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Shift Scheduling Problem In A Public Service Sector: A Fuzzy Integer Programming Approach

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Abstract— Many Industries are functionally in shift basis. On each shift the companies are in need of optimum number of employees to operate it effectively. In this article the writer considers the Trichy unit of the Tamil Nadu State Transport Corporation [TNSTC] to consider the employee scheduling process to run the Government Buses in an efficient way. The author is going to study this problem based on fuzzy environment using the modified Zimmermann's Approach.

Keywords— Decision Making, Shift Scheduling Problem, Fuzzy Integer Programming Problem, Modified Zimmermann's Approach.

1. INTRODUCTION

Work shift scheduling problems can be picked up in assigning postal employees of post offices, nurses in hospitals, pilots or crew, staff in the airline industry, or police officers to specific work days and shifts. The scheduling is done when customers demand quick response and total demand can be forecasted with reasonable accuracy. In such cases, capacity is adjusted to fit the expected loads or customer demand on the avail system. Several factors to be considered in work shift scheduling or the agents which result in different work shift scheduling problems are presented at a lower place. [4]

- Demand modeling: determining the number of staff required at each time interval over planning period or rostering horizon. The planning period can be a shift of some number of hours, number of shifts per day or number of shifts per week. Various forecasting tools can be applied to forecast the demand.
- Scheduling workforce in time periods or intervals: Service organizations need to schedule workforce daily and/or weekly with consideration of a number of days off taking by employees, consecutiveness and non-consecutiveness of the number of days away. Similar considerations apply to the number of shifts an employee can do, Minimum break between shifts etc.

- Cyclic or acyclic rosters: In case of repeating demand patterns, all employees of same class perform same line of employment for the same length of duty period but maybe with different starting time of first shift or duty. Such sort of practice can be seen in transport services like bus and railroad tracks. Whereas, in a cyclic rosters, the line of work for employees is independent and possess a different starting time of shift and different duration of breaks. Such rosters can be largely experienced in the service sectors having different demand patterns like call centres and Information Technology enabled services.
- Performance measures: Minimize the number of workers involved, Cost of operating number of employees, Minimize customer response time, target service level (attending some target percentage of claims in specified time)
- Regulations: Total working hours, number of shifts per day or per week, number of hours per shift, leave and other eventualities
- Other constraints: Minimum number of employees desired each period; demand to be played at each time interval, preferences of employees based on qualification and seniority and so forth [4].

I. SHIFT SCHEDULING PROBLEM

A. *Work Shift Applications in Transport related service (Airline/ Railways and Bus)*

Transport related services sector have temporal and spatial feature involved in starting time and location and stopping time and position. There can be many decisions regarding scheduling of resources in the transport sector like giving below [4]:

- Assign buses to depots or lines
- Number of trips per line
- Frequency of buses
- Deadheads in different paths
- Number of drivers required in different shifts
- Rest period of drivers

Similar decisions are required for railways where the trip segments are railway stations.

B. *Work Shift Scheduling with Days-Off Constraints*

Let's suppose a service organization need to schedule employees where the system works for seven days a week and provides each employee two consecutive days off. The service organization can be a retail store, hospital, Airline sector and police services. Such services are available 24 hours of the day and 7 days of week round the year. The service organization incurs cost of operation for hiring so many employees. The organization wants to minimize the total number of employees or to minimize the total slack capacity. At the same time based on the predicted demand each day involves a minimum number of employees. As per regulations, each employee wants two consecutive days off from employment not necessarily weekend. In such scenario the problem of service governance is to schedule workforce [4].

2. REVIEW OF LITERATURE

Mariappan P, Jenifer Christinal A, Maragatham M, Eswari P has discussed about optimizing the workforce Management Schedule by using Integer Programming in 2019. In this report the authors saw a shift scheduling problem in a Multi-Specialty Hospital and Transport Corporation service sectors [4]. An application of Operations Research Model in Hospital Management by Mariappan, et al., investigated the shift scheduling methodology in an effective manner [5]. In 2015, the Shift Scheduling problem for fuzzy environment was studied [6] by Muhittin Sagnak, Yigit Kazancoglu in their research article entitled as Shift Scheduling with Fuzzy Logic: An Application with an Integer Programming Model.

In this case study the scheduling of drivers in Trichy Transport Corporation [TNSTC] has been investigated. The proposed methodology reduces the number of drivers required for each day and also the formulated problem used to evaluate the uncertainty of shift scheduling problems in the transport sector.

3. METHODOLOGY

C. General Model of Integer Programming Problem

Let the decision variables be $x_1, x_2, x_3, \dots x_n$. The objective function of an LPP is a mathematical representation in terms of measurable quantity such as profit, cost, revenue, etc.

Optimize (Minimize or Maximize) $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Subject to

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n (<=>) b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n (<=>) b_2$

…

$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n (<=>) b_1$

$x_1, x_2, x_3 \dots x_n \geq 0$ and all are integers.

Where $c_1, c_2, c_3, \dots, c_n$ are parameters that give contribution to decision variables. The constraints these are the set of linear inequalities which impose restriction of the limited resources [3]

D. Fuzzy Set Theory

Zadeh established Fuzzy set theory in 1965. The major contribution of fuzzy set theory is the ability to describe vague data. According to Zadeh approach "Fuzzy set is a class of targets with a continuum of degrees of membership. Such a set is characterized by a membership function". [8]

E. Fuzzy Integer Programming Problem

In reality, certain coefficients that appear in classical Linear Programming problems may not be well-defined, either because their values depend on other parameters. Fuzzy Linear Programming follows from the fact that classical Linear Programming is often efficient in practical results. Now we consider the Fuzzy Linear Programming Problem [1], [2].

$Max\ Z = \tilde{c}x$ (1)

Subject to

$\tilde{A}x \leq \tilde{b}$

$x \geq 0$

The solution of this problem is to find the possibility distribution of the optional objective function Z.

F. Zimmermann Method

A Linear Programming Problem with a fuzzy function and fuzzy inequalities shown by Zimmermann is indicated as follows: [2], [3]

$c^T x \ \tilde{\geqq}\ b_0$ (2)

$(AX)_i \leq \tilde{b_i}, i = 1,2, \dots, m; x \geq 0$

Inequality is a symmetrical model, of which the objective function becomes one constraint. To compose a general formulation, inequality is converted to a matrix form as:

$-c^T x \ \tilde{\leqq}\ b_0$

In which

$B = \begin{bmatrix} -C \\ (Ax)_i \end{bmatrix}$ $b = \begin{bmatrix} -b_0 \\ bi \end{bmatrix}$ The inequalities of constraint signify "be as small as possible or equal" that can be permitted to violate the right-hand side b by extending more or less value The degree of violation is represented by membership function as:

$$\mu_0(x) = \begin{cases} 0 & ;if \quad Cx \le b_0 - d_0 \\ 1 - \frac{b_0 - Cx}{d_0} & ;if \quad b_0 - d_0 \le Cx \le b_0 \\ 1 & ;if \quad Cx \ge b_0 \end{cases}$$

$$\mu_i(x) = \begin{cases} 0 & ;if \quad (Ax)_i \ge b_i + d_i \\ 1 - \frac{(Ax)_i - b_i}{d_i} & ;if \quad b_i \le (Ax)_i \le b_i + d_i \\ 1 & ;if \quad (Ax)_i \le b_i \end{cases}$$

In which d is a matrix of admissible violation.

Introducing the auxiliary variable λ, the model changes into

$\mu_0(x) \ge \lambda;\ \mu_i(x) \ge \lambda; \lambda \in [0, 1]$

This problem can be stated as Linear Programming as follows:

$Max\ \lambda$

Subject to

$\mu_0(x) \ge \lambda;\ \mu_i(x) \ge \lambda; \lambda \in [0, 1]$. This problem was shown with membership function of fuzzy objective function and fuzzy constraints as follows:

$Max\ \lambda$

Subject to

$$1 - \frac{b_0 - Cx}{d_0} \ge \lambda$$

$$1 - \frac{(Ax)_i - b_i}{d_i} \ge \lambda\ ; \forall i$$

$\lambda \in [0, 1]$

$x \ge 0$

After some simplification, fuzzy linear programming model obtain as follows:

$Max\ \lambda$

Subject to

$Cx - \lambda d_0 \ge b_0 - d_0$

$(Ax)_i + \lambda d_i \le b_i + d_i\ ; \forall i$

$\lambda \in [0,1]$

$x \ge 0$

G. *Modified Zimmermann Method*

The best solution regarding the modified Zimmermann's approach is obtained by changing the intervals in membership functions. The modified membership function as follows: [1], [2]

$$\mu_0(x) = \begin{cases} 0 & ;if \quad Cx \le b_0 - d_0 \\ \frac{Cx - (b_0 - d_0)}{d_0} & ;if \quad b_0 - d_0 \le Cx \le b_0 \\ 1 & ;if \quad Cx \ge b_0 \end{cases}$$

$$\mu_i(x) = \begin{cases} 0 & ;if \quad (Ax)_i \ge b_i \\ 1 - \frac{(Ax)_i - (b_i - d_i)}{d_i} & ;if \quad b_i - d_i \le (Ax)_i \le b_i \\ 1 & ;if \quad (Ax)_i \le b_i - d_i \end{cases}$$

Where, d is a matrix of admissible violation. Using the auxiliary variable λ, the model is changed into

$\mu_0(x) \ge \lambda;\ \ \mu_i(x) \ge \lambda;\ \ \lambda \in [0,1]$

This problem can be stated as Linear Programming as follows:

$Max\ \lambda$

Subject to

$\mu_0(x) \ge \lambda;\ \ \mu_i(x) \ge \lambda;\ \ \lambda \in [0,1]$

This problem was shown with membership function of fuzzy objective function and fuzzy constraints as follows:

Max λ

Subject to

$$\frac{Cx-(b_0-d_0)}{d_0}\geq\lambda$$

$$1-\frac{(Ax)_i-(b_i-d_i)}{d_i}\geq\lambda\ ;\forall i$$

$\lambda\in[0,1]$

$x\geq 0$

The reduced model leads to the required fuzzy LPP:

Max λ (3)

Subject to

$Cx-\lambda d_0\geq b_0-d_0$

$(Ax)_i+\lambda d_i\leq b_i\ ;\forall i$

$\lambda\in[0,1]$

$x\geq 0$

II. CASE STUDY

The primary data was collected from Trichy unit of Tamil Nadu State Transport Corporation. There are 177 drivers are working in the unit, and known that 16 drivers were taking over time duty. They have to work six consecutive days and have one day off. Their six days of work can start on any day of the week and schedule rotates indefinitely.

TABLE I. NUMBER OF DRIVERS WORKING PER DAY

Day	No. of Drivers Required
Sunday	118
Monday	98
Wednesday	102
Thursday	108
Friday	126
Saturday	120

The problem is to find the minimum number of drivers required.

A. *Mathematical Model of Driver Scheduling Problem*

Let x_1 be the number of drivers starting their duty on Sunday to Friday, x_2 be the number of drivers starting their duty on Monday to Saturday, x_3 be the number of drivers starting their duty on Tuesday to Sunday, x_4 be the number of drivers starting their duty on Wednesday to Monday, x_5 be the number of drivers starting their duty on Thursday to Tuesday

Let x_6 be the number of drivers starting their duty on Friday to Wednesday, x_7 be the number of drivers starting their duty on Saturday to Thursday

And $x_i\geq 0$ and are integers, where $i=1,2,3,4,5,6,7$

The work schedule is given in the following table.

TABLE II. WEEKLY WORK SCHEDULE

No of Drivers	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
x_1	⋆	⋆	⋆	⋆	⋆	⋆	-
x_2	-	⋆	⋆	⋆	⋆	⋆	⋆
x_3	⋆	-	⋆	⋆	⋆	⋆	⋆
x_4	⋆	⋆	-	⋆	⋆	⋆	⋆
x_5	⋆	⋆	⋆	-	⋆	⋆	⋆
x_6	⋆	⋆	⋆	⋆	-	⋆	⋆

x_7	⋆	⋆	⋆	⋆	⋆	-	⋆
Requirement b_i	118	122	98	102	108	126	120
Drivers working over time d_i	15	16	10	9	7	12	14

TABLE III. NUMBER OF DRIVERS WORKING FOR EACH SHIFT

Days	Number of Drivers						Total
	Shifts						
	I	OT	II	OT	III	OT	
Sunday	32	3	35	5	51	7	118
Monday	54	9	32	4	36	3	122
Tuesday	26	2	52	5	20	3	98
Wednesday	53	4	27	2	22	3	102
Thursday	50	4	25	1	33	2	108
Friday	39	4	36	2	51	6	126
Saturday	39	5	43	5	38	4	120

Since the requirement of staffs are in uncertain situations, so that we are using the Zimmermann's modified approach to solve the problem. The objective of the problem is to minimize the total number of drivers. The constraints are formulated with the data from the table 2.

B. Drivers Scheduling Problem

$\min \lambda$

Subject to

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 - 16\lambda \geq 161$

$x_1 + x_3 + x_4 + x_5 + x_6 + x_7 - 15\lambda \geq 118$

$x_1 + x_2 + x_4 + x_5 + x_6 + x_7 - 16\lambda \geq 122$

$x_1 + x_2 + x_3 + x_5 + x_6 + x_7 - 10\lambda \geq 98$

$x_1 + x_2 + x_3 + x_4 + x_6 + x_7 - 9\lambda \geq 102$

$x_1 + x_2 + x_3 + x_4 + x_5 + x_7 - 7\lambda \geq 108$

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 - 12\lambda \geq 126$

$x_2 + x_3 + x_4 + x_5 + x_6 + x_7 - 14\lambda \geq 120$

$\forall\, x_i \geq 0$, and are integers, $i = 1,2,3,4,5,6,7$

And each day has three shifts, by applying the modified Zimmermann's fuzzy model, then seven sub problems are formed as follows.

Let y_i be the number of drivers working their duty in i^{th} shift.

C. Subproblem 1 (Sunday)

$\min \lambda$

Subject to

$y_1 + y_2 + y_3 - 15\lambda \geq 103$

$y_1 - 3\lambda \leq 32$

$y_2 - 5\lambda \leq 35$

$y_3 - 7\lambda \leq 51$

$\forall\, y_i \geq 0$ and are integer, $i = 1,2,3$

D. Subproblem 2 (Monday)

$\min \lambda$

Subject to

$y_1 + y_2 + y_3 - 16\lambda \geq 122$

$y_1 - 9\lambda \leq 54$

$y_2 - 4\lambda \leq 32$

$y_3 - 3\lambda \leq 36$

$\forall\, y_i \geq 0$ and are integer, $i = 1,2,3$

E. Subproblem 3 (Tuesday)

$\min \lambda$

Subject to

$y_1 + y_2 + y_3 - 10\lambda \geq 98$

$y_1 - 2\lambda \leq 26$

$y_2 - 5\lambda \leq 52$

$y_3 - 3\lambda \leq 20$

$\forall\, y_i \geq 0$ and are integer, $i = 1,2,3$

F. Subproblem 4 (Wednesday)

$\min \lambda$

Subject to

$y_1 + y_2 + y_3 - 9\lambda \geq 102$

$y_1 - 4\lambda \leq 52$

$y_2 - 2\lambda \leq 27$

$y_3 - 3\lambda \leq 22$

$\forall\, y_i \geq 0$ and are integer, $i = 1,2,3$

G. Subproblem 5 (Thursday)

$\min \lambda$

Subject to

$y_1 + y_2 + y_3 - 7\lambda \geq 101$

$y_1 - 4\lambda \leq 50$

$y_2 - 1\lambda \leq 25$

$y_3 - 2\lambda \leq 33$

$\forall\, y_i \geq 0$ and are integer, $i = 1,2,3$

H. Subproblem 6 (Friday)

$\min \lambda$

Subject to

$y_1 + y_2 + y_3 - 12\lambda \geq 114$

$y_1 - 4\lambda \leq 39$

$y_2 - 2\lambda \leq 36$

$y_3 - 6\lambda \leq 51$

$\forall\, y_i \geq 0$ and are integer, $i = 1,2,3$

I. Subproblem 7 (Saturday)

$\min \lambda$

Subject to

$y_1 + y_2 + y_3 - 14\lambda \geq 106$

$y_1 - 5\lambda \leq 39$

$y_2 - 5\lambda \leq 43$

$y_3 - 4\lambda \leq 38$

$\forall\, y_i \geq 0$ *and are integer,* $i = 1,2,3$

4. RESULT ANALYSIS

The above problems have been solved by using the TORA optimization technique. The minimum requirement of drivers for the Trichy Transport Corporation is 130, which is followed by the solution of the Drivers Scheduling problem. And the solution of all sub problems has been given in table 4.

Days	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Shift I	30	49	25	50	48	39	36
Shift II	32	30	49	26	26	36	40
Shift III	41	27	14	17	17	39	30
Total	103	106	88	93	101	114	106

For the collected data, the above result shows that the required minimum number of drivers needed to start their shift on each day over a week. By using the proposed model the transport corporation can get the optimal allocation of drivers on each day.

5. CONCLUSION

In this study, Modified Zimmermann's approach examined the uncertainty of allocating drivers to work in shifts. Also, the solution obtained by using integer linear programming problem in fuzzy environment reveals that the minimum required drivers needed for each shift. This study can be used to minimize the number of workers as an effective performance measure in any service sectors.

6. REFERENCES

[1]. Maragatham M, Surjith Jiji D, Mariappan P, "A Case Study On Production Industry Using Fuzzy Linear Programming", Our Heritage, ISSN: 0474-9030.Vol 68, Issue 4, Jan 2020.

[2]. Maragatham M, Surjith Jiji D, Mariappan P, "A Fuzzy Integer Programming Approach for Vendor Selection Problem", Our Heritage, ISSN: 0474-9030.Vol 68, Issue 4, Jan 2020.

[3]. Mariappan P, "Operations Research an Introduction", Pearson Publications, 2013.

[4]. Mariappan P, Jenifer Christinal A, Maragatham M, Eswari P, "Optimizing the workforce Management Schedule – An Integer Programming Approach", International Journal on Mathematics and Actuarial Science, Vol 2, ISSN: 978-620-0-31243-3

[5]. Mariappan P, Thakar. M. B, Jenifer Christinal. A, "Application of Operations Research Model in Hospital Management", Journal of Research & Readings in Applied Statistics, ISSN: 2321-0877, 2019.

[6]. Muhittin Sagnak, Yigit Kazancoglu, "Shift Scheduling with Fuzzy Logic: An Application with an Integer Programming Model", Procedia Economics and Finance 827-832, doi: 10.1016/S2212-567(15)00888-6. 2015.

[7]. Rahidas Kumar, Sahedeo Mahto, “Decision Making in Hospital Scheduling through Linear Optimization Approach”, Journal of Computer and Mathematical Science, Vol-9(11), 1609-1615, ISSN: 0976-5427 (Print) , ISSN: 2319-8133 (Online). 2018.
[8]. Zadeh, L. A., Fuzzy Sets. Infomation and Control, 8, 338 – 353. 1965.
[9]. Zimmermman, H. J., Application of fuzzy set theory to mathematical programming. 1985.