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An Application of Rough Topology in Bacterial Wilt of Vegetable Plants

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Abstract: This paper presents the study of new structure in rough topology. The aim of this study are to examine the rule of potential risk factors for some disease. Using the concept of rough topology, we find the deciding factors for the most common Leaf spot diseases in Plant.

Keywords: Rough sets, Rough topology, Upper Approximation, Lower Approximation, Damping off and Leaf spot diseases.

1. INTRODUCTION

The concept of Rough Sets (RS) was proposed by Zdzisław Pawlak in 1982 [6], but their origins are in his previous work from 1981, when he introduced the rough relations, the classification of objects by attributes and information systems. The main idea behind RS is that an amount of information is associated to each object from the universe of discourse. The theory is based on the concept of information system, which is a tabularized data set. The columns are labeled as “attributes”, while rows are labeled as “objects” or “events”. This theory deals with the approximation of sets or concepts by means of equivalence relations and is considered as one of the first non-statistical approaches in data analysis. Several interesting applications of the theory have come up, in particular, in Artificial Intelligence and Cognitive Sciences. The main difference between rough sets and fuzzy sets is that the rough sets have precise boundaries whereas fuzzy set theory is generally based on ill-defined sets of data, where the bounds are not precise and hence fuzzy predictions tend to deviate from exact values. The lower and upper approximations of a set are analogous to the interior and closure operations in a topology called rough topology in terms of lower and upper approximations of a rough set and we have applied the concept of topological basis to find the deciding factor for Bacterial Wilt of Vegetable Plants.

2. PRELIMINARIES

Definition 2.1 [7]:

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. The pair (U, R) is called the approximation space. Let X be a subset of U.

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i) The lower approximation of X with respect to R is the set of all objects, which can be certain classified as X with respect to R and is denoted by $R_*(X)$.
That is, $R_*(X) = \bigcup_{x\in U}\{R(X) : R(X) \subseteq X\}$
where $R(X)$ denotes the equivalence class determined by X.

ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and is denoted by $R^*(X)$.
That is, $R^*(X) = \underset{x\in U}{U}\{R(X) :$
$R(X) \cap X \neq \phi\}$

iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and is denoted by $B_R X$.
That is, $B_R(X) = R^*(X) - R_*(X)$.
The set X is said to be rough with respect to R if $R^*(X) \neq R_*(X)$. That is, $B_R(X) \neq \phi$.

Preposition 2.2[7]:
If (U, R) is an approximation space and X and Y are subsets of U, then

i) $R_*(X) \subseteq X \subseteq R^*(X)$
ii) $R_*(\phi) = R^*(\phi) = \phi$ and $R_*(U) = R^*(U) = U$
iii) $R*(X \cup Y) = R*(X) \cup R^*(Y)$
iv) $R_*(X \cup Y) \supseteq R_*(X) \cup R_*(Y)$
v) $R_*(X \cap Y) = R_*(X) \cap R_*(Y)$
vi) $R^*(X \cap Y) \subseteq R^*(X) \cap R^*(Y)$
vii) $R_*(X) \subseteq R_*(Y)$ and $R^*(X) \subseteq R^*(Y)$ whenever $X \subseteq Y$
viii) $R_*(X^C) = [R^*(X)]^C$ and $R^*(X^C) = [R_*(X)]^C$
ix) $R_*R_*(X) = R_*R^*(X) = R_*(X)$
x) $R^*R^*(X) = R_*R^*(X) = R^*(X)$

Remark 2.3:
$R^* : P(U) \to P(U)$ satisfies the Kuratowski closure axioms that

i) $R^*R^*(\phi) = \phi$
ii) $X \subseteq R^*(X)$
iii) $R^*(X \bigcup Y) = R^*(X) \bigcup R^*(Y)$
iv) $R^*R^*(X) = R^*(X)$ for all subsets X and Y of U

Remark 2.4:
Since $R^* : P(U) \to P(U)$ satisfies the following properties that

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i) $R^*(U) = U$
ii) $R_*(X) = X$
iii) $R_*(X \cap Y) = R_*(X) \cap R_*(Y)$
iv) $R_* R_*(X) = R_*(X)$ for all subsets X and Y of U ,the operator R_* is the interior operator.

3. ROUGH TOPOLOGY

In, this section we study about new topology called rough topology in terms of lower and upper approximations.

Remark 3.1:[3]

Let U be the universe of objects and R be an equivalence relation on U .

For $X \subseteq U$,we define $\tau_R = \{U, \phi, R_*(X), R^*(X), B_R(X)\}$ where $R^*(X)$, $R_*(X)$ and $B_R(X)$ are respectively the upper approximation, lower approximation and the boundary region of X with respect to R .We note that U and $\phi \in \tau_R$.

Since,

$R_*(X) \subseteq R^*(X)$, $R_*(X) \cup R^*(X) = R^*(X) \in \tau_R$

Also $R^*(X) \cup B_R(X) = R^*(X) \in \tau_R$ and $R_*(X) \cup B_R(X) = R_*(X) \in \tau_R(X)$

Also, $R_*(X) \cap R^*(X) = R_*(X) \in \tau_R; R^*(X) \cap B_R(X) = B_R(X) \in \tau_R$ and $R_*(X) \cap B_R(X) = \phi \in \tau_R$.

Definition 3.2:[3]

Let U be the universe, R be an equivalence relation on U and

$\tau_R = \{U, \phi, R_*(X), R^*(X), B_R(X)\}$

where $X \subseteq U$ satisfies the following axioms:

i) U and $\phi \in \tau_R$
ii) The union of the elements of any sub collection of τ_R is in τ_R .
iii) The intersection of the elements of any finite sub collection of τ_R is in τ_R .

τ_R forms a topology on U called as the rough topology on Uwith respect to X

We call (U, τ_R, X) as the **rough topological space**.

Example 3.3:

Let $U = \{a,b,c,d,e\}$, $U/R = \{\{a,b\},\{c,d\},\{e\}\}$

the family of equivalence class of U by the equivalence relation R and $X = \{a,c,d\}$

Then, by the definition of upper approximation $R^*(X) = \bigcup_{x \in U} \{R(X) : R(X) \cap X \neq \phi\}$

$R^*(X) = \{a,b,c,d\}$

By the definition of lower approximation, $R_*(X) = \bigcup_{x \in U} \{R(X) : R(X) \subseteq X\}$

$R_*(X) = \{c,d\}$

By the definition of boundary region,

$$B_R(X) = R^*(X) - R_*(X)$$
$$= \{a,b,c,d\} - \{c,d\}$$

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$B_R(X) = \{a,b\}$
Therefore, the rough topology $\tau_R = \{U, \phi, \{a,b,c,d\}, \{c,d\}, \{a,b\}\}$.

Preposition 3.4:[3]
If τ_R is the rough topology on U with respect to X, then the set $B = \{U, R_*(X), B_R(X)\}$ is the basis for τ_R.
Definition 3.5:[3]
Let U be the universe and R be an equivalence relation on U .Let τ_R be the rough topology on U and β_R be the basis for τ_R. A subset M of A, the set of attributes is called the **Core** of R if $\beta_M \neq \beta_{R-(r)}$ for every r in M .That is, a core of R is a subset of attributes which is such that none of its elements can be removed without affecting the classification power of attributes.

4. ROUGH TOPOLOGY IN BACTERIAL WILT OF BRINJAL

Plant	Yellowing of foliage in Leaves	Vascular discoloration	Drooping of Leaves	Temperature	Disease
P_1	Yes	Yes	Yes	Very High	Yes
P_2	Yes	No	No	High	No
P_3	Yes	No	Yes	Very High	Yes
P_4	Yes	No	Yes	Very High	No
P_5	No	Yes	Yes	High	Yes
P_6	Yes	Yes	Yes	Very High	Yes
P_7	No	No	Yes	Normal	No
P_8	Yes	No	No	High	No

Here $U = \{P_1, P_2, P_3, ..., P_8\}$ and Yellowing of foliage in Leaves,Vascular discoloration, Drooping of Leaves, Temperature form the condition attributes.
Case 1:
Let $X = \{P_1, P_3, P_4, P_6\}$ the set of plants having plant disease.
$U/I(R) = \{\{P_1, P_6\}, \{P_2, P_8\}, \{P_3, P_4\}, \{P_5\}, \{P_7\}\}$ The lower and upper approximations of X with respect to R are given by
$R_*(X) = \{P_1, P_5, P_6\}$ and $R^*(X) = \{P_1, P_3, P_4, P_5, P_6\}$.
$B_R(X) = \{P_3, P_4\}$
Therefore the rough topology on U with respect to X is given by
$\tau_R = \{U, \phi, \{P_1, P_5, P_6\}, \{P_1, P_3, P_4, P_5, P_6\}, \{P_3, P_4\}\}$ The basis for the topology τ_R is given by

$\beta_R = \{U, \{P_1, P_5, P_6\}, \{P_3, P_4\}\}$.

Removal of Attributes:

1) Yellowing of foliage in Leaves:

If we remove the attribute 'Yellowing of foliage in Leaves' from the set of condition attributes the family of equivalence classes corresponding to the resulting set of attributes is given by

$U/I(R-(Y)) = \{\{P_1, P_6\}, \{P_2, P_8\}, \{P_3, P_4\}, \{P_5\}, \{P_7\}\}$ The lower and upper approximations of X with respect to R are given by

$R_*(X) = \{P_1, P_5, P_6\}$ and $R^*(X) = \{P_1, P_3, P_4, P_5, P_6\}$.

$B_R(X) = \{P_3, P_4\}$

Therefore the rough topology on U with respect to X is given by

$\tau_{R-Y} = \{U, \phi, \{P_1, P_5, P_6\}, \{P_1, P_3, P_4, P_5, P_6\}, \{P_3, P_4\}\}$ The basis for the topology τ_{R-Y} is given by

$\beta_{R-Y} = \{U, \{P_1, P_5, P_6\}, \{P_3, P_4\}\}$

and hence $\tau_{R-Y} = \tau_R$ and its basis $\beta_{R-Y} = \beta_R$ 2. Vascular discoloration:

If we remove the attribute 'Vascular discoloration' from the set of condition attributes,the family of equivalence classes corresponding to the resulting set of attributes is given by

$U/I(R-(V)) = \{\{P_1, P_3, P_4, P_6\}, \{P_2, P_8\}, \{P_5\}, \{P_7\}\}$ The lower and upper approximations of X with respect to R are given by

$R_*(X) = \{P_5\}$ and $R^*(X) = \{P_1, P_3, P_5, P_6\}$.

$B_R(X) = \{P_1, P_3, P_6\}$

Therefore the rough topology on U with respect to X is given by

$\tau_{R-V} = \{U, \phi, \{P_5\}, \{P_1, P_3, P_5, P_6\}, \{P_1, P_3, P_6\}\}$

The basis for the topology τ_{R-V} is given by $\beta_{R-V} = \{U, \{P_5\}, \{P_1, P_3, P_6\}\}$

and hence $\tau_{R-V} \neq \tau_R$ and $\beta_{R-V} \neq \beta_R$.

2) Drooping of Leaves:

If we remove the attribute 'Drooping of Leaves' from the set of condition attributes the family of equivalence classes corresponding to the resulting set of attributes is given by

$U/I(R-(D)) = \{\{P_1, P_6\}, \{P_2, P_8\}, \{P_3, P_4\}, \{P_5\}, \{P_7\}\}$

The lower and upper approximations of X with respect to R are given by

$R_*(X) = \{P_1, P_5, P_6\}$ and $R^*(X) = \{P_1, P_3, P_4, P_5, P_6\}$.

$B_R(X) = \{P_3, P_4\}$

Therefore the rough topology on U with respect to X is given by

$\tau_{R-D} = \{U, \phi, \{P_1, P_5, P_6\}, \{P_1, P_3, P_4, P_5, P_6\}, \{P_3, P_4\}\}$ The basis for the topology τ_{R-Y} is given by

$\beta_{R-D} = \{U, \{P_1, P_5, P_6\}, \{P_3, P_4\}\}$
and hence $\tau_{R-D} = \tau_R$ and its basis $\beta_{R-D} = \beta_R$.

3) Temperature:
When the attribute 'Temperature' is omitted,

$U/I(R-(T)) = \{\{P_1, P_6\}, \{P_2, P_8\}\{P_3, P_4\}, \{P_5\}, \{P_7\}\}$ The lower and upper approximations of X with respect to R are given by
$R_*(X) = \{P_1, P_5, P_6\}$ and $R^*(X) = \{P_1, P_3, P_4, P_5, P_6\}$.
$B_R(X) = \{P_3, P_4\}$
Therefore the rough topology on U with respect to X is given by
$\tau_{R-T} = \{U, \phi, \{P_1, P_5, P_6\}, \{P_1, P_3, P_4, P_5, P_6\}, \{P_3, P_4\}\}$ The basis for the topology τ_{R-T} is given by
$\beta_{R-T} = \{U, \{P_1, P_5, P_6\}, \{P_3, P_4\}\}$
and hence $\tau_{R-T} = \tau_R$ and its basis $\beta_{R-T} = \beta_R$
If $M = \{V\}$
$U/I(R-(M)) = \{\{P_1, P_5, P_6\}, \{P_2, P_4, P_7, P_8\}, \{P_3\}\}$ The lower and upper approximations of X with respect to R are given by
$R_*(X) = \{P_1, P_3, P_5, P_6\}$ and $R^*(X) = \{P_1, P_3, P_5, P_6\}$.
$B_R(X) = \phi$
Therefore the rough topology on U with respect to X is given by
$\tau_M = \{U, \phi, \{P_1, P_3, P_5, P_6\}\}$.
The basis for the topology τ_M is given by
$\beta_M = \{U, \phi, \{P_1, P_3, P_5, P_6\}\}$
and hence $\tau_{R-V} \neq \tau_M$ and its basis $\beta_{R-V} \neq \beta_M$.
Therefore, $CORE(R) = \{V\}$

Case 2:

Let $X = \{P_2, P_4, P_7, P_8\}$, the set of plants not having disease.
Then $U/I(R) = \{\{P_1, P_6\}, \{P_2, P_8\}, \{P_3, P_4\}, \{P_5\}, \{P_7\}\}$
The lower and upper approximations of X with respect to R are given by
$R_*(X) = \{P_2, P_7, P_8\}$
$R^*(X) = \{P_2, P_3, P_4, P_7, P_8\}$
$B_R(X) = \{P_3, P_4\}$
Therefore the rough topology on U with respect to X is given by
$\tau_R = \{U, \phi, \{P_2, P_7, P_8\}, \{P_2, P_3, P_4, P_7, P_8\}, \{P_3, P_4\}\}$
The basis for the topology τ_R is given by
$\beta_R = \{U, \{P_1, P_5, P_6\}, \{P_3, P_4\}\}$.

1) Yellowing of foliage in Leaves:

If we remove the attribute 'Yellowing of foliage in Leaves' we get $U/I(R-(Y))=\{\{P_1,P_6\},\{P_2,P_8\},\{P_3,P_4\},\{P_5\},\{P_7\}\}$ The lower and upper approximations of X with respect to R are given by

$R_*(X)=\{P_2,P_7,P_8\}$

$R^*(X)=\{P_2,P_3,P_4,P_7,P_8\}$

$B_R(X)=\{P_3,P_4\}$

Therefore the rough topology on U with respect to X is given by

$\tau_{R-Y}=\{U,\phi,\{P_2,P_7,P_8\},\{P_2,P_3,P_4,P_7,P_8\},\{P_3,P_4\}\}$ The basis for the topology τ_{R-Y} is given by

$\beta_{R-Y}=\{U,\{P_1,P_5,P_6\},\{P_3,P_4\}\}$.

and hence $\tau_{R-Y}=\tau_R$ and its basis $\beta_{R-Y}=\beta_R$.

2) Vascular discoloration

On the removal of attribute 'Vascular discoloration' we get

$U/I(R-(V))=\{\{P_1,P_3,P_4,P_6\},\{P_2,P_8\},\{P_5\},\{P_7\}\}$ The lower and upper approximations of X with respect to R are given by

$R_*(X)=\{P_2,P_7,P_8\}$

$R^*(X)=\{P_1,P_2,P_3,P_4,P_6,P_7,P_8\}$

$B_R(X)=\{P_1,P_3,P_4,P_6\}$

Therefore the rough topology on U with respect to X is given by

$\tau_{R-V}=\{U,\phi,\{P_2,P_7,P_8\},\{P_1,P_2,P_3,P_4,P_6,P_7,P_8\},$
$\{P_1,P_3,P_4,P_6\}\}\neq\tau_R$ The basis for the topology τ_{R-V} is given by

$\beta_{R-V}=\{U,\{P_2,P_7,P_8\},\{P_1,P_3,P_4,P_6\}\}\neq\beta_R$

3) Drooping of Leaves:

On the removal of attribute 'Drooping of Leaves' we get

$U/I(R-(D))=\{\{P_1,P_6\},\{P_2,P_8\},\{P_3,P_4\},$ $\{P_5\},\{P_7\}\}$ The lower and upper approximations of X with respect to R are given by

$R_*(X)=\{P_2,P_7,P_8\}$

$R^*(X)=\{P_2,P_3,P_4,P_7,P_8\}$

$B_R(X)=\{P_3,P_4\}$

Therefore the rough topology on U with respect to X is given by

$\tau_{R-D}=\{U,\phi,\{P_2,P_7,P_8\},\{P_2,P_3,P_4,P_7,P_8\},$ $\{P_3,P_4\}\}$

The basis for the topology τ_{R-D} is given by

$\beta_{R-D}=\{U,\{P_1,P_5,P_6\},\{P_3,P_4\}\}$.

and hence $\tau_{R-D} = \tau_R$ and its basis $\beta_{R-D} = \beta_R$.

4) Temperature:

On the removal of attribute 'Temperature' we get

$U/I(R-(T)) = \{\{P_1, P_6\}, \{P_2, P_8\}\{P_3, P_4\}, \{P_5\}, \{P_7\}\}$

The lower and upper approximations of X with respect to R are given by

$R_*(X) = \{P_2, P_7, P_8\}$

$R^*(X) = \{P_2, P_3, P_4, P_7, P_8\}$

$B_R(X) = \{P_3, P_4\}$

Therefore the rough topology on U with respect to X is given by

$\tau_{R-T} = \{U, \phi, \{P_2, P_7, P_8\}, \{P_2, P_3, P_4, P_7, P_8\}, \{P_3, P_4\}\}$ The basis for the topology τ_{R-T} is given by

$\beta_{R-T} = \{U, \{P_1, P_5, P_6\}, \{P_3, P_4\}\}$.

and hence $\tau_{R-T} = \tau_R$ and its basis $\beta_{R-T} = \beta_R$.

If $M = \{V\}$

$U/I(R-(M)) = \{\{P_1, P_5, P_6\}, \{P_2, P_4, P_7, P_8\}, \{P_3\}\}$ The lower and upper approximations of X with respect to R are given by

$R_*(X) = \{P_2, P_4, P_7, P_8\}$ and $R^*(X) = \{P_2, P_4, P_7, P_8\}$.

$B_R(X) = \phi$

Therefore the rough topology on U with respect to X is given by

$\tau_M = \{U, \phi, \{P_2, P_4, P_7, P_8\}\}$.

The basis for the topology τ_M is given by

$\beta_M = \{U, \phi, \{P_2, P_4, P_7, P_8\}\}$

and hence $\tau_{R-V} \neq \tau_M$ and its basis $\beta_{R-V} \neq \beta_M$.

Therefore, $CORE(R) = \{V\}$

Therefore, $CORE(R) = \{V\}$

Observation:

From both cases we could conclude that 'Vascular discoloration' is the key attribute that has close connection to the plant disease.

Algorithm:

Step 1: Given a finite universe U, a finite set A of attributes that is divided into two classes, C of condition attributes and D of decision attributes, an equivalence relation R on U corresponding to C and a subset X of U, represent the data as an information table, columns of which are labelled by attributes, rows by objects and entries of the table are attribute values.

Step 2: Find the lower approximation, upper approximation and the boundary region of X with respect to R.

Step 3: Generate the rough topology τ_R on U and its basis β_R.

Step 4: Remove an attribute X from C and find the lower and upper Approximation and the boundary region of X with respect to the equivalence

On $C-(X)$.

Step 5: Generate the rough topology $\tau_{R-(X)}$ on U and its basis $\beta_{R-(X)}$.

Step 6: Repeat steps 3 and 4 for all attributes in C.

Step 7: Those attributes in for which form the

4. CONCLUSION:

In this work, we have shown that real world problems can be dealt with the rough topology. The concept of basis has been applied to find the deciding factors of '**Bacterial Wilt**' which had been reported in worldwide. We could find that 'Vascular discoloration' is the deciding factor for '**Bacterial Wilt**' .It is also seen that from a clinical point of view, the rough topological model in pair with the medical experts with respect to the diseases analysed here. The proposed rough topology can be applied to more general and complex information systems for future research .The rough set model is based on the original data only and does not need any external information, unlike probability in statistics or grade of membership in the fuzzy set theory, it is also tool suitable for analysing not only quantitative attributes but also qualitative ones. The result of the rough set model are easy to understand, while the results from other models need an interpretation of the technical parameters. Thus it is advantageous to use rough topology in real life situations.

5. REFERENCES

[1] Jansi Rani, P.G., and Bhaskaran, R., Computation of reducts using topologyand measure of significance of attributes, Journal of Computing 2 (2010)50-55.

[2] Lashin, E.F., and Medhat, T., Topological reduction of information systems,Chaos, Solitons and Fractals 25 (2005) 277-286.

[3] Lellis Thivagar, M., Carmel Richard and Nirmala Rebecca Paul, Mathematical Innovations of a Modern Topology in Medical Events,International Journal of Information Science 2012, 2(4):33-36.

[4] Rady, E.A., Kozae, A.M. and Abd El-Monsef, M.M.E., Generalized rough sets, Chaos, Solitons and Fractals 21 (2004) 49-53.

[5] Salama,A.S., Some Topological properties of rough sets with tools for datamining, IJCSI International Journal of Computer Science issues 8(2011)588-595.

[6] Yuhuaqian, Chuangy in Dang, Jiye Liang, and Dawei Tang, Set-valued information systems, Information Sciences 179(2009) 2809-2832.

[7] Zdzislaw Pawlak, Rough set theory and its applications, Journal of Telecommunications and Information Technology 3(2002)7-10.

[8] Zuqiang Meng, and Zhongzhi Shi, A fast approach to attribute reduction in incomplete decision systems with tolerance relation – based rough sets, information 179 (2009) 2774-2793.