

RECENT TRENDS IN LINEAR PROGRAMMING:
VEHICLE ROUTING PROBLEM

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Abstract:

In this paper we discuss the *Vehicle Routing Problem (VRP)*, the goal is to find optimal routes for multiple vehicles visiting a set of locations. We mean by "optimal routes" for a *Vehicle Routing Problem* is the routes with the least total distance. However, if there are no other constraints, the optimal solution is to assign just one vehicle to visit all locations, and find the shortest route for that vehicle. This is essentially the same problem as the Transportation Salesmen Problem. A better way to define optimal routes is to minimize the length of the longest single route among all vehicles. This is the right definition if the goal is to complete all deliveries as soon as possible. The *Vehicle Routing Problem* finds optimal routes defined. Also in this paper we discuss, why is it a challenge to solve the Vehicle Routing Problem?

Keywords:

Vehicle Routing Problem, Heuristic, The Clarke and Wright algorithm, Approximation algorithms.

Introduction:

One problem in the field of transportation related OR that has been given a lot of attention in the scientific literature is the so called vehicle routing problem (VRP). The **vehicle routing problem** (VRP) is a combinatorial optimization and integer programming problem which asks "What is the optimal set of routes for a fleet of vehicles to traverse in order to deliver to a given set of customers?" In the vehicle routing problem we are given a fleet of vehicles and a set of customers to be visited. The vehicles are often assumed to have a common home base, called the depot. The cost of travelling between each pair of customers and between the depot and each customer is given. Our task is to find a route for each vehicle, starting and ending at the depot, such that all customers are served by exactly one vehicle, and such that the overall cost of the routes are minimized. Typically, the solution has to obey several other restrictions, such as capacity of the vehicles or desired visit times at customers.

The class of vehicle routing problems contains all the problems that involve creating one or more routes, starting and ending in one or more common depots or at predefined start and end terminals. In the literature the term vehicle routing problem is occasionally used for the specific problem that is called the capacitated vehicle routing problem. A subclass of vehicle routing problems is pickup and delivery problems. In this class of problems, we are given a number of requests and a fleet of vehicles to serve the request. Each request consists of a pickup at some location and a delivery at another location. The cost of travelling between each pair of locations is given. The problem is to find routes for each vehicle such that all pickups and deliveries are served and such that the pickup and delivery corresponding to one request is served by the same vehicle and the pickup is served before the delivery. Again a number of additional constraints are often enforced, the most typical being capacity and time window constraints.

Types of solution



Three types of solution methods are typically employed to solve these types of problems.

• **Heuristics:**

Heuristics are solution methods that typically relatively quickly can find a feasible solution with reasonable quality. There are no guarantees about the solution quality though, it can be arbitrarily bad. The heuristics are tested empirically and based on these experiments comments about the quality of the heuristic can be made. Heuristics are typically used for solving real life problems because of their speed and their ability to handle large instances. A special class of heuristics that has received special attention in the last two decades is the meta-heuristics. Meta-heuristics provides general frameworks for heuristics that can be applied to many problem classes. High solution quality is often obtained using metaheuristics.

• **Approximation algorithms:**

Approximation algorithms are a special class of heuristic that provide a solution and an error guarantee. For example one method could guarantee that the solution obtained is at most k times more costly than the best solution obtainable. Two classes of approximation algorithms called polynomial time approximation scheme (PTAS) and fully polynomial time approximation scheme (FPTAS) are of special interest as they can approximate the solution with any desired precision. An FPTAS is in a certain sense “stronger” than a PTAS. An example of a problem that admits an FPTAS is the Knapsack problem. For some problems it is not possible to design a FPTAS, PTAS or even a polynomial time approximation algorithm with constant error guarantee unless $P = NP$ and approximation can be impractical: the error guarantee can be too poor or the running time of the algorithm can be too high

• **Exact methods:**

Exact methods guarantee that the optimal solution is found if the method is given sufficiently time and space. As stated initially, a simple enumeration is out of the question, so exact methods must use cleverer techniques. The worst case running time for NP-Hard problems are still going to be high though. We cannot expect to construct exact algorithms that solve NP-hard problems in polynomial time unless $NP = P$. For some classes of problems there is hope of finding algorithms that solve problem instances occurring in practice in reasonable time though.

Objective Functions in Vehicle Routing Problems

- Minimization of Travel-Dependent Parameters
- Minimization of the Number of Utilized Vehicles
- Minimization of the Sum of Tour Durations
- Minimization of the Completion Time
- Minimization of Lateness Costs
- Minimization of the Number of unserved Customers
- Minimization of Customer Inconvenience and Request Response Time

Heuristic Algorithms:

Heuristic algorithms for the VRP can often be derived from procedures derived from the TSP(Travelling Salesman Problem). The nearest neighbour algorithm, insertion algorithms and tour improvement procedures can be applied to CVRPs and DVRPs almost without modifications. However, when applying these methods to VRPs care must be taken to ensure that only feasible vehicle routes are created.

The Clarke and Wright algorithm:

This classical algorithm was first proposed in 1964 by Clarke and Wright to solve CVRPs in which the number of vehicles is free. The method starts with vehicle routes containing the depot and one other vertex. At each step, two routes are merged according to the largest saving that can be generated.

Step 1. Compute the savings $s_{ij} = C_{il} + C_{lj} - C_{ij}$ for $i, j = 2, \dots, n$, and $i \neq j$. Create $n-1$ vehicle routes $(1, i, 1)$ ($i = 2, \dots, n$).

Step 2. Order the savings in a non-increasing fashion.

Step 3. Consider two vehicle routes containing arcs (i, l) and (l, j) , respectively. If $s_{ij} > 0$ tentatively merge these routes by introducing arc (i, j) and by deleting arcs (i, l) and (l, j) . Implement the merge if the resulting route is feasible. Repeat this step until no further improvement is possible. Stop.

The Clarke and Wright algorithm implicitly ignores vehicle fixed costs and fleet size. Vehicle costs f can easily be taken into account by adding this constant to every C_{ij}

($j = 2, \dots, n$). Solutions with a fixed number of vehicles can be obtained by repeating Step 3 until the required number of routes has been reached, even if the savings become negative.

The sweep algorithm:

The origins of the sweep algorithm can be traced back to the work of Wren (1971) and Wren and Holliday (1972) for CVRPs with one or several depots, and vertices located in the Euclidean plane. In order to ease the implementation of this method, it is preferable to represent vertices by their polar coordinates (θ_i, ρ_i) , where θ_i is the angle and ρ_i is the ray length. Assign a value $\theta_i = 0$ to an arbitrary vertex i^* and compute the remaining angles from (l, i^*) . Rank the vertices in increasing order of their θ_i . A possible implementation of the method is the following.

Step 1. Choose an unused vehicle k .

Step 2. Starting from the unrouted vertex having the smallest angle, assign vertices to the vehicle as long as its capacity is not exceeded. If unrouted vertices remain, go to Step 1.

Step 3. Optimize each vehicle route separately by solving the corresponding Travelling Salesman Problem (exactly or approximately). Perform vertex exchanges between adjacent routes if this saves distance. Re-optimize and stop.

Why is it a challenge to solve the Vehicle Routing Problem?

In real-life scenarios, Vehicle Routing Problem is not about providing the shortest possible route. There are many other factors that contribute to planning the optimized routes. The complexity in VRP arises due to the presence of multiple constraints and its unpredictable nature.

Massive computational effort required

Vehicle Routing Problem requires a massive computational effort.

Even experienced dispatchers and drivers cannot solve it manually as various factors contribute to it.

Unpredictable factors:

There are uncertainties attached to real-life route planning that makes VRP very hard to solve. They can be changing customer demands, traffic jams, unexpected roadblocks due to maintenance, and so on.

Why is it crucial to have an effective solution for VRP?

The primary reason for an effective VRP solution is to reduce logistics costs.

The other reasons are:

VRP helps businesses attain sustainable growth

It enhances the productivity and efficiency of the fleet

Beyond saving time, it increases customer satisfaction, thereby boosting profit rates

What is the most effective solution to address VRP?

There are three solutions to address VRP, but only one among them is the most effective.

Manual solving:

Manual solving is the most inefficient method to address VRP. Solving VRP is a nerve-racking task and even experienced fleet managers find it tough to solve.

Preset Solvers:

Preset solvers cannot address VRP as it can only solve some basic constraints. It can suit academic settings that require in-depth research and is not ideal for real-world business settings.

Route Optimization Solution:

Route Optimization solution is the most effective tool to tackle VRP as it considers real-world constraints.

What kind of route optimization software should you choose to solve VRP?

Businesses should prefer a route optimization solution that has the following features:

Ensures the safety of deliveries

Has an updated and adaptable set of algorithms

Is economical and scalable

Enables multi-stop route planning and multi-vehicle routing

The next-gen solution

Negligible dependency on human intelligence: Using Advanced algorithms:

Make sure that you buy a route planning software that requires minimum human support. An effective vehicle route optimization solution provides automated route recommendations based on the type of vehicle, vehicle capacity, traffic, stop duration, delivery time, among other constraints. It helps fleet drivers save their fuel and reduce driving time.



Intelligent allocation capabilities:

An accurate parcel sorting capability is essential for fleet route optimization. It helps fleet managers intelligently allocate orders based on their priority.

Automatic rerouting capabilities:

On-demand orders are testing the delivery capabilities of logistics businesses. New orders are added every second, minute, and hour, therefore rerouting is necessary to manage, reschedule and reassign on-demand deliveries.

A capable route planning software dynamically reroutes orders as and when new orders are added. This automatic rerouting ability helps fleet managers in handling their on-demand orders optimally.

Superior Geocoding capabilities:

Confusion in addresses can lead to delayed deliveries. Competent geocoding capability refines and converts inaccurate addresses into precise geographic coordinates. This capability ensures on-time deliveries for fleet drivers.

Conclusions:

Vehicle routing problem forms an integral part of supply chain management, which plays a significant role for productivity improvement. The Vehicle Routing Problem lies at the heart of distribution management. There exist several versions of the problem and a wide variety of exact and approximate algorithms have been proposed for its solution. Exact algorithms can only solve relatively small problems, but a number of approximate algorithms have proved very satisfactory.

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