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Mathematical Modeling And Simulation On SIR Epidemic Model In Fuzzy Environment – Case Study On Tamil Nadu COVID Outbreak

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Abstract— The purpose of this paper is to investigate the SIR epidemic model with nonlinear incidence rate in fuzzy environment. The methodology proposes to assess the spread of infection through statistics for the state of Tamil Nadu COVID outbreak. Also, the researcher evaluated the stability of the model and stated that the model for disease-free equilibrium is stable.

Keywords— Epidemic model, Incidence rate, Disease Free Equilibrium, Pentagonal Fuzzy Number, Stability Analysis.

1. INTRODUCTION

Epidemic models have started to explore the part of disease infection and treatment function in their dynamic equations [10]. Barros et al, Bassanezi and Barros have suggested a new approach to the treatment of an outbreak model using fuzzy parameters in differential equations describing the dynamical system [4]. The recent fuzzy parameter analysis conducted in [6, 7, 10] identified the interval valued parameter set in a prey-predator model harvested and an epidemic model with fuzzy parameters studied for the computer network. The complex behaviour of an epidemic model with fuzzy transmission has recently been studied by Mondal et al [8] and The Fuzzy epidemic model for influenza virus spread and its potential control was studied by Verma et al. [11].
The major objectives of this paper are as follows:

- Develop the SIR Epidemic model with nonlinear incidence rate with uncertain parameters β,b,μ.
- Examine the proposed model and exhibit the existence and stability analysis. The model was interpreted based on secondary data collected on the official website of the Tamil Nadu Ministry of Health.
- Finally, the spread of infection with fuzzy parameters were analysed by the result, and the paper ends with conclusions and references.

2. METHODOLOGY

A. SIR Epidemic Model

The outbreak and spread of disease are questioned and studied for many years. The ability to make disease predictions will allow scientists to test inoculation, and many of them have a huge impact on the morality rate of a specific epidemic. Infectious disease modeling is a technique that has been used to research the processes by which disease spreads, forecast the likely path of an outbreak, and test infection management strategies.

These three compartments S (for susceptible), I (for infectious) and R are the normal conversion marks (for recovered). This model is called the SIR model.

$$\frac{dS}{dt} = \alpha SI + kN$$

$$\frac{dI}{dt} = \varepsilon \mathrm{I} - \alpha SI$$

$$\frac{dR}{dt} = -\varepsilon I$$

For many infectious diseases like Cancer, Measles, Mumps, Rubella and Cholera the model can be applied. The letters also represent the number of people in each compartment at a particular time. To indicate the number might vary over time (each if the total population size remains constant). We make the precise numbers a function of t (time): S(t),I(t),R(t). These functions can be worked out for a particular disease in a specific population in order to anticipate potential breaks and keep them under control. The incident rate is the number of new cases in a given period of time per population at risk. Different incidence rates have been suggested by several scholars. Nidhi et al [9] have investigated an epidemic model with nonlinear incidence rate. To control the spread of infection, the rate and nature of incidence play an important role.

B. Conditions for Stability

The stability analysis procedure requires the proposed model to meet the following conditions.

i. If the Eigenvalues have all real parts less than zero, then the model is stable.
ii. If the Eigenvalues have all real parts greater than zero, then the model is unstable.
iii. If at least one of the Eigenvalues has real part equal to zero, then no conclusion can be made. This is a borderline case between stability and instability.

C. Pentagonal Fuzzy Number

A pentagonal fuzzy number, which represented with five points as follows,

$A = (a_1, a_2, a_3, a_4, a_5)$, $a_i \in R$, This representation is interpreted as membership function μ_A

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ \frac{x-a_2}{a_3-a_2} & a_2 \le x \le a_3 \\ 1 & x = a_3 \\ \frac{a_4-x}{a_4-a_3} & a_3 \le x \le a_4 \\ \frac{a_5-x}{a_5-a_4} & a_4 \le x \le a_5 \\ 0 & \text{otherwise} \end{cases}$$

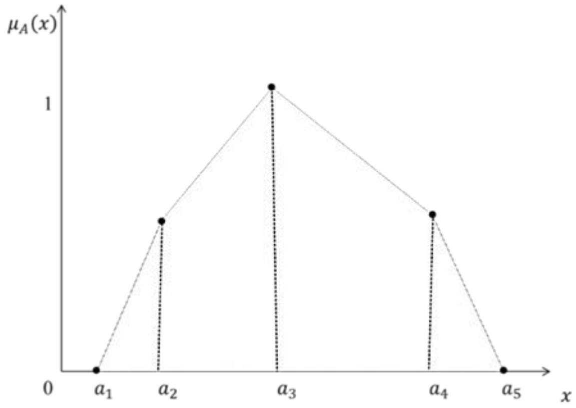

Fig. 1. Pentagonal Fuzzy Number

D. Defuzzification

Since technical processes require clear control actions, a procedure which generates net values from one or several given fuzzy numbers [15].

Graded Mean Integration representation

Chen and Hseih [11, 12, 13] propose graded mean integration representation for representing generalized fuzzy number.

If the generalized fuzzy number $A = (a_1, a_2, a_3, a_4, a_5 : w)$, then the graded mean h-level is $\frac{h[L^{-1}(h)+R^{-1}(h)]}{2}$. Where L^{-1} and R^{-1} are inverse functions of L and R.

And, the defuzzified value of the Fuzzy number A by the graded mean integration representation $\Re(A)$ is defined as [14].

$$\Re(A) = \frac{\int_0^h \left[\frac{L^{-1}(h) + R^{-1}(h)}{2}\right] dh}{\int_0^w h\, dh}$$

Where $h \in (0, w)$, and $0 < w \leq 1$.[14]

If $A = (a_1, a_2, a_3, a_4, a_5)$ is a pentagonal fuzzy number. Chen and Hsieh [11, 12, 13] have already found the general formulae of the representation of generalized pentagonal fuzzy number as follows: [14]

$$\Re(A) = \frac{a_1 + 4a_2 + 2a_3 + 4a_4 + a_5}{6}$$

For this study, the researcher used Graded Mean Integration representation method for defuzzification.

3. PROPOSED MODEL

In the SIR model, it has been described that the dynamics of directly transmitted disease with interaction between susceptible and infected individuals in the absence of vital dynamics (i.e., the rates of birth and mortality are not considered). In this section, we propose a fuzzy SIR model. Following is the system of differential equations describing the proposed model. The uncertain parameters are $\tilde{\beta}, \tilde{b}, \tilde{\mu}$.

The following nonlinear fuzzy system is proposed for the study.

$$\frac{dS}{dt} = A - \tilde{\beta}S - \frac{a\lambda SI}{1 + \alpha I} + \tilde{\mu}R$$

$$\frac{dI}{dt} = \frac{a\lambda SI}{1 + \alpha I} - \left(\tilde{\beta} + \tilde{b}\right)I$$

$$\frac{dR}{dt} = \tilde{b}I - \left(\tilde{\mu} + \tilde{\beta}\right)R$$

where

TABLE I. INDEX OF PARAMETERS

Variables	Description
S(t)	Susceptible rate at time t
I(t)	Infected rate at time t
R(t)	Recovered rate at time t
A	Recruitment rate
$\tilde{\beta}$	Uncertain Natural Death rate
$\tilde{b}$	Uncertain Death rate due to disease
$\tilde{\mu}$	Uncertain Recovery rate
α	Hospital bed density
a	Literacy rate
λ	Proportionality Constant

The above system has a disease-free equilibrium at the point $\left(\frac{A}{\tilde{\beta}}, 0,0\right)$.

By resolving the system, the following values are obtained.

$$\left.\begin{aligned} S &= \frac{(\tilde{\beta}+\tilde{\mu})(1+aI)}{a\lambda} \\ I &= \frac{(\tilde{\beta}+\tilde{\mu})[\tilde{\beta}(\tilde{\beta}+\tilde{b}) - Aa\lambda]}{a\lambda\tilde{\mu} + (\tilde{\beta}+\tilde{\mu})(\tilde{\beta}+\tilde{b})(a\lambda - \alpha\tilde{\beta})} \\ R &= \frac{bI}{\tilde{\mu}+\tilde{\beta}} \end{aligned}\right\} \quad \ldots(1)$$

4. STABILITY ANALYSIS FOR DISEASE FREE EQUILIBRIUM

To test the stability of the disease's free equilibrium, frame the Jacobean matrix.
Let us see

$$J(E_0) = \begin{bmatrix} -\tilde{\beta} & -\frac{a\alpha\lambda}{\tilde{\beta}} & \tilde{\mu} \\ 0 & \frac{a\alpha\lambda}{\tilde{\beta}} - (\tilde{\beta}+\tilde{b}) & 0 \\ 0 & \tilde{b} & -(\tilde{\beta}+\tilde{\mu}) \end{bmatrix}$$

The characteristic equation of the system as follows.

$x^3 + c_1 x^2 + c_2 x + c_3 = 0$

Where, the Eigenvalues written as

$$c_1 = 3\tilde{\beta} + \tilde{b} + \tilde{\mu} - \frac{a\alpha\lambda}{\tilde{\beta}}$$

$$c_2 = \tilde{\beta}(\tilde{\beta}+\tilde{\mu}) - (2\tilde{\beta}+\tilde{\mu})\left(\frac{a\alpha\lambda}{\tilde{\beta}} - (\tilde{\beta}+\tilde{b})\right)$$

$$c_3 = \tilde{\beta}(\tilde{\beta}+\tilde{\mu})\left[\frac{a\alpha\lambda}{\tilde{\beta}} - (\tilde{\beta}+\tilde{b})\right]$$

Using the collected crisp parameters in the Eigenvalues, we fix

$c_1 = -0.37274$

$c_2 = -2.48387 + i1.47103$

$c_2 = -2.48387 - i1.47103$

Here we can state that, all the Eigenvalues have the negative real root. Therefore, the proposed model is stable for the parameters gathered.

CASE STUDY

This paper discusses the issue of calibrating the epidemiological parameters of an SIR model describing the evolution over time of the current COVID-19 pandemic. The collected secondary data was given below,

TABLE II. PARAMETERS COLLECTED FROM TAMIL NADU MINISTRY OF HEALTH WEB

Variables	Description	Values
A	Recruitment rate	2.4×10^{-5}
$\tilde{\beta}$	Natural Death rate	0.72 – 0.81
$\tilde{b}$	Death rate due to disease	0.035 – 0.0434
$\tilde{\mu}$	Recovery rate	0.292 – 0.582
α	Hospital bed density	0.3244
a	Literacy rate	0.803
λ	Proportionality Constant	10

We know that the natural mortality rate and the natural recovery rate changed because of the seasons. Disease-related death is also uncertain. Therefore, the parameters β, b, μ are considered as fuzzy ranges [β= (0.72 – 0.81), b= (0.035 – 0.0434), μ= (0.292 – 0.82)]. Based on these intervals, we can generate the Pentagonal fuzzy numbers.

Thus, the parameters are as follows.

$\tilde{\beta}$=(0.72,0.74,0.7661,0.78,0.81)

$\tilde{b}$=(0.035,0.037,0.0392,0.041,0.0434)

$\tilde{\mu}$=(0.292,0.35,0.437,0.582,0.582)

And all the other parameters are crisp, so that we can proceed and find the values of S, I and R using (1), we obtain.

S=(0.1287,0.13867,0.1472,0.1629,0.17724)

I=(0.06624,0.06659,0.06716,0.06724,0.06912)

R=(0.00229,0.00226,0.002188,0.002154,0.002153)

We need to defuzzify the fuzzy numbers into crisp values for more outcome analysis.

Using the proposed way of defuzzification, we obtain.

S=0.3011,I=0.13416,R=0.0044125

Thus, the rate of the susceptible population is 0.3011. The infected population rate is 0.1341. The rate of recovered population is 0.0044125.

5. CONCLUSION

The SIR epidemic model has been investigated and analyzed for stability in this article. It concludes that it is stable in the model. Also, in the proposed model, the COVID outbreaks of the Tamil Nadu state were interpreted, and the result shows that the spread of infection in the model is comparable to the results. In order to monitor the spread of infection, we should do the next process in the proposed model.

6. REFERENCE

[1]. Alessandro Comunian, Romina Gaburro, Mauro Giudici, "Inversion of a SIR-based model: A critical analysis about the application to COVID-19 epidemic". Physica D, https://doi.org/10.1016/j.physd.2020.132674.

[2]. Barros. L.C., Bassanezi. R.C., Leite, M.B.F.: The SI epidemiological models with a fuzzy transmission parameter. Comput. Math. Appl. 45, 1619–1628 (2003)

[3]. Barros. L.C., Bassanezi. R.C.: About fuzzy dynamical systems: theory and applications, Ph.D. thesis, University of Campinas, Sao Paulo, brazil (1997)

[4]. Barros L.C, Oliveira R.Z.G, Leite M.B.F, and Bassanezi R.C, "Epidemiological Models of Directly Transmitted Diseases: An Approach via Fuzzy Sets Theory". International

Journal of Uncertainty, Fuzziness and Knowledge-Based Systems. Vol. 22, No. 05, pp. 769-781 (2014) https://doi.org/10.1142/S0218488514500408
[5]. Bassanezi. R.C., Barros. L.C.: A simple model of life expectancy with subjective parameters. Kybernets 24, 91–98 (1995)
[6]. Massad. E., Ortega. N.R.S., Barros. L.C., Struchiner, C.J.: Fuzzy Logic in Action: Applications and Epidemiology and Beyond. In: Studied in Fuzziness and Soft Computing, vol. 232 (2008)
[7]. Mishra. B.K., Pandey. S.K.: Fuzzy epidemic model for the transmission of worms in computer network. Nonlinear Anal.: Real World Appl. 11, 4335–4341 (2010)
[8]. Mondal. P.K., Jana. S., Haldar. P., Kar, T.K.: Dynamical behavior of an epidemic model in a fuzzy transmission. Int. J. Uncertainty, Fuzziness Knowl.-Based Syst. 23, 651–665 (2015)
[9]. Nidhi Nirwani, V.H.Badshah, R. Khandelwal, "Stability Analysis of an SIR model with immunity and modified transmission function". International Journal of Applied Mathematical Research, 4(2) (2015) 228-233, DOI: 10.14419/ijamr.v4i2.4257.
[10]. Pal. D., Mahaptra, G.S., Samanta, G.P.: Optimal harvesting of prey-predator system with interval biological parameters: a bioeconomic model. Math. Biosci. 241, 181–187 (2013).
[11]. Renu Verma, S.P. Tiwari, Ranjit Upadhyay, "Dynamical Behaviors of Fuzzy SIR Epidemic Model". Conference paper in Advances in Intelligent Systems and Computing. Sept 2018. DOI: 10.1007/978-3-319-66827-7_45.
[12]. Shan-Huo Chen and Chin Hsun Hseih, (2000), Graded Mean Integration Representation of Generalized Fuzzy Number journal of the chunese Fuzzy System Association, Taiwan, 5(2): 1-7.
[13]. Shan-Huo Chen, and Chih Hsun Hsieh, (2000), Graded Mean Integration Representation of Generalized Fuzzy Number, Proceeding of Conference of Taiwan Fuzzy system Association, Taiwan.
[14]. Shan-Huo chen, and Chin Hsun Hseih,(2000), Representation, Ranking, Distance and Similarity of L-R Type Fuzzy Number and Application, Australia Journal of Intelligent Information Processing Systems, Australia, 6(4): 217 – 229.
[15]. Sk. Khadar Babu, Rajesh Anand. B, Madhusudhan Reddy. K, M.V. Ramanaiah, Karthikeyan.K, "Statistical Optimization for Generalised Fuzzy Number". International Journal of Modern Engineering Research (IJMER), Vol.3, Issue.2, March-April. 2013 pp-647-651 ISSN: 2249-6645.
[16]. Zimmermann H.J, "Fuzzy Set Theory and its Applications". Kluwer Academic Publishers, 2001, ISBN 978-94-010-0646-0.
[17]. Source of secondary data: https://tnhealth.tn.gov.in/