International Journal of Aquatic Science
ISSN: 2008-8019
Vol 12, Issue 02, 2021

Sum Graphs & Its Related Concepts

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Abstract— In Graph Labeling, vast amount of literature is available. The concept of Sum Graphs and Integral Sum Graphs was introduced by F. Harary. In this paper, we introduce Even sum graphs and few graphs on it have been verified.

Keywords— even sum graph, even sum number, path, cycle, star graph,

1. INTRODUCTION

Graph theory has become one of the most rapidly growing areas in Mathematics. There are several areas in Graph theory which have received good attention from mathematics. Labeling of graphs is one of the most interesting problem in the area of Graph theory. Graph Labeling is an assignment of integers to the vertices or edges or both is subject to certain conditions motivated by practical problems .The concept of Sum Graphs & Integral Sum Graphs was introduced by F. Harary [3, 4, 5]. The properties of Sum Graphs are investigated by many authors, including Chen. Z [1], Mary Florida. L [7], Nicholas. T [6], Soma Sundaram. S [6], Vilfred.V [7, 8, 9], Surya Kala.V [9] and Rubin Mary. K [9]. In this paper we introduce a new type of graph called Even sum graph. All graphs used in this paper are simple graphs. For all basic ideas, we refer [2, 3].

2. BASIC DEFINITIONS

Definition 2.1[1]
A graph G is said to be an integral sum graph if the vertices of G can be labeled with distinct integers so that $e = uv$ is an edge of G if and only if the sum of the labels of the vertex u and the vertex v is also a label in G.
Definition 2.2[10]
The minimum number of isolated vertices required to make the graph G, an integral sum graph is called the sum number of G and is denoted by $\xi(G)$.

Definition 2.3[2]

A graph G is said to be a sum graph if the vertices of G can be labeled with distinct positive integers so that $e = uv$ is an edge of G if and only if the sum of the labels of the vertex u and the vertex v is also a label in G.

Definition 2.4[10]

The minimum number of isolated vertices required to make the graph G, a sum graph is called the sum number of G and is denoted by σ(G).

3. MAIN RESULTS

Definition 3.1

A graph G is called an even sum graph if there is a labeling θ of its vertices with distinct even positive integers, so that for any two distinct vertices u and v, uv is an edge of G if and only if θ(u) + θ(v) = θ(w) for some vertex w in G.

Definition 3.2

The minimum number of isolated vertices required to make the graph G, an even sum graph is called the even sum number of G and is denoted by γ(G).

Theorem 3.3

The path P_n is an even sum graph.

Proof:

Let $v_1, v_2, \ldots, v_n$ be the vertices of the path P_n . Then $V(P_n) = \{v_i / 1 \leq i \leq n\}$ is the vertex set of the path P_n and $E(P_n) = \{ v_iv_{i+1} / 1 \leq i \leq n-1\}$ is the edge set of P_n. Define a function $\theta : V(P_n) \rightarrow E^+$ by $\theta(v_1) = v_{n-1} + v_n$, $\theta(v_2) = 0$, $\theta(v_3) = 2$, $\theta(v_4) = 4$ and $\theta(v_i) = \theta(v_{i-2}) + \theta(v_{i-1})$, where $5 \leq i \leq n$. Then the labels are distinct and θ(u) + θ(v) = θ(w) for some vertices u, v, w in P_n. Hence the path P_n is an even sum graph.

Example 3.4

P_5 is an even sum graph.

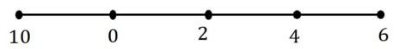

Figure: 3.1

Theorem 3.5

The star graph $K_{1,n}$ is an even sum graph.

Proof:

Let $v_0, v_1, v_2, \ldots, v_n$ be the vertices of $K_{1,n}$. Then $V(G) = \{v_i / 0 \leq i \leq n\}$ be the vertex set of $K_{1,n}$ and $E(G) = \{ v_0v_i / 1 \leq i \leq n \}$ be the edge set of $K_{1,n}$. Define a function $\theta : V(G) \rightarrow E^+$ by $\theta(v_0) = 0$, $\theta(v_i) = 2i$, where $1 \leq i \leq n$. Then the labels are distinct and θ(u) + θ(v) = θ(w) for some vertices u, v, w in $K_{1,n}$.

Example 3.6

$K_{1,6}$ is an even sum graph.

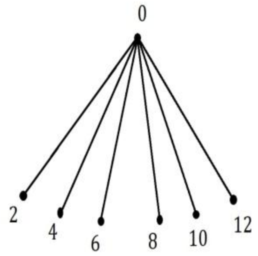

Figure: 3.2

Theorem 3.7

The graph G = $C_n \cup 1K_1$ is an even sum graph.

Proof:

Let $v_1, v_2, \ldots, v_n$ be the vertices of the graph G and E(G) = { v_iv_{i+1} / $1 \leq i \leq n-1$} be the edge set of G. Define a function θ :V(G) → E^+ by $\theta(v_1) = 0$, $\theta(v_2) = 2$, $\theta(v_3) = 4$ and $\theta(v_i) = \theta(v_{i-2}) + \theta(v_{i-1})$, where $i \geq 4$. Then the labels are distinct and θ(u) + θ(v) = θ(w) for some vertices u, v, w in C_n. Hence the graph G is an even sum graph.

Example 3.8

$C_7 \cup 1K_1$ is an even sum graph.

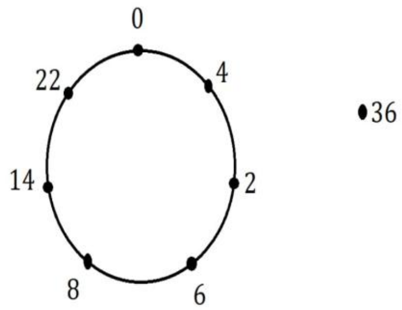

Figure: 3.3

Theorem 3.9

The cycle C_n is not an even sum graph.

Proof:

Let $v_1, v_2, \ldots, v_n$ be the vertices of the cycle C_n and E(C_n) = { v_iv_{i+1} / $1 \leq i \leq n-1$} be the edge set of C_n. If we label the vertices of C_n from E^+, then there exists atleast one edge uv such that the sum of the labeling of the vertices u and v is not a label of a vertex in C_n, since the starting and ending point are same. .Hence the cycle C_n is not an even sum graph.

Example 3.10

C_4 is not an even sum graph .

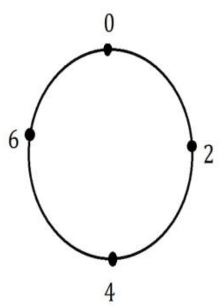

Figure: 3.4

4. CONCLUSION

In this paper, we have explored the concept of even sum graphs and we investigated path, star graph and cycle to be an even sum graph. This paper motivates to derive similar results on other type of graphs to be an even sum graph and to investigate the characterization of even sum graphs.

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