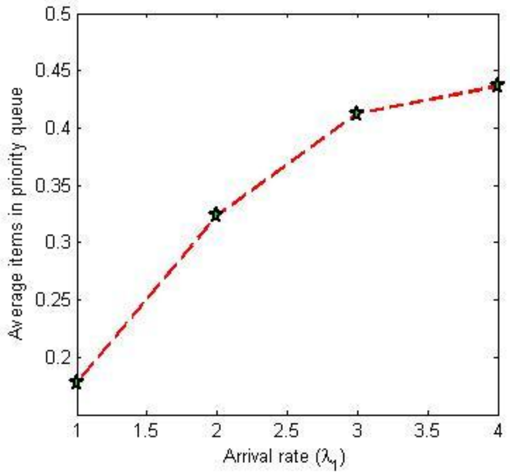


Figure 3 Arrival rate (λ_1) Vs
Average items in Priority queue

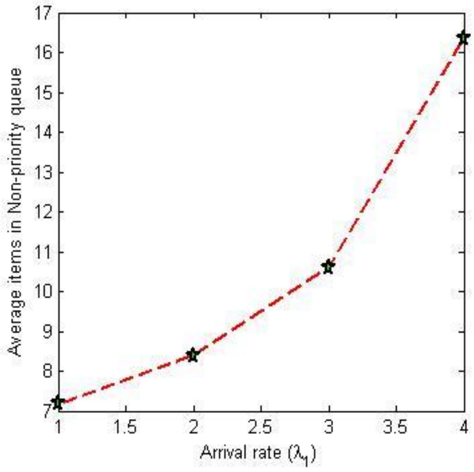


Figure 4 Arrival rate (λ_1) Vs
Average items in Non-Priority

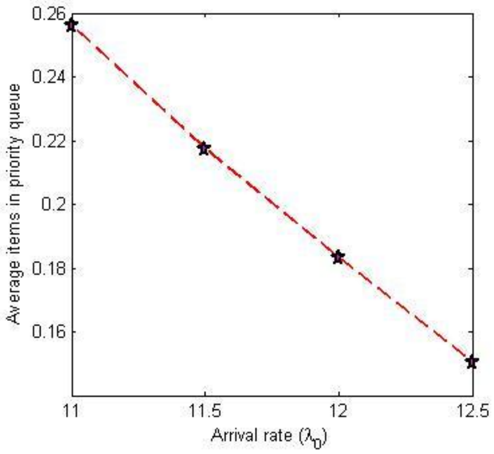


Figure 5 Arrival rate (λ_0) Vs
Average items in Priority queue

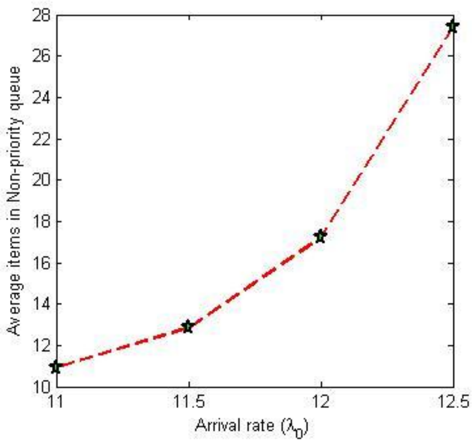
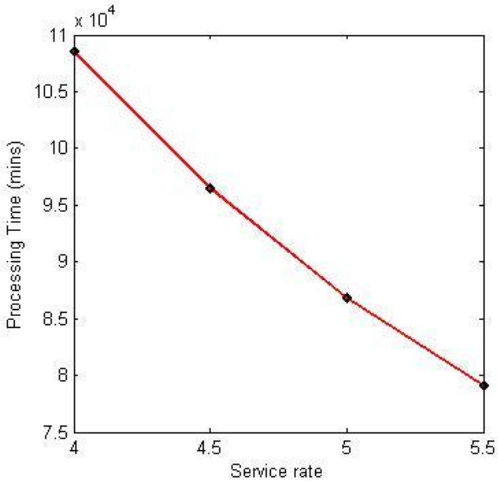


Figure 6 Arrival rate (λ_0) Vs
Average items in Non-Priority



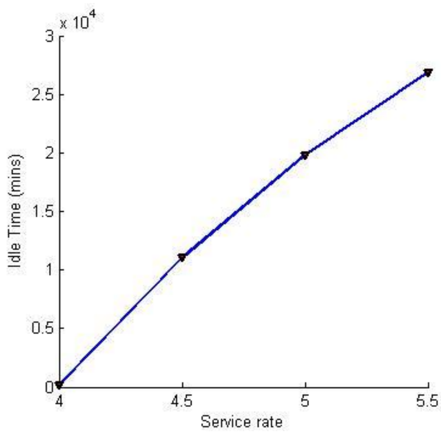


Figure 7 Service rate VsIdle time time

Figure 8 Service rate VsIdle time Processing Processing time

5. CONCLUSION

A queueing system with multiple vacations, priority and non-priority queue has been considered in this paper. A real time scenario for the proposed model has also been provided. The probability generating function of the queue size at an arbitrary time epoch has been derived. Some of the performance metrics have been discussed using simulation.

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